

The exponentiated Hencky strain energy in modeling tire derived material

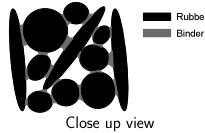
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1 Introduction

The mechanical characterization of a new Tire Derived Material (TDM) for structural applications is proposed. TDM is a composite made by cold forging a mix of rubber fibers and grains, obtained by grinding scrap tires, and polyurethane binder, which has been used mainly in railway applications for vibration reduction [1].



TDM pad



Close up view

Common hyperelastic material models fail in describing TDM behavior in different deformation modes with a unique set of parameters [1]. Also fitting polynomial-like strain energy functions to experimental data of elastomeric solids can lead to oscillating functions with parameters that may not have physical meaning.

Here, a variation of the **exponentiated Hencky strain energy** is proposed to capture the high nonlinearity of TDMs in volumetric deformation:

$$W_{\text{eHm}}(F) := \underbrace{\frac{\mu}{k} e^{k \|\text{dev}_v \log U\|^2} + \frac{\kappa}{2k} e^{\tilde{k} (\log U)^2}}_{\text{Neff et al., 2014 [5]}} + \frac{\kappa_1}{m k} e^{\tilde{k} (\log U)^2} \quad (1)$$

where μ and κ are the shear and the bulk modulus at infinitesimal strain, κ_1 is a bulk modulus for large deformations and k, \tilde{k}, \bar{k} and m are non-dimensional parameters. The exponentiated Hencky energy was recently introduced by Neff et al. [5] and possesses a number of interesting properties [3, 6, 4].

2 Rate-independent response

We consider three types of TDMs with the same composition (90% SBR fibers + 10% SBR grains) but different densities (500, 600, 800) and three modes of deformation: shear, uniaxial compression, and pseudo-hydrostatic compression.

First, we consider a lubricated cylindrical specimen that is inserted into a rigid (steel) cavity of the same radius and then axially compressed [1]. The experiment is designed to test the pressure-volume relation. However, the pressure is approximated as $p \approx \sigma_{11}$, which is only valid for $p \gg s_{11}$.

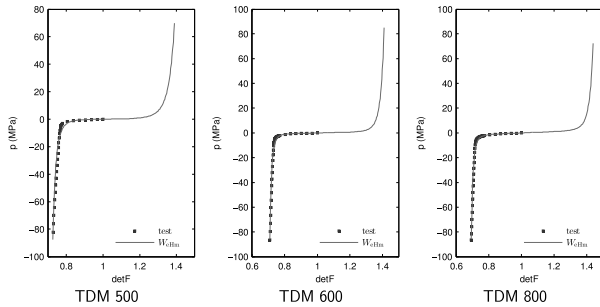


Figure 1: Comparison between pseudo-volumetric response and experimental tests $p \approx \sigma_{11}$.

Uniaxial compression tests were performed using a multi-step relaxation procedure. Between each loading step there is a 600 s dwell to allow for relaxation of the material. To evaluate the transverse behavior, digital image correlation was performed at the end of every relaxation period, allowing us to define a **non-linear Poisson's coefficient**

$$\bar{\nu} := \frac{\log \lambda_2}{\log \lambda_1}$$

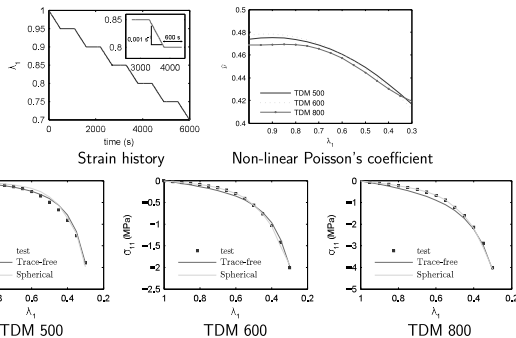


Figure 2: Compression stress for the exponentiated Hencky energy W_{eHm} and experimental tests.

Finally, shear samples were tested with the classical dual lap test configuration [2].

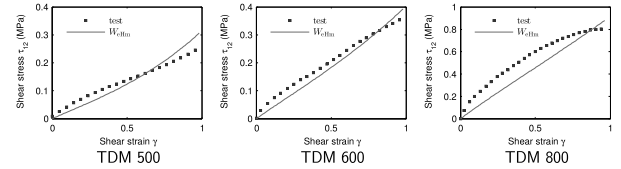


Figure 3: Shear stress corresponding to the exponentiated Hencky energy W_{eHm} and experimental tests.

3 Nonequilibrium response

A finite strain model of viscoelasticity is constructed considering the multiplicative decomposition of the deformation gradient F into elastic (F_e) and inelastic (F_i) parts [7]. We assume the existence of two viscous mechanisms, intermolecular resistance at the microscale level (A) and grain interactions at the macroscale level (B) [2]:

$$W_{\text{eHm}}^v = W_{\text{eHm}}^{\text{EQ}} + W_{\text{NEQ}}^A(b_e^A) + W_{\text{NEQ}}^B(b_e^B), \quad (2)$$

where $W_{\text{eHm}}^{\text{EQ}}$ represents the strain energy in the equilibrium spring Eq. (1) and W_{NEQ}^k is the strain energy of each relaxation mechanism and is associated to the "elastic" left Cauchy deformation tensor $b_e^k = F_e^{-k} \cdot [F_e^k] \cdot F_e^{-k}$ ($k = A, B$), also called the Finger tensor. For most polymer based materials, the volumetric deformation is purely elastic and the viscous effects are restricted to the isochoric component of the deformation. Following this assumption the strain energy for the relaxation mechanisms can be written as:

$$W_{\text{NEQ}}^A(b_e^A) = \frac{\mu_A}{k_A} e^{k_A \|\text{dev}_v \log b_e^A\|^2}, \quad W_{\text{NEQ}}^B(b_e^B) = \mu_B \|\text{dev}_v \log b_e^B\|^2. \quad (3)$$

Dynamic shear tests in a dual lap set up and dynamic compression tests consisting of a static pre-strain of 10% and a superimposed sinusoidal excitation were both performed at different frequencies and amplitudes. Figure 4 shows a few tests from the many performed as they are representative of the overall abilities of the model.

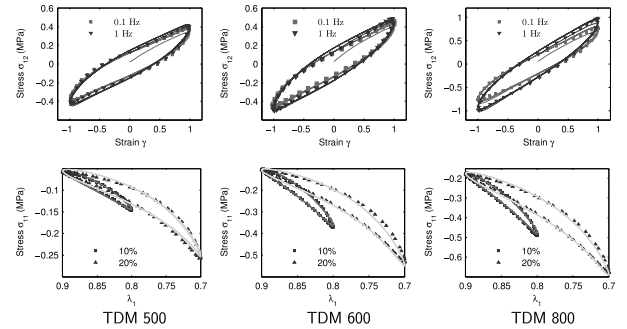


Figure 4: Comparison between cyclic compression and shear tests (markers) and the viscoelastic model based on the modified exponentiated Hencky energy W_{eHm}^v (solid line).

4 Conclusion

A hyper-visco-elastic constitutive model for TDMs based on an exponentiated Hencky strain energy is presented. The model describes different deformation modes with a unique set of parameters in the equilibrium range. The predicted results are in excellent agreement with the presented data, from both static and dynamic tests, and thus give a viable model for engineering applications of TDMs.

5 References

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