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HÖLDER CONTINUITY FOR CONTINUOUS SOLUTIONS OF THE SINGULAR MINIMAL SURFACE EQUATION WITH ARBITRARY ZERO SET

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ABSTRACT. In this paper we prove the following theorem: Let $\Omega \subset \mathbb{R}^n$ be a bounded open set, $\psi \in C_c^2(\mathbb{R}^n)$, $\psi > 0$ on $\partial\Omega$, be given boundary values and u a nonnegative solution to the problem

$$u \in C^0(\overline{\Omega}) \cap C^2(\{u > 0\})$$
$$u = \psi \text{ on } \partial\Omega$$
$$\operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = \frac{\alpha}{u\sqrt{1+|Du|^2}} \text{ in } \{u > 0\}$$

where $\alpha > 0$ is a given constant. Then $u \in C^{0,\frac{1}{2}}(\overline{\Omega})$.

Furthermore we prove strict mean convexity of the free boundary $\partial\{u = 0\}$ provided $\partial\{u = 0\}$ is assumed to be of class C^2.

1. Introduction

Consider the problem of minimizing the energy

$$\mathcal{F}(u) := \int_\Omega u^\alpha \sqrt{1+|Du|^2}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open subset of $\mathbb{R}^n$ with Lipschitz boundary and $\alpha > 0$. When $\alpha \in \mathbb{N}$, $\mathcal{F}(u)$ coincides with, up to a constant factor, the area of the rotated graph

$$\mathcal{M}_{rot} := \{(x, u(x)\omega) : x \in \Omega,\ \omega \in S^\alpha \subset \mathbb{R}^{\alpha+1}\} \subset \mathbb{R}^{n+\alpha+1}.$$

$\mathcal{F}(u)$ may be defined for $u \in BV_+^{1+\alpha}(\Omega)$, where

$$BV_+^{1+\alpha}(\Omega) := \left\{u \in L^\alpha(\Omega) : u \geq 0,\ u^{1+\alpha} \in BV(\Omega)\right\}.$$

It was shown by U. Dierkes and J. Bemelmans [1], that, given $\psi \in L^{1+\alpha}(\partial\Omega)$, solutions to

$$\mathcal{F}^*(u) = \mathcal{F}(u) + \frac{1}{1+\alpha}\int_{\partial\Omega} |u^{1+\alpha} - \psi^{1+\alpha}|\, d\mathcal{H}^{n-1} \to \min \text{ in the class } BV_+^{1+\alpha}(\Omega)$$

exist and fulfill the weak maximum principle

$$||u||_{\infty,\Omega} \leq ||\psi||_{\infty,\partial\Omega}.$$

Explicit examples of such minimizers were constructed by U. Dierkes [2][3]. In particular he showed the existence of Hölder continuous local minimizers u of $\mathcal{F}$ with Hölder exponent $\frac{1}{2}$ that fail to be in the class $C^{0,\frac{1}{2}+\epsilon}$ for any $\epsilon > 0$. These examples led to the formulation of the conjecture that all minimizers must be $\frac{1}{2}$-Hölder continuous functions [5], which in turn inspired the present paper.

2. Statement of theorems

In this section Ω will be a bounded open subset of $\mathbb{R}^n$ and $\alpha > 0$ a positive constant. Our goal is to prove the following main result:

Theorem 1. *Let* $\psi \in C_c^2(\mathbb{R}^n)$, $\psi > 0$ *on* $\partial\Omega$, *be given boundary values and* $u \in C^0(\overline{\Omega}) \cap C^2(\{u > 0\})$, $u \geq 0$, *fulfill the relations*

$$u \in C^0(\overline{\Omega}) \cap C^2(\{u > 0\})$$
$$u = \psi \text{ on } \partial\Omega$$
$$\operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = \frac{\alpha}{u\sqrt{1+|Du|^2}} \text{ in } \{u > 0\}.$$

Then $u \in C^{0,\frac{1}{2}}(\overline{\Omega})$.

Remarks.

(i) Note that here we do not require any regularity of the boundary $\partial\Omega$

(ii) Theorem 1 applies to local minimizers u of $\mathcal{F}$ that are continuous in $\overline{\Omega}$. The minimizing property yields $u \in C^\omega(\{u > 0\})$, while the equation

$$\operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = \frac{\alpha}{u\sqrt{1+|Du|^2}} \tag{1}$$

is equivalent to the Euler equation for $\mathcal{F}$ wherever $u > 0$. See the author's paper [10] for a proof.

(iii) If $n \leq 6$ local minimizers u are continuous in the interior of Ω [1]. It is still unknown whether interior continuity holds in general.

(iv) Continuity at the boundary for minimizers may be achieved by requiring that Ω has nonnegative inward mean curvature in the sense of Caccioppoli sets, i.e. for every $\xi \in \partial\Omega$ there exists a neighborhood U_ξ such that

$$\int_{U_\xi} |D\chi_\Omega| \leq \int_{U_\xi} |D\chi_{\Omega\cup E}|$$

for every set E of finite perimeter and $E\Delta\Omega \subset\subset U_\xi$. This was shown by Dierkes [4].

In view of these remarks we have the following

Theorem 2. *Let* $n \leq 6$, Ω *be a mean convex, bounded open set with Lipschitz boundary,* $\psi \in C_c^2(\mathbb{R}^n)$, $\psi > 0$ *on* $\partial\Omega$, *be given boundary values and* $u \in BV_+^{1+\alpha}(\Omega)$ *a minimizer of* $\mathcal{F}^*$ *in the class* $BV_+^{1+\alpha}(\Omega)$. *Then* $u \in C^{0,\frac{1}{2}}(\overline{\Omega})$.

In [10] the author proved the mean convexity of the zero set $\{u = 0\}$ of local minimizers of $\mathcal{F}$. We will here show a connection between the *strict* mean convexity of $\{u = 0\}$ and Hölder continuity of u.

Theorem 3. *Let Ω be a bounded open set with Lipschitz boundary, $u \in C^0(\overline{\Omega}) \cap BV_+^{1+\alpha}(\Omega)$ be a minimizer of $\mathcal{F}^*$ with boundary values $\psi \in C_c^2(\mathbb{R}^n)$, $\psi > 0$ on $\partial\Omega$. Additionally assume that $\partial\{u = 0\} \in C^2$. Then there exists a constant $c > 0$ so that*

$$\inf_{\partial\{u=0\}} H \geq c$$

where H denotes the inward mean curvature of $\partial\{u = 0\}$

Remark. In particular, Theorem 3 says that the zero set of minimizers of $\mathcal{F}^*$ is strictly mean convex in the classical sense, provided the classical mean curvature exists.

3. Proof of Theorem 1

Theorem 1 is a consequence of the following proposition.

Proposition 1. *Let $\psi \in C_c^2(\mathbb{R}^n)$, $\psi > 0$ on $\partial\Omega$, be given boundary values and $u \in C^0(\overline{\Omega}) \cap C^2(\{u > 0\})$, $u \geq 0$, solve*

$$u \in C^0(\overline{\Omega}) \cap C^2(\{u > 0\})$$
$$u = \psi \text{ on } \partial\Omega$$
$$\operatorname{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = \frac{\alpha}{u\sqrt{1+|Du|^2}} \text{ in } \{u > 0\}.$$

Then $u^2 \in C^{0,1}(\{u \leq \delta\})$ for all $0 < \delta < \inf_{\partial\Omega} \psi$ and in addition $u \in C^{0,\frac{1}{2}}(\overline{\Omega})$.

The proof employs a method originally due to N. Korevaar and L. Simon [8].

Proof. We work on the of graph of $u \llcorner \{u > 0\}$ using the coordinates $x = \operatorname{proj}(x, u(x))$. ∇ and Δ denote the tangential gradient and Laplace operators also on graph u respectively. Note that in $\{u > 0\}$, $u \in C^\infty$ by Schauder theory. For any function $f \in C^2(\{u > 0\})$ we have

$$\nabla f = (Df, 0) - D_i f \nu^i \nu = \left(Df - \frac{Df \cdot Du}{1+|Du|^2} Du, \frac{Df \cdot Du}{1+|Du|^2}\right) \tag{2}$$

and therefore

$$|\nabla f|^2 = |Df|^2 - (D_i f \nu^i)^2 \tag{3}$$

as well as

$$\Delta f = g^{ij} D_i D_j f + H \nu^i D_i f \tag{4}$$

where we used the symbol $\nu = \frac{(-Du,1)}{\sqrt{1+|Du|^2}}$ to denote the upward unit normal of graph(u), $g^{ij} = \delta^{ij} - \nu^i \nu^j$ stands for the inverse of the first fundamental form and $H(x) = -\nabla_i \nu^i = \frac{\alpha}{u\sqrt{1+|Du|^2}}$ denotes the mean curvature of the graph of u. Because of (4) we have

$$\Delta u = H \nu^{n+1}. \tag{5}$$

Additionally we have the Jacobi equation (cmp. [6] chapter 3.4, proposition 2):

$$\Delta\nu^{n+1} = -\nu^{n+1}|A|^2 - e_{n+1}\cdot\nabla H, \tag{6}$$

where $|A| = \sqrt{\sum_{i=1}^{n+1}|\nabla\nu^i|^2}$ indicates the norm of the second fundamental form of graph(u). Obviously,

$$\nabla u = \left(\frac{Du}{1+|Du|^2}, \frac{|Du|^2}{1+|Du|^2}\right),$$

so that also

$$|\nabla u|^2 = \frac{|Du|^2}{1+|Du|^2} = e_{n+1}\cdot\nabla u. \tag{7}$$

Let now $\delta > 0$ be such that $\{u < \delta\} \subset\subset \Omega$. Since $u \in C^0(\overline{\Omega})$ such a δ exists and will in general depend on the solution u. δ will be fixed throughout the proof. Further let $\phi \in C_c^2(\mathbb{R}^n)$ be such that $\phi = 0$ in $\{u < \delta\}$, $\phi = \psi$ in a neighborhood of $\partial\Omega$ and $||\phi||_{C^2(\mathbb{R}^n)} \le \gamma = \gamma(\delta, \psi) < \infty$. Now we define the auxiliary function $\eta\colon \mathbb{R}_+ \to \mathbb{R}_+$,

$$\eta(t) := (e^{Kt} - 1)e^{-2\gamma K}$$

with a constant $K > 0$ to be chosen later. If we denote with $(u-\phi)^+$ the maximum of $u-\phi$ and 0, then we get $0 \le \eta((u-\phi)^+) \le 1$, as by the weak maximum principle $u \le \gamma$. Let $\epsilon > 0$ and M be the maximum of

$$f(x) := \frac{\eta((u-\phi)^+)}{\nu^{n+1}+\epsilon}$$

on $\overline{\{u > 0\}}$. Clearly, f is continuous on $\overline{\{u > 0\}}$, nonnegative, $f = 0$ on $\partial\{u > 0\}$ and positive in $\{0 < u < \delta\}$, so that f must achieve its maximum M in a point $x_0 \in \{u > 0\}$. Define the function

$$\Psi(x) := \eta((u-\phi)^+) - M(\nu^{n+1}+\epsilon) \le 0,$$

which, at x_0, fulfills the relations

$$\Psi(x_0) = 0,\ \nabla\Psi(x_0) = 0,\ \text{and } \Delta\Psi(x_0) \le 0. \tag{8}$$

We calculate

$$\nabla\Psi = \eta'\nabla(u-\phi) - M\nabla\nu^{n+1} = 0 \tag{9}$$

in x_0 (Obviously, in x_0, $u > \phi$). Since u solves the differential equation

$$H = \frac{\alpha\nu^{n+1}}{u} \tag{10}$$

in $\{u > 0\}$, there we also have $\nu^{n+1} = \frac{uH}{\alpha}$ and thus

$$\nabla\nu^{n+1} = \frac{1}{\alpha}(\nabla uH + u\nabla H).$$

Inserting this into (9) we get

$$\eta'\nabla(u-\phi) - \frac{M}{\alpha}(\nabla uH + u\nabla H) = 0,$$

or after rearrangement

$$\nabla H = \frac{1}{Mu}\left(\alpha\eta'\nabla(u-\phi) - MH\nabla u\right). \tag{11}$$

Now we calculate

$$\begin{aligned}
\Delta\Psi &= \eta''|\nabla(u-\phi)|^2 + \eta'\Delta(u-\phi) - M\Delta\nu^{n+1} \\
&\quad \text{using (5) and (6):} \\
&= \eta''|\nabla(u-\phi)|^2 + \eta' H\nu^{n+1} - \eta'\Delta\phi + M(\nu^{n+1}|A|^2 + e_{n+1}\cdot\nabla H) \\
&\quad \text{using (10) and (11):} \\
&= \eta''|\nabla(u-\phi)|^2 + \alpha\eta'\frac{(\nu^{n+1})^2}{u} - \eta'\Delta\phi + \frac{e_{n+1}}{u}\cdot(\alpha\eta'\nabla(u-\phi) - MH\nabla u) \\
&\quad + M\nu^{n+1}|A|^2 \\
&\quad \text{We make use of (7) and (10):} \\
&= \eta''|\nabla(u-\phi)|^2 + \alpha\eta'\frac{(\nu^{n+1})^2}{u} - \eta'\Delta\phi + \alpha\eta'\frac{|\nabla u|^2}{u} - \alpha\eta'\frac{e_{n+1}\cdot\nabla\phi}{u} \\
&\quad - \alpha\frac{M|\nabla u|^2}{u^2}\nu^{n+1} + M\nabla^{n+1}|A|^2 \\
&\quad \text{Recall that } (\nu^{n+1})^2 + |\nabla u|^2 = 1. \\
&= \eta''|\nabla(u-\phi)|^2 + \alpha\frac{\eta'}{u} - \eta'\Delta\phi - \alpha\eta'\frac{e_{n+1}\cdot\nabla\phi}{u} - \alpha\frac{M\nu^{n+1}}{u^2}|\nabla u|^2 \\
&\quad + M\nu^{n+1}|A|^2 \\
&\le 0.
\end{aligned}$$

We have $\Psi(x_0) = 0$, so that in x_0

$$\nu^{n+1} = \frac{\eta - M\epsilon}{M} \le \frac{1}{M} - \epsilon \le \frac{1}{M}$$

or equivalently $\sqrt{1+|Du|^2} \ge M$. In addition we get using (3)

$$\begin{aligned}
&|\nabla(u-\phi)|^2 \\
&= |D(u-\phi)|^2 - \frac{|Du\cdot D(u-\phi)|^2}{1+|Du|^2} \\
&= |Du|^2 - 2Du\cdot D\phi + |D\phi|^2 - \frac{|Du|^4 - 2|Du|^2 Du\cdot D\phi + (Du\cdot D\phi)^2}{1+|Du|^2} \\
&= \frac{|Du|^2 - 2Du\cdot D\phi + |D\phi^2| + |Du|^2|D\phi|^2 - (Du\cdot D\phi)^2}{1+|Du|^2} \\
&\ge \frac{|Du|^2 - 2\gamma|Du|}{1+|Du|^2} \\
&\to 1, \text{ as } |Du| \to \infty.
\end{aligned}$$

Whence there exists a constant M_0, depending only on γ, so that in case $M > M_0$ also $|\nabla(u-\phi)|^2 > \frac{1}{2}$. Assume for a moment that $M > M_0$. Then we may estimate further

$$\eta'' + 2\alpha\frac{\eta'}{u} - 2\eta'(\Delta\phi + \alpha\frac{e_{n+1}\nabla\phi}{u}) - 2\alpha\frac{\eta}{u^2} \le 0,$$

since in x_0:

$$-M\nu^{n+1} = -\eta + M\epsilon \geq -\eta.$$

Let us further assume that also $x_0 \in \{u < \delta\}$. There we have $\phi \equiv 0$, which means

$$u^2\eta'' + 2\alpha u\eta' - 2\alpha\eta \leq 0$$

or rather

$$(u^2K^2 + 2\alpha uK - 2\alpha)e^{Ku} + 2\alpha \leq 0.$$

$s := Ku$ thus fulfills the inequality

$$(2\alpha - 2\alpha s - s^2)e^s \geq 2\alpha.$$

However, this can only be the case if $s = 0$. As s is strictly positive, we obtain a contradiction and therefore $x_0 \in \{u \geq \delta\}$ or $M \leq M_0$ must be true. Let us now continue to assume that $M > M_0$ and therefore $x_0 \in \{u \geq \delta\}$. Because of (4), (2), (10) and $u(x_0) \geq \delta$ we get

$$\left|\Delta\phi + \alpha\frac{e_{n+1}\nabla\phi}{u}\right| \leq C = C(\gamma, \delta).$$

This yields

$$\eta'' - 2C\eta' - 2\alpha\frac{\eta}{u^2} \leq 0,$$

which implies

$$\left(K^2 - 2CK - \frac{2\alpha}{\delta^2}\right)e^{K(u-\phi)^+} \leq \frac{-2\alpha}{\gamma^2}.$$

By choosing K large so that $K^2 - 2CK - 2\frac{\alpha}{\delta^2} > 0$ we obtain

$$0 < -\frac{2\alpha}{\gamma^2},$$

an obvious contradiction. We conclude that $M \leq M_0$ must hold and hence

$$\frac{\eta((u-\phi)^+)}{\nu^{n+1}+\epsilon} \leq M_0. \tag{12}$$

By applying the above procedure to the function

$$g(x) := \frac{\eta((\phi-u)^+)}{\nu^{n+1}+\epsilon}$$

in place of f, we get the estimate (12) also for $(\phi - u)^+$. Note that here it is immediately clear that the maximum M of g must be attained in $\{u > \delta\}$, since g vanishes in $\{0 < u < \delta\}$. Again we define

$$\Phi(x) := \eta((\phi - u)^+) - M(\nu^{n+1} + \epsilon) \leq 0.$$

When calculating $\Delta\Phi$ one easily recognizes that $\Delta\Phi$ and $\Delta\Psi$ differ only on the sign of the term $\frac{\alpha}{u}\eta'$, which in turn is bounded by $\frac{\alpha}{\delta}\eta'$. The remaining calculations are identical to the above, so we refrain from repeating the argument. Concluding we obtain

$$\frac{\eta(|u-\phi|)}{\nu^{n+1}+\epsilon} \leq M_0$$

for all $\epsilon > 0$. Letting $\epsilon \to 0$ it follows

$$\eta(|u-\phi|)\sqrt{1+|Du|^2} \leq M_0.$$

Consequently

$$|u-\phi||D(u-\phi)| \leq \frac{1}{K}e^{2\gamma K}\eta(|u-\phi|)(\sqrt{1+|Du|^2}+|D\phi|) \leq \frac{1}{K}e^{2\gamma K}(M_0+\gamma).$$

The function $|u-\phi|$ may be extended continuously by 0 outside of $\{u > 0\}$ and it follows that $(u-\phi)^2 \in C^{0,1}(\overline{\Omega})$, which clearly implies $(u-\phi) \in C^{0,\frac{1}{2}}(\overline{\Omega})$. Since $\phi \in C^2$ we thus conclude that $u \in C^{0,\frac{1}{2}}(\overline{\Omega})$. Finally $u^2 \in C^{0,1}(\{u \leq \delta\})$ follows since $\phi \equiv 0$ on this set. □

4. Proof of Theorem 3

Proposition 2. *Let u be a minimizer of $\mathcal{F}^*$ in Ω, $\partial\{u > 0\} \in C^2$ and $\{u = 0\} \subset\subset \Omega$. Then the inward mean curvature $H \in C^0(\partial\{u = 0\})$ fulfills the following inequality:*

$$\inf_{\{u>0\}} \frac{\alpha}{u\sqrt{1+|Du|^2}} \leq \inf_{\partial\{u>0\}} H$$

Proof. Let $\epsilon > 0$, $\phi \in C^1_c(\Omega)$ be nonnegative and ν denote a continuously differentiable extension of the outward unit normal ν of $\partial\{u = 0\}$. Then

$$\Phi_\epsilon \colon \Omega \to \Omega, \quad \Phi_\epsilon \colon x \mapsto x + \epsilon\phi(x)\nu(x)$$

is a variation of $\{u > 0\}$ into its interior. From the known formula for the first variation of perimeter (see [7] Theorem 10.4) and the mean convexity of the zero set $\{u = 0\}$ (see [10] Theorem 2) it follows that

$$\begin{aligned}&\int_{\partial\{u>0\}} H\phi \, d\mathcal{H}^{n-1}\\ &= \lim_{\epsilon\to 0}\frac{1}{\epsilon}\left(\int |D\chi_{\Phi_\epsilon(\{u>0\})}| - \int |D\chi_{\{u>0\}}|\right)\\ &\geq \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_{\{u>0\}\setminus\Phi_\epsilon(\{u>0\})} \frac{\alpha}{u\sqrt{1+|Du|^2}}\,dx\end{aligned}$$

Upon setting $f := \frac{\alpha}{u\sqrt{1+|Du|^2}}$ this implies

$$\begin{aligned}
&\int_{\partial\{u>0\}} H\phi\, d\mathcal{H}^{n-1} \\
&\geq \lim_{\epsilon\to 0} \frac{1}{\epsilon} \int_{\{u>0\}\setminus\Phi_\epsilon(\{u>0\})} ||f||_{-\infty,\{u>0\}}\, dx \\
&= \lim_{\epsilon\to 0} \frac{||f||_{-\infty,\{u>0\}}}{\epsilon} \left(|\{u>0\}| - \int_{\{u>0\}} |\det D\Phi_\epsilon|\, dx \right) \\
&= \lim_{\epsilon\to 0} \frac{||f||_{-\infty,\{u>0\}}}{\epsilon} \left(\int_{\{u>0\}} 1\, dx - \int_{\{u>0\}} 1 + \epsilon \operatorname{div}(\phi\nu)\, dx \right) \\
&= -||f||_{-\infty,\{u>0\}} \int_{\{u>0\}} \operatorname{div}(\phi\nu)\, dx \\
&= ||f||_{-\infty,\{u>0\}} \int_{\partial\{u>0\}} \phi\, d\mathcal{H}^{n-1}
\end{aligned}$$

so that also

$$\frac{\int_{\partial\{u>0\}} H\phi\, d\mathcal{H}^{n-1}}{\int_{\partial\{u>0\}} \phi\, d\mathcal{H}^{n-1}} \geq ||f||_{-\infty,\{u>0\}}.$$

Now for every $x_0 \in \partial\{u>0\}$ one can choose a sequence of radially symmetric $\phi_j \in C_c^\infty(\Omega)$ such that

$$\lim_{j\to\infty} \frac{\int_{\partial\{u>0\}} H\phi_j\, d\mathcal{H}^{n-1}}{\int_{\partial\{u>0\}} \phi_j\, d\mathcal{H}^{n-1}} = H(x_0).$$

In particular

$$\inf_{\partial\{u>0\}} H \geq ||f||_{-\infty,\{u>0\}}.$$

□

Proof of Theorem 3. Theorem 3 follows by combining propositions 1 and 2. Lipschitz continuity of u^2 in a neighborhood of $\partial\{u=0\}$ implies the boundedness of $u|Du|$ from above, which in turn yields the bound from below for $\frac{\alpha}{u\sqrt{1+|Du|^2}}$. □

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