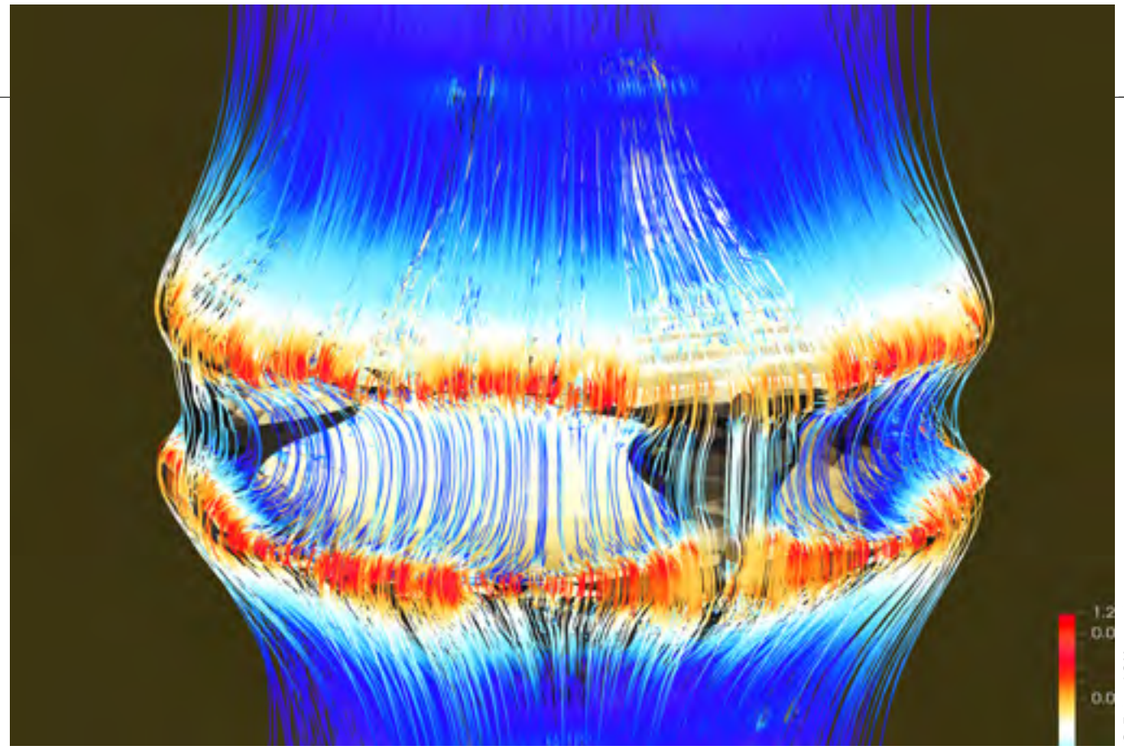

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Shape optimisation of type-II superconductors

Faculty of Mathematics

The Faculty of Mathematics consists of 29 research groups, making it one of Germany's largest mathematics faculties. Its research represents a broad cross-section of the many developments taking place in the discipline. They range from application-motivated questions to curiosity-driven basic research, all closely connected by the universal nature of mathematical concepts. With a wide range of externally funded research projects across the full spectrum of specialisms, the Faculty offers an attractive and international environment for early-career academics to contribute to the latest advances in mathematics. The ongoing pandemic also highlights our work in the field of mathematics education, which explores the opportunities inherent to digital tools.

All our research groups belong to one of the following key areas:

- Algebraic geometry and arithmetic
- Analysis, numerical analysis and optimisation
- Stochastics
- Mathematics education

They are closely interconnected. Our research groups have been able to secure external funding for their projects across the board, which confirms the success of our strategic orientation. During the reporting period, our faculty members were funded within the scope of five Priority Programmes of the German Research Foundation (DFG), three research training groups, two Collaborative Research Centres (SFB) and one European Research Council (ERC) Advanced Grant, in which they were involved either as leaders or scientific contributors. Further funding sources include other DFG projects, the Humboldt Foundation, various programmes of the Federal Ministry of Education and Research and other research institutions. The past two years saw several highlights, including Professor Marc Levine's acquisition of the ERC Advanced Grant 'QUADAG – Quadratic refinements in algebraic geometry' and the establishment of the new research training group 'symmetries and Classifying Spaces: Analytic, Arithmetic and Derived' at the Essen Seminar for Algebraic Geometry and Arithmetic.

Funded by a multitude of external sources, our research projects allow us to integrate our research groups into national and international networks and organise international conferences, workshops and summer schools in Essen. They also attract many academics from Germany and abroad, who bring their own projects with them. Two Heisenberg fellows, Andreas Nickel and André Chatzistamatiou, have chosen our Faculty as the basis for their research. The international researchers Professor Paul Arne Østvær, Professor Kazim Büyükboduk, Dr Daniel Kohen and Dr Mingshuo Zhou came to our faculty as visiting scholars with funds from the Humboldt Foundation, other exchange programmes and their own contributions. A very high percentage of researchers working on the projects of our research groups have an international background. Two Mercator fellows at the newly established research training group further strengthen this international exchange. The international master's degree programme of the ALGANT Network attracts outstanding, advanced students from all over the world to Essen. They join us at an early stage of their university education and often go on to have a successful academic career after graduating.

In the following section, we take our readers on a tour of our key research areas by presenting a select few projects. A full overview would exceed the scope of this report. The universal nature of mathematical structures is a recurring topic at our faculty. Although it is often applied to specific issues in a narrow field, it can express and explain a wide variety of phenomena across disciplines. Our tour takes us from basic research in the field of number theory to partial differential equations and our application-motivated projects. On the way, we will pass financial mathematics, superconductivity and projects focusing on digital transformation and inclusion in mathematical education.

The ERC project QUADAG

Professor Marc Levine has been awarded the ERC Advanced Grant on quadratic refinements in algebraic geometry for a five-year research project in the field of motivic homotopy theory. Levine has been an influential contributor to this field for many years. His project is based on the combination of methods in algebraic geometry with those of homotopy theory. The algebraic foundation of the former ensures that a good local understanding of objects always facilitates global conclusions through long-distance effects. In the latter, geometry facilitates highly flexible deformations at the local level without global effects. A combination of these opposed approaches, which were originally developed to solve issues in number theory, has been applied to many fields of mathematics, including mathematical physics.

For his ERC project, Professor Levine has developed new methods that open up additional fields of application. His goal is to use quadratic refinements to establish new connections between real algebraic geometry, tropical geometry, singularities and objects in number theory.

The project has already attracted four new international members – two post-doctoral researchers and two doctoral candidates – to the Essen-based research group. It originated from the DFG Priority programme 1786, 'Homotopy Theory and Algebraic Geometry', whose researchers from the eponymous areas have been working to find new connections such as those studied in the new ERC project since 2015. Multiple research groups at our faculty are involved in this programme. Over the reporting period, we were able to organise two international conferences and one international doctoral research school on the topic of motives and stacks in Essen.

Algebraic Geometry and Arithmetic: the research training group 'symmetries and Classifying Spaces'

This year, the research training group 'symmetries and Classifying Spaces: Analytic, Arithmetic and Derived' was established at our faculty. Its research groups from the Essen Seminar for Algebraic Geometry and Arithmetic aim to capitalise on the numerous interfaces of their research projects for doctoral training in an extraordinarily dynamic field. The group leaders are Massimo Bertolini, Ulrich Görtz, Daniel Greb, Georg Hein, Jochen Heinloth, Jan Kohlhaase, Marc Levine, Ursula Ludwig, Andreas Nickel and Vytautas Paskunas.

Symmetries and the classification of geometric objects are key issues in mathematics and, in particular, the various approaches to algebraic geometry: classical algebraic geometry, complex geometry, arithmetic geometry, derived algebraic geometry and other fields at the intersection of algebraic geometry, analysis and topology. Our research focuses on groups that describe symmetries of geometric, analytic and number-theoretic objects and classifying spaces, i.e., spaces that parametrise all objects of a given type, to varying degrees. The two topics are often closely connected.

Enormous progress has been made over the past years: the theory of perfectoid spaces, the Langlands programme, progress on the Birch and Swinnerton-Dyer conjecture and the minimal-model programme are famous examples. The development of new methods allows the field to advance rapidly, and new breakthroughs are on the horizon. This makes it a promising field of research for young mathematicians looking to start their careers. With a multitude of methods in use, doctoral candidates benefit from working in an environment where they have access to expertise in many of the numerous approaches. The Faculty of Mathematics offers such a stimulating environment to its early-career researchers in Essen. Our doctoral candidates are supported in their transitions from students to researchers and get to establish themselves in a fascinating mathematical field.

We will outline some of the results produced by the research groups who have contributed to this project and emphasise the interfaces between them. The construction of integer or rational solutions to equations is a fundamental problem of number theory. The Birch and Swinnerton-Dyer conjecture describes a relationship between the structure of rational solutions and the analytically defined invariants of the underlying properties. In order to understand that mysterious relationship, it seems inevitable that we must develop methods for constructing arithmetic solutions using analytical objects. Professor Bertolini's research group has proven new results of this type for p-adic L-functions.

Professor Paskunas's group was also able to use p-adic methods to produce new global, arithmetic results, also taking advantage of the local geometry of classifying spaces. These classifying spaces formally resemble the spaces of spaces of representations of groups arising in geometry, which the research groups of Professors Greb, Hein and Heinloth study. In both situations, the same issue occurs: the global geometry of the problems frequently exhibits pathologies which lead to stability conditions for the objects parametrized by these spaces. Surprisingly, analytic descriptions, which may often be formulated in terms of stability results for solutions of certain differential equations, also yield conditions that can be understood in purely algebraic terms. To understand the relationship between these stability conditions and the geometry of the parameter spaces, the researchers were able to prove results that allow them to reduce the study of stability properties to a few key conditions. Because the spaces thus obtained often exhibit singularities, it is difficult to approach them with analytic methods. Dr Ursula Ludwig's work focuses on this obstacle and expands it to include fundamental analytical methods for interesting classes of singularities.

Analysis – nonlinearity, memory effects and randomness

In the field of analysis, we introduce Professor Petra Wittbold's and PD Dr Aleksandra Zimmerman's work on nonlinear evolution equations. Evolution equations are used to describe the temporal development of systems. Complex issues in biology, medicine, physics, engineering, economics and the social sciences require nonlinear models for adequate description.

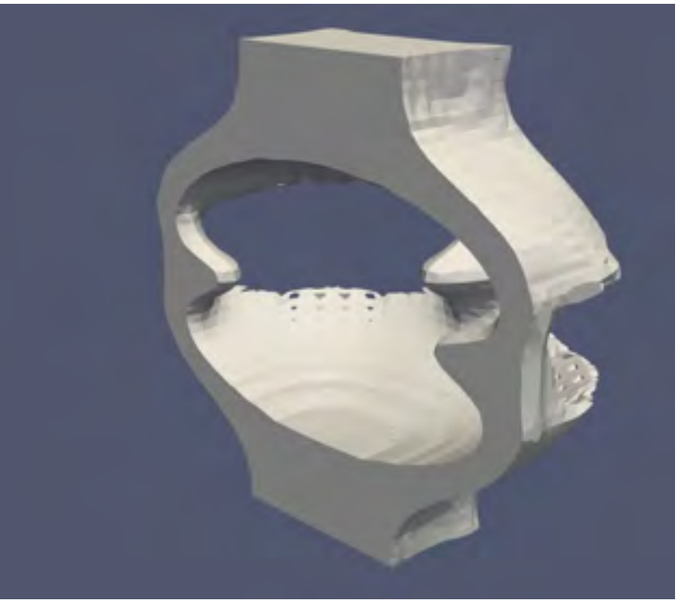
Memory effects and random effects are an important factor in describing the development of such processes, as they influence the present and the future of humans and materials alike. Take filtration processes as an example: if a liquid flowing through a porous medium contains particles that interact with that medium (e.g., by settling in its pores), its flow behaviour will change with time. This is a typical memory effect. Random parameters are introduced into the models in order to account for the heterogeneity of the medium, interactions on various scales and the inaccuracy of measured data. Over the past two years, there has been a lot of research into whether nonlinear evolution equations with memory and random effects are well posed. Many projects were carried out in collaboration with mathematical institutes in Brazil and France. Within the scope of her own externally funded project, Dr Aleksandra Zimmerman has worked with an interdisciplinary research group from the Laboratoire de Mécanique et d'Acoustique (LMA) of Aix-Marseille University on modelling the interfaces of composite materials in consideration of stochastic effects.

Professor Paola Pozzi's research group focuses on geometric aspects of evolutionary processes. In particular, its members were able to produce new results in the field of curvature flows. These flows describe changes in surfaces occurring due to physical processes, which are often described in terms of the minimisation of surface tensions. They can also be used to obtain classification results on the shape of geometrical structures in cases where it can be proven that the limits of the flows have special geometrical properties. A collaborative project between mathematicians from Australia and Japan was able to obtain new existence results for such flows. In the years ahead, the researchers will work on questions on the long-term evolution of networks under various flows, working with research groups in Ulm and Taiwan within the scope of the DFG project on the flow of elastic networks.

Dean: Professor Dr Georg Weiss

Stochastics: evolving networks and nonlinear dynamic processes

The research group on probability theory focuses on networks for modelling complex systems with many interdependencies. Interactions in complex systems are often represented by networks. One problem inherent to this approach is that in real-world problems, the structure of



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Shape optimisation of type-II superconductors

those networks changes with time due to interdependencies. The processes themselves influence the topology of the network, as connections disappear and new ones appear. At the same time, the structure of the network also has a considerable impact on the processes. Within the scope of multiple DFG-funded projects, Professor Anita Winter's research group has developed analytical and probability theory methods that facilitate the mathematically rigorous description of both effects. In the project 'Evolving Pathogen Phylogenies: a Two-Level Branching Approach', funded under the DFG Priority Programme 'Probabilistic Structures in Evolution', the researchers developed a model for describing virus populations influenced by cell division processes, in particular, their long-term behaviour.

Additional projects have been launched in the same field. They are taking place within the scope of the DFG Priority Programme 'Random Geometric Systems'. One of its members, Dr Anton Klimovsky, is an early-career researcher from our faculty. He is contributing an independent project to the programme.

The research group on applied stochastics has secured funding for various projects within the scope of their membership of the CRC 'statistical Modelling of Nonlinear Dynamic Processes'. Their work also focuses on similar mathematical structures that may be applied to problems in financial mathematics as well as hearing acoustics. One of the group's recent projects has produced new results on variance reduction in Markov chains. Variance reduction methods are important tools for reducing complexity in simulation-based, numerical algorithms, such as various Monte Carlo methods. They are also widely used in Bayesian statistics and machine learning.

Issues in machine learning are also a key topic of other research groups that focus on numerical methods, such as Professor Martin Hutzenthaler's group (stochastics) and Professor Johannes Kraus's group (numerics).

Numerics and optimisation: non-smooth variational problems for modelling superconductivity and friction

Below a critical temperature, electrical resistance vanishes in superconducting materials. Electricity is transported virtually without losses.

Disc brakes, such as those used in bicycles, generate the braking action through friction between the brake pads and the brake disc. This friction depends on the surface structure of the brake pads.

With the exception of the present report, these two topics are unlikely to be discussed in the same context.

But mathematical modelling of the physical processes of superconductivity and frictional contact leads to similar mathematical problems. Both are the focus of the DFG Priority Programme 'Non-Smooth and Complementarity-Based Distributed Parameter Systems: Simulation and Hierarchical Optimization'. Professor Irwin Yousept's and Professor Gerhard Starke's research groups are participating in the priority programme throughout its entire period from October 2016 until October 2022. The mathematical similarity between the questions lies in the term 'non-smooth'.

The relevant process variables – current density in the case of superconductivity, tension in the brake pad in the case of friction – are not clearly influenced by the created fields in all parameter areas. Instead, they have a 'kink' in the most interesting area: when reaching the critical current density and during the transition from adhesion to sliding, respectively.

In the mathematical description of the processes, these 'kinks' are variational inequalities. In the past decades, a comprehensive solution theory and numerical methods have been developed to construct approximations of such variational inequalities efficiently. Adaptive mesh refinements based on error estimators are an important component of the solution strategy, as they ensure that the dimension of discretised problems does not become excessive. The strategy further involves suitable iterative processes for approximating a solution to the discretised problems, which are still highly non-linear and non-smooth. Complementarity conditions incorporating Lagrange multipliers play a role in both sub-aspects.

The established methods for variational inequalities are not directly applicable to the problems our sub-projects seek to solve, however. We are working with hyperbolic evolution variational inequalities (in the superconductivity problem) and quasi-variational inequalities (in the friction problem). In the first case, time-dependent irregularities in current densities and singularities in the electromagnetic fields may occur; in the second case, the variation formulation itself depends on the solution. Both projects yielded enough research topics for one doctoral dissertation each, followed by plenty of post-doctoral research.

Mathematics education: new students, digital media and inclusion

The four research groups in the field of mathematics education are closely interlinked with each other within the University of Duisburg-Essen and at the national and international level. They participate in the several joint projects, such as the programs 'Bildungsgerechtigkeit im Fokus' and 'ProViel' funded by the Federal Ministry of Education and Research and the German Centre

Selected Publications

Bögelein, V., F. Duzaar, C. Scheven (2020): *Higher integrability for the singular porous medium system. J. Reine Angew. Math. 767, 203–230.*

Burtscheidt, J., M. Claus, S. Dempe (2020): *Risk-Averse Models in Bilevel Stochastic Linear Programming SIAM Journal on Optimization, 30(1), 377–406.*

Drijvers, P., D. Thurm, E. Vandervieren, M. Klinger, F. Moons, H. van der Ree, A. Mol, B. Barzel, M. Doorman (2020): *Distance mathematics teaching in Flanders, Germany and the Netherlands during COVID-19 lockdown. Educational Studies in Mathematics.*

Emerton, M., V. Paškūnas (2020): *On the density of supercuspidal points of fixed regular weight in local deformation rings and global Hecke algebras. J. Éc. polytech. Math., 337–371.*

Greb, D., S. Kebekus, T. Peternell, B. Taji (2019): *Nonabelian Hodge Theory for klt spaces and descent theorems for vector bundles Compositio Mathematica 155(2), 289–323.*

Hutzenthaler, M., A. Jentzen (2020): *On a perturbation theory and on strong convergence rates for stochastic ordinary and partial differential equations with nonglobally monotone coefficients. Annals of Probability 48(1), 53–93.*

Levine, M. (2020): *Some recent trends in motivic homotopy theory. Notices Amer. Math. Soc. 67(1) 9–20.*

Löhr, W., L. Mytnik, A. Winter (2020): *The Aldous chain on cladograms in the diffusion limit, Annals of Probability 48(5), 2565–2590.*

Ludwig, U. (2020): *An Extension of a Theorem by Cheeger and Müller to Spaces with Isolated Conical Singularities, Duke Math. J. 169(13), 2501–2570.*

Scherer, P., M. Nührenbörger, L. Ratte (2020): *Reflexionen von Multiplikatorinnen und Multiplikatoren zum Gestaltungsprinzip der Teilnehmendenorientierung – Fachspezifische Professionalisierung beim Design von Fortbildungen. Journal für Mathematik-Didaktik. online first. doi 10.1007/s13138-020-00179-8*

Winckler, M., I. Yousept, J. Zou (2020): *Adaptive edge element approximation for H(curl) elliptic variational inequalities of second kind. SIAM J. Numer. Anal. 58(3), 1941–1964.*

Professors

Professor Dr Bärbel Barzel
Professor Dr Denis Belomestny
Professor Dr Massimo Bertolini
Professor Dr Mircea Birsan
Professor Dr Andreas Büchter
Professor Dr Christian Clason
Professor Dr Ulrich Dierkes
Professor Dr Andreas Gastel
Professor Dr Heiner Gonska
Professor Dr Ulrich Görtz
Professor Dr Daniel Greb
Professor Dr Lisa Hefendehl-Hebeker
Professor Dr Georg Hein
Professor Dr Jochen Heinloth
Professor Dr Martin Hutzenthaler
Professor Dr Volker Krätschmer
Professor Dr Johannes Kraus
Professor Dr Jan Kohlhaase
Professor Dr Marc Levine
Professor Dr Frank Müller
Professor Dr Patrizio Neff
Professor Dr Vytautas Paskunas
Professor Dr Paola Pozzi
Professor Dr Arnd Rösch
Professor Dr Florian Schacht
Professor Dr Petra Scherer
Professor Dr Christoph Scheven
Professor Dr Rüdiger Schultz
Professor Dr Gerhard Starke
Professor Dr Heinz Steinbring
Professor Dr Mikhail Urusov
Professor Dr Georg Weiss
Professor Dr Anita Winter
Professor Dr Petra Wittbold
Professor Dr Irwin Yousept

for Mathematics Teacher Education (DZLM), initiated by the Deutsche Telekom Stiftung.

Digital media are an important aspect that connects the activities of all research groups. Various research and development projects are dedicated to the role and utilisation of digital media in mathematics lessons at primary and secondary level, in higher education and in teacher education. Their topics include potentials for the (summative and formative) diagnosis of learning progress and processes, the use of digital media in teacher education, and the role of digital textbooks in mathematics teaching. Due to the coronavirus pandemic, these issues are more relevant today than they have ever been. This is reflected in various activities of the research groups: they rapidly provided new supporting materials for mathematics teachers and analysed the currently used technologies in the international study 'Math@Distance', a joint endeavour with colleagues from Belgium and the Netherlands. The project highlighted stark differences in the way the countries approach remote teaching.

The new project 'Digimal.nrw', which focuses on digital mathematics teacher education in North Rhine-Westphalia, was awarded to the Faculty as early as May. Professor Petra Scherer and Professor Florian Schacht's research groups will be leading the research consortium. The project seeks to achieve targeted improvements to mathematics teacher education at the primary level.

Other projects of our research groups focus on the growing importance of professionalism in teaching in order to prepare budding teachers for the challenges they will face in their work (e.g., digital transformation and inclusion). In collaboration with the German Centre for Mathematics Teacher Education (DZLM), our groups are organising professional development opportunities for multipliers and a nationwide programme to furnish them with basic qualifications. The Centre's developmental department is co-managed by our Faculty's Professor Bärbel Barzel. Various doctoral and post-doctoral projects explore issues of professionalism in mathematics education.

Others address the topic of inclusion and inclusive mathematics teaching. Apart from the development of class-based learning environments, our research covers the design and evaluation of courses, competence development in students, and studies into professional education for teachers.

It takes place in the form of multiple projects, including doctoral dissertation projects, which are closely interlinked. Besides seeking to find ways to gauge the learning capabilities of new students, our scholars focus on research-based methods for advance teaching practices. They include the systematic development of exercises in consideration of learning and problem-solving strategies and the use of supplementary e-assessment tasks or interactive, dynamic visualisations. The Faculty was able to establish a structured range of digital services, which are now used in a wide range of courses for new students. Acceptance of the new services is high. During their development, the researchers behind the project analysed many traditional exercise formats in terms of their digitisation potential and implemented them accordingly.

Conferences, workshops, summer schools

- 'Motivic Homotopy Theory and Refined Enumerative Geometry'
- 14. Doctoral candidates' meeting: stochastics
- Autumn meeting of the group 'Lehr-Lern-Labore'
- Autumn meeting of the group 'Mathematikunterricht digitale Werkzeuge'
- Workshop on prismatic cohomology
- Hausdorff Trimester Programme 'Evolution of Interfaces'
- Workshop 'stochastic Optimization and Related Topics'
- 14th International Conference on Technology in Mathematics Teaching (ICTMT 14)
- Summer School 'Motives and Stacks'
- PP 1786 annual conference
- PP 1590 workshop 'Evolutionary Forces and Genealogical Trees'
- Workshop 'Fundamentals of Complex Networks: From Static Towards Evolving'

Awards and honours

In 2019, Professor Rüdiger Schultz was awarded the research prize of the Gesellschaft für Operations Research.

Outlook

The projects described above lay the groundwork for further research in the years ahead. The Essen Seminar for Algebraic Geometry and Arithmetic (ESAGA) will be focusing on the establishment of a research training group and the associated research programme. We have received an extraordinarily high number of outstanding applications for our new doctoral studentships and are optimistic that the young academics will make interesting contributions to our faculty. Within the scope of the ERC project, Marc Levine has put together an active, international group of early-career researchers who will be shaping this active field in future. The DigiMal.nrw project is a new collaboration; its first members have been confirmed recently.

At the Faculty itself, two appointment processes are nearing completion: that for the tenure track junior professorship of algebra and number theory and that for Marc Levine's successor. Both will influence the character of our faculty.

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At a mathematics seminar

Our researchers in the fields of analysis and numerical analysis are working on further collaborative projects, and a new call for papers will be issued in analysis. The coming years will offer many new catalysts for exciting developments.

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