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Einladung zum Seminar

Mechanik und Numerische Mathematik

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FETI methods for multiscale PDEs

In this talk we consider a Poisson-type equation in two and three dimensions with a highly varying coefficient, i. e., $-\nabla \cdot [\alpha \nabla u] = f$ in Ω, and with some Dirichlet and/or Neumann boundary conditions on $\partial\Omega$. We are interested in solvers of the underlying finite element system which are robust with respect to the variation in $\alpha(\cdot)$. A great success has been made with FETI-type domain decomposition methods: If the domain Ω can be decomposed into regular subdomains Ω_i where $\alpha(\cdot)$ is constant (or at least only slightly varying) on each of the subdomains, one can construct robust preconditioners for the iterative solution of the discretized PDE. It has been shown that the condition number of the preconditioned system behaves like $\mathcal{O}(\max_i (1 + \log(H_i/h_i))^2)$, where H_i denotes the subdomain diameter and h_i the subdomain mesh size. This estimate is independent of the values of α, and in particular of the jumps across subdomain interfaces.

In the present work, we generalize these standard results on FETI methods. We have two applications in mind: (i) the case of coefficient jumps not aligned with the subdomain interfaces, and (ii) highly varying coefficients within the subdomains. The latter situation appears for example when considering Newton linearizations of nonlinear magnetic field problems. We propose modifications of the standard FETI preconditioners which depend only (very mildly) on the variation of the coefficients near the interfaces.

If we assume for each subdomain Ω_i, that the coefficient varies only mildly near the boundary, i. e., $\frac{\alpha(x)}{\alpha(y)} \leq \alpha_i^*$ for all x, y that are less than η_i away from the boundary $\partial\Omega_i$, but varies arbitrarily otherwise, we can even give a rigorous analysis. In this case we can show that the condition number can be bounded by $C \max_i \left(\alpha_i^* \left(\frac{H_i}{\eta_i}\right)^2 (1+\log(H_i/h_i))^2\right)$, both in 2D and 3D. Provided the minimum of α in each subdomain Ω_i is attained in the boundary layer, this bound can be improved to linear dependence on H_i/η_i. This is confirmed in numerical experiments.

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