

University Duisburg - Essen Duisburg Campus

ISE Laboratory Physics Course

Directions for Experiments

Experiments

No. Title

B8 Determination of the Specific Charge of the Electron.....

B10 Experiment with X-Rays.....

D5 Spectral Analysis.....

D8 Radiation from Heated Solids.....

Appendix: Analysis of Errors

Edition:

Prof. Dr. M. Farle
Dr. R. Meckenstock
Fakultät für Physik,
Experimentalphysik,
Universität Duisburg - Essen,
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Introduction

This section of the guide to the laboratory physics course refers to general and binding rules and security instructions.

The second part of the guide, which can be found on <http://www.uni-due.de/agfarle/grundlagenpraktikum/ANLEITUNGEN/ISE.pdf> presents a short comprehensive description of the basic physics of the experiments to be performed in this laboratory course including instructions on carrying out the experiments and on the evaluation of results. The last section gives a short introduction to the qualitative and quantitative estimation of experimental errors.

The experiments in this laboratory course have been selected and modified from a series of experiments of the basic laboratory course for physics students. The aim of this laboratory course is to introduce the participant to major principles of how to perform and analyse experimental methods in physics. In this sense the important questions to each physical experiment are:

- What is the purpose of the experiment (which physical property should be determined or which physical principal should be proven)
- How is the accuracy of the experiment (Is the experiment suitable to achieve its goals)
- Are the measured quantities within the expected order of magnitude and how are they related to corresponding literature.

The description of each experiment contains some major keywords to the subject of the course and should be considered for further studies in the available literature. Please note that the basic physical descriptions given afterwards are only introductory, i.e., they do not cover the subject exhaustively. Further reading of textbooks on undergraduate physics is strongly recommended. Questions presented at the end of each experiment are intended to serve for self-checking. You have to be able to answer them before the explicit course.

The participant in the physics laboratory course should be as conscientious as possible in carrying out the experiments. It does not matter if an experimental value obtained by many students before is just reproduced. Instead, one should learn to take measurements as reliable as possible independent of the well known result. The error inherent to the result obtained from the experiment is decisive for the quality of the experiment performed and the reliability of the result. In future situations the participant might be confronted with problems, for which the solution is not known a-priori and thus, have to prove and validate his results thoroughly so that the physical outcome of the experiment is understandable and reproducible for a greater community.

It is therefore required to study the introduction to the evaluation of errors given in the appendix and to include the estimated error to every experimental result obtained. A quantitative experimental result presented without the estimated error is not meaningful. Furthermore, the numerical result should be rounded to the extent given by the errors involved. Significant to the latter are usually 2 or 3 decimal places. The large number of decimal places displayed by a pocket calculator have to be justified with respect to their physical significance. The sources of striking large errors obtained (sometimes of striking small ones as well) are to be discussed. They must originate from the conditions of the experiments performed.

Advice for Laboratory Course Participants

In the Laboratory physics course, the experiments are carried out in groups (usually two to three students per group). The time schedule and selection of the experiments is determined by grouping the participants. The grouping is carried out together with handing out these experiment instructions after registering for the laboratory course (usually at the beginning of the semester). Please note the “ Vorlesungsverzeichnis” and the posting before room ME 142.

With your registration data two lab cards are filled out for each participant with: family name, first name, matriculation number, discipline of study and group number. One lab card remains by the laboratory course administration. The other card is issued to the participant at the first Lab course and is to be presented at every discussion before the experiments and at the final discussion at the end of the semester as well. At the end it is the prove of successful participation until this is given to the Prüfungsamt by the lab course administration.

One should prepare thoroughly for every experiment and consider the physical questions arising in the particular context, e.g., by using the keywords for further consulting textbooks. To verify the trainee's preparation a discussion is given by a lecturer before each experiment. This discussion serves, additionally, as an insurance that the participant's are well aware of the security aspects of each individual experiment and do not damage the equipment do to lag of knowledge.

One should take care, that every discussion finished successfully is registered on the lab card with the date and signature of the lecturer. If the discussion has not been finished successfully, the experiment (including the discussion) is to be repeated at the given extra date. Out of the four experiments to be carried out in the ISE laboratory physics course, only one experiment can be repeated for any reason. If more than one discussion has ended unsuccessfully, or the student is absent for any reason more than once the whole laboratory course has to be repeated. There is only one extra date!

A record including all measured data and parameters is to be prepared immediately during the experiment. The forms provided at the experiments by the assisting lecturer (tutor) should help to get all relevant results. It is to be signed by the tutor at the end of the experiment as a prove of successful participation to the explicit experiment in general on the same day. It is important that at the end of the form the free space called experimental errors and discussion is filled out hand- written in a reasonable way in the frame of the experimental coals formulated above. This part has to be checked and signed separately by the Tutor of the experiment.

Be aware that the physical background of the experiment is NOT part of the written record, but is part of the first testing. The students are advised to prepare a short presentation of the experimental setup and the carrying out of the experiment in advance of the experiment to be sure to be well prepared for the experimental workflow of each lab course.

Every experimental record has to be checked and signed by the tutor at the end of the experiment or if there is a lag of time at an explicit date provided by the tutor. If this signature is not on the record at least 2 weeks after the experiment or extra date the experiment is considered as not valid and is to be repeated. (Once again: only one repetition is possible and no repetition of an extra date).

All records approved by the tutors are to be presented at the final discussion. If all those records and especially there experimental discussion are once again tested and proven by the lecturer

of the final discussion, the laboratory course is considered as successfully finished and is noted by the lecturer on the lab cards. The date and place of the final discussion is announced on the information board (at room ME 142) before the end of the principal laboratory courses and on the web page of the Grundlagenpraktikum:

<http://www.uni-due.de/agfarle/grundlagenpraktikum>.

The lab cards and all reports are to be presented at this date. The successful participation is reported to the Prüfungsamt.

Laboratory Security Instructions

General behaviour in the laboratory physics course

Every course participant should observe caution in the laboratory in order to prevent any personal hazard or damage of laboratory equipment.

Advice given by the lecturers and assisting students responsible for the laboratory course and given in the guide to the experiments, i.e., how to handle the experimental instruments, should be explicitly obeyed.

Before starting an experiment, one should wait for the instructions of the assisting student responsible for the experiment (see name list on the information board). After the experiment, the space should be left tidy. Eating, drinking, smoking, and the use of cellular telephones are not allowed in the laboratory.

Working with electric circuits

Contacting live parts of current sources with DC or AC voltages above 50 V can be lethal or, in the least, can cause personal damage. Surges caused by short circuits can cause instrument damages and fire.

When working with electric circuits with high voltages, at least two persons are to be present. In cases of emergency, the mains voltage is to be switched off. This is done either by switching off the local block distributor or by pressing the emergency button (red button on the terminal box next to the door).

All experimental setups with electrical circuits are to be checked by the assisting student before operating. Completing, dismantling or alteration of electric circuits is to be done with the line disconnected.

To avoid errors, circuits should be clearly built up using cables with proper length and color. When working with electric measurement devices, one should pay attention that the correct polarity, measuring range, and measuring input is used (danger of overload).

After the end of an experiment, all electric instruments and the terminal blocks at the experiment are to be switched off. Batteries are to be separated from circuits, circuits previously completed are to be dismantled.

Using ionizing radiation (x-rays)

High energy electromagnetic radiation (x-rays) penetrates human tissue and causes damage due to its ionizing effect; the amount of damage corresponds to the absorbed radiation dose. The x-ray apparatus used in the laboratory course is an instrument designed to be operated in schools. It has a complete protection enclosure and, therefore, a very low external radiation power, being at the lowest limit to be detectable at all. Opening of the protection enclosure automatically switches off the radiation. Every trial to bypass the protection circuit is strictly forbidden.

Behaviour in emergency cases

Keep calm! - Warn persons running into danger! - During help pay attention to your own security!

Accident

For immediate help for injuries, there are first aid boxes containing bandaging material in room ME 142 (room of the assisting students) and room MD 147.

For further assistance, **call the emergency (phone no. 92211)** or the fire brigade (phone no. 112).

Explain:

- *Who is reporting the accident?*
- *What has happened?*
- *Where did it happen?*
- *Are there injured persons?*
- *When did it happen?*

In Case of Fire

If an alarm is heard, leave the building using the shortest way out (see site plan on the information board ME 142 and escape route signs in every floor and building entrance). Do not use the elevators! The meeting place for the M-area is the parking lot of the school at Geibelstraße (the street across the Mensa).

In the case of a fire in the rooms of the laboratory physics course, the following measures should be taken in the given order of appearance:

- Make sure not to be hurt or injured
- Rescuing other people
- Fire fighting using fire extinguishers

Reporting the fire to the fire brigade (phone no. 112).

- *Explain: Who is reporting? (Name, institution, phone no.)*
- *Where is the fire? (Building, room)*
- *What is burning?*
- *Are there any injured people?*

In case of fire alarm not involved directly. Close windows and doors. Make sure that you and all people in your surrounding leave the room and the building and gather at the emergency meeting place provided at the emergency plans located at the elevators.

Laboratories with telephones: ME 142, MD 147, MD 163, MD 165

Exp. B 8: Determination of the specific Charge e/m of the Electron

1. Literature:

Keywords: Electron tubes, forces of electric and magnetic fields on electric charges, magnetic field strength and magnetic induction, ionization and recombination, emission of light, Millikan experiment

2. Basics

The aim of this experiment is the determination of the specific electric charge e/m of the electron by the deflection of an electron beam in a magnetic field. Similar experiments had been performed first by H. Busch (1922).

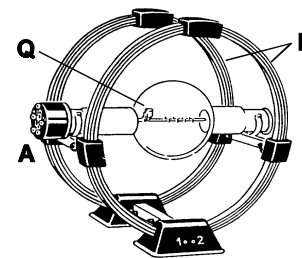


Fig.1:
Beam tube with Helmholtz coils H,
electron source Q, base and connection
socket A

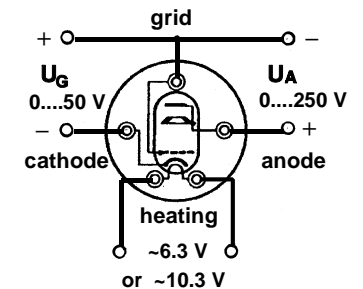


Fig.2:
Connection socket with circuit
scheme for beam generation unit

Fig. 1 shows the beam tube with Helmholtz coils, Fig. 2 shows the end side of the beam tube with circuit scheme and connection sockets for the hot cathode, the grid, and the anode with a hole. The electron beam is generated by thermal emission of electrons from the heated cathode and by their subsequent acceleration and focussing. If an accelerating voltage U is applied across the cathode and anode, the electrons (mass m , charge e) leave the anode through the hole with a certain velocity \underline{v} . According to the law of energy conservation, the kinetic energy

of the electron is equal to the electric work of acceleration:

$$\frac{1}{2}mv^2 = eU \quad (1)$$

Having left the anode, the electrons fly in a straight beam with constant velocity \underline{v} in free space in the absence of electric or magnetic fields. If, however, a magnetic field with induction \underline{B} is applied, the *Lorentz force* \underline{F}_L acts on the electrons, whereby

$$\underline{F}_L = -e(\underline{v} \times \underline{B}). \quad (2)$$

Since by definition of the vector product $\underline{v} \times \underline{B}$ the force \underline{F}_L always acts in a direction perpendicular to the common plane of the vectors \underline{v} and \underline{B} , only the direction of the velocity \underline{v} , but not the absolute of \underline{v} is changed.

Considering the path of an electron moving with velocity \underline{v} perpendicular to the B-vector of a homogeneous magnetic field, the absolute value of \underline{F}_L is simply

$$F_L = e v B. \quad (3)$$

The Lorentz force acts permanently with constant value F_L perpendicular to \underline{v} , and the electron is forced into a circular path with centripetal force equal to F_L , which means that

$$e v B = m \frac{v^2}{r}, \quad (4)$$

with r the radius of the circle. Combining eqs. (1) and (4) gives

$$\frac{e}{m} = \frac{2U}{r^2 B^2}. \quad (5)$$

The acceleration voltage U and the path radius r are to be measured directly in this experiment. The B -value of the magnetic field has to be determined from the used geometry for the field generating coil system and the current I through the coils. In free space (vacuum), the relation between magnetic induction B and magnetic field

strength H is

$$B = \mu_0 H, \quad (6)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ VsA}^{-1}\text{m}^{-1}$ is the *magnetic permeability constant*. In principle one can calculate the field strength H of any particular coil system using *Biot-Savart's law*. For the field strength or induction in the center of a ring coil with n windings and radius R , one gets, if the length $l \ll R$

$$H = \frac{nI}{2R} \quad \text{and} \quad B = \mu_0 \frac{nI}{2R}. \quad (7)$$

To create a homogeneous magnetic field, one uses two circular coils (*Helmholtz coils*) on a common axis with coil center distance equal to the coils radius R (see Fig. 1). Provided that the current through both coils with equal number n of windings is the same, the magnetic induction B within the center region of the coil system can be easily calculated from the mean radius R , the number n and the current I from the equation

$$B = 0.715 \mu_0 \frac{nI}{R}. \quad (8)$$

The factor 0.715 results from the geometry of the coil system. Using eqs. (5) and (8) we finally get

$$\frac{e}{m} = C \frac{U}{(rI)^2}, \quad (9)$$

with the constant

$$C = 2 \left(\frac{R}{0.715 \mu_0 n} \right)^2. \quad (10)$$

The radius of the coil system used is $R = 20 \text{ cm}$; the number of windings in each coil is $n = 154$. From the experimental results for U , I and r , one can calculate the specific charge e/m using eq. (9).

The path of the electron beam becomes visible, because the electrons collide on

their way with inert gas molecules filled into the tube at some reduced pressure $p \approx 10^{-1}$ Pa. The ionization of the molecules results from the impact. After recombining with electrons, the gas molecules are found in energetically excited states. During subsequent transitions into energetically lower states or into the ground state, the molecules emit electromagnetic radiation mostly in the visible range (light).

Since the value of the electronic charge $e = -1.6021 \times 10^{-19}$ C can be determined independently (e.g. in the experiment of R. A. Millikan (1910), charged oil droplets within a capacitor), the electron mass $m_e = 9.1091 \times 10^{-31}$ kg can be calculated from the ratio e/m .

The deflection of electron beams by magnetic fields is technically used in TV or conventional computer monitors as well as in magnetic lenses of electron microscopes. The technique can be applied also to other charged particles (protons, ions, etc.). This is done, e.g., in mass spectrometers and acceleration installations in the field of elementary particle and nuclear physics (cyclotron, synchrotron).

3. Experimental Tasks

- 1. Task:** Using the electron beam tube, the specific charge e/m has to be determined from several measurements of the coil current and the acceleration voltage for different given radii of the circular electron beam.
- 2. Task:** Possible errors in the determination of e/m have to be critically discussed and a calculation of the statistical error of e/m due to the errors in the single measured quantities U, I, r, \dots has to be performed.

4. Carrying out the Experiment

To provide the magnetic field, both coils (see Fig. 1) are connected in series with a stabilized variable DC current source. The current I through the coils is measured using a DC Ammeter.

The beam tube is electrically connected with the DC voltage sources for the anode voltage U_A and the grid voltage U_G according Fig. 2 (*Check the polarity!*) and the AC voltage source for the cathode heating. This AC voltage may be different for the different tubes in operation, i.e., 6.3 V or 10.3V, as indicated on the tube socket. The acceleration voltage U to be determined is the sum of U_A and U_G . U is

measured using a DC voltmeter connected across the cathode and the anode of the tube.

Important: Before the voltage and current sources are switched on, the circuits have to be checked by the assisting student! Furthermore, the potentiometers for the variation of U_A and U_G have to be checked for zero voltage positions before the voltage source is put into operation. Wait about 1 min. after the start of cathode heating before varying U_A between 0 and 250V and U_G between 0 and 50V.

The sharpness and brightness of the electron beam is optimized by varying the voltage U_G . During longer breaks, U_A and U_G have to be set to zero, but not the cathode heating voltage. If after the heating up time the beam is visible, a certain acceleration voltage $U (= U_A + U_G)$ is adjusted, the current through the coils switched on, and the tube cautiously rotated until the beam leaves the anode plate perpendicular to the direction of the magnetic field to complete a full circle. The current through the coils is now varied until the beam completes a circle with radius $r = 2, 3, 4$ and 5 cm, as indicated by different fluorescent marks within the tube.

The following are to be considered for the measurements:

For a given acceleration voltage (e.g. 150V), the current I through the coils has to be measured 4 times for each of the 4 radii. Afterwards, the mean current for each radius has to be determined.

The experiment has to be performed for at least 4 different acceleration voltages (e.g. 150 V, 200 V, 250 V and 300 V).

5. Analysis and Discussion

First, the constant C is to be calculated according eq. (10), then e/m according eq. (9) for each set of corresponding values U, r and the mean of I . Finally, the mean value of e/m is to be calculated and compared with the value known from literature.

An error calculation for e/m is to be performed (see appendix). Use the following errors: $\Delta r = \pm 0.5$ mm, $\Delta R = \pm 2$ mm, μ_0 and n without error. ΔU and ΔI depend on the grade of the instruments used (Please ask the assisting student). In addition, ΔI is determined from the standard deviation of the average value (statistical uncertainty as calculated for every 4 measured I values). The calculation is to be continued using the larger of both values ΔI . The maximum relative error of e/m is to be calculated according to the law of statistical error propagation for each

radius r and voltage U . Since e/m is the product of different powers of R , U , r and I , the calculation of the maximum relative error is quite simple (see appendix).

The results are to be discussed, and possible systematic errors (e.g. electrostatic charging of the tube, electrical potentials of the metal pieces) are to be taken into account.

6. Questions for Self-checking

- 1) How can free electrons be generated (except by thermal emission)?
- 2) How does the radius and the orbit frequency of the electron beam depend on the velocity or acceleration voltage?
- 3) How can one prove that the circular beam consists of negatively charged particles?
- 4) Which path do charged particles follow when entering the magnetic field in an angle?
- 5) Why do the values of e/m become smaller for very high acceleration voltages?
- 6) What is understood by the terms *impact ionization* and *recombination*?
- 7) Describe the microscopic mechanism of atomic light emission.

Exp. B 10: Experiment with X-Rays

1. Literature

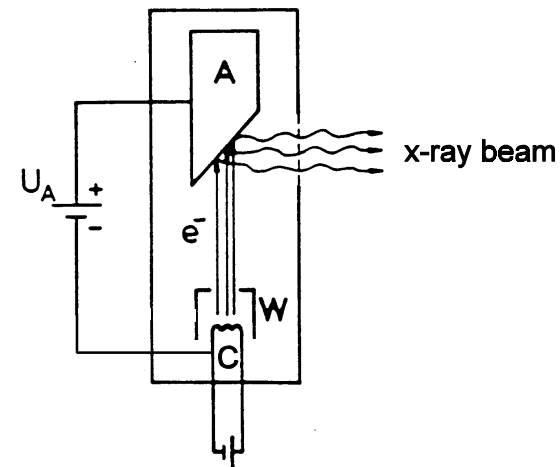
Keywords: Generation of x-rays, continuous x-ray spectrum, Bohr's model of the atom, Term scheme of atoms, characteristic spectrum, Moseley's law, absorption spectrum, crystal lattice, Bragg's law of reflection, detection of x-rays, x-ray counting tube

2. Basics

2.1 Continuous X-Ray Spectrum

X-rays can be generated using an (evacuated) x-ray tube (see schematic view in Fig. 1). Free electrons are generated by thermionic emission from a hot cathode C, focussed with the aid of a *Wehnelt cylinder* W and accelerated towards an anode A by a high anode voltage U_A .

When the electrons reach the anode, most of them gradually lose their kinetic



energy ($\frac{1}{2} mv^2 = eU_A$) during many collisions with the atoms of the anode material (eg. Tungsten or Copper) due to the Coulomb interaction (what is this?). The main part of their kinetic energy is converted simply into heat (i.e. atomic vibrational energy of the anode material), the remaining (smaller) part is converted into electromagnetic radiation. Among the latter, x-rays are generated within a continuous spectrum, the so called *Bremsstrahlung* ("white x-ray light"), first

discovered by W. Röntgen (1895).

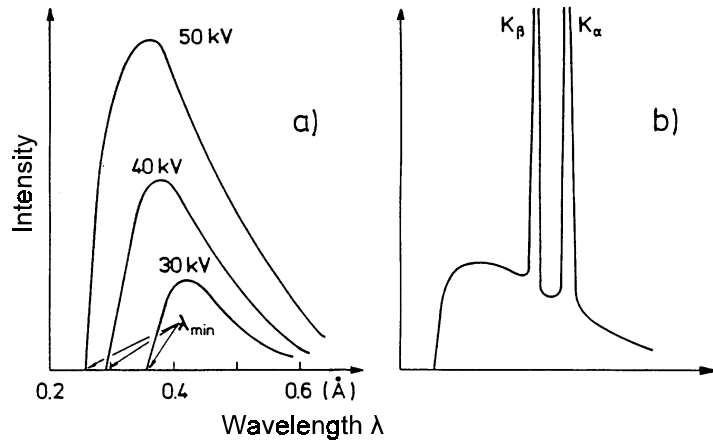


Fig. 2: a) continuous, b) continuous and characteristic x-ray spectrum (schematic)

Fig. 2a shows the intensity of continuous x-ray spectrum vs. wavelength for various accelerating voltages. The spectral intensity is independent of the anode material and exhibits a short wave length cutoff λ_{\min} , where the intensity drops to zero. At the cutoff wavelength λ_{\min} , depending only on the accelerating voltage U_A , the kinetic energy of an electron is converted into radiation energy within a single process according to the following relation

$$\frac{1}{2}mv^2 = eU_A = h\nu_{\max} = \frac{hc}{\lambda_{\min}} \quad (1)$$

This means that the maximum frequency ν_{\max} or the minimum wave length $\lambda_{\min} = c/\nu_{\max}$ of the continuous x-ray spectrum for a certain voltage U_A are solely determined by the charge of the electron e and Planck's constant h .

For many technical purposes, as in the present experiment, the acceleration voltages U_A are in the range 20 to 50 kV; thus the wave lengths are (according eq. 1) in the range of less than 1 \AA ($= 10^{-10} \text{ m} = 0.1 \text{ nm} = 100 \text{ pm}$).

2.2 Characteristic Spectrum

The x-ray spectrum observed often exhibits in addition to the continuous spectrum shown in fig. 2a very high intensities at certain discrete wave lengths, which are characteristic for the atoms of the anode material (see eg. fig. 2b). These *spectral lines* are caused by energetic transitions of electrons across the different energetic states within the atoms of the anode material.

The physical origin of the characteristic x-ray spectrum is easily explained with the aid of Bohr's model of the atom (compare with Fig. 3). At the impact of the

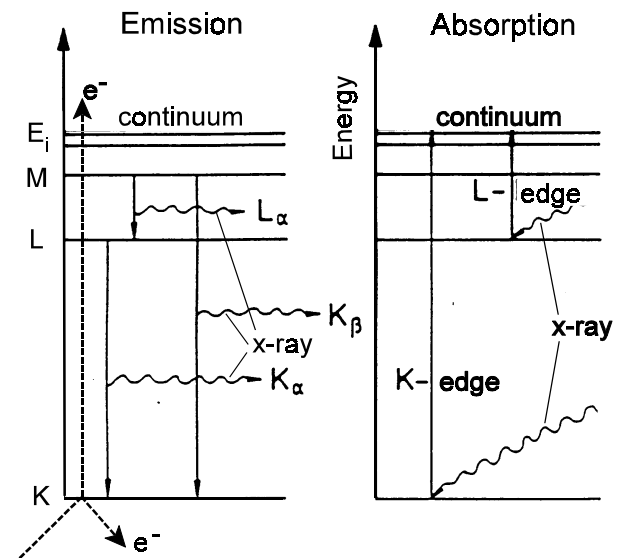


Fig. 3: term scheme with electronic transitions of the atom, left: at emission, right: at absorption of x-rays

accelerated electron with an atom of the anode material, another electron, e.g., from the (energetically lowest, inner) K-shell, is removed and struck into the high energy continuum of the metal. The remaining hole in the K-shell is filled up by an electron from, e.g., the (energetically next higher) L-shell. At this transition, a certain amount of energy ΔE corresponding to the energy difference between the L- and the K-shell is released and emitted as electromagnetic radiation with the energy quantum $\Delta E = h\nu = hc/\lambda$ (x-ray photon). At the transition from, e.g., L to

K, the so called K_α x-ray line appears in the spectrum.

Transitions from the M-shell to the K-shell generate the K_β line, similarly the transitions from M to L generate the L_α line, etc. The characteristic spectrum is thus generated according to the energy levels of the atoms of the anode material. Both frequency or wavelength of these x-ray lines depend on the square of the atomic number Z of the considered element. For the K_α line the eq.

$$\nu_{K_\alpha} = \frac{3}{4} (Z - \sigma)^2 Ry, \quad \sigma = 1 \quad (\text{Moseley's Law}), \quad (2)$$

holds with the *Rydberg frequency* $Ry = 3.29 \cdot 10^{15} \text{ s}^{-1}$. The *screening constant* σ accounts for the screening of the charge $+Ze$ of the nucleus by the remaining K-shell electron.

2.3 Absorption Spectrum

The weakening of x-rays in material is caused, first, by *classical* (elastic) *scattering* (changing the photon's momentum without changing its energy), second, by *Compton scattering* (partial transfer of photon energy to free or weakly bound electrons), and, third, by absorption. The capability of matter to absorb x-rays is described by the coefficient of absorption τ_A . It strongly depends on the atomic number Z (number of atomic electrons) and on the wavelength λ of the radiation ($\tau_A \sim Z^4 \lambda^3$). At the absorption of x-rays, the whole photon energy is transferred to inner shell electrons being excited into higher energy states. From Fig. 3, it is evident that at the absorption of x-rays, no characteristic absorption lines - like the emission lines - can be observed. Otherwise, this would mean that, e.g., for the K_α line the excitation of one K-electron to the L-shell would be necessary. This shell and the other shells, M, N etc., are usually occupied. Therefore, only the excitation into states with $E \geq E_i$ (ionization energy), into the so called *continuum* is possible, and an absorption spectrum, as shown schematically in Fig. 4, is observed. Besides the increase of $\tau_A \sim \lambda^3$, the spectrum exhibits K, L, M, .. absorption edges (corresponding the excitation energies of K, L, M, ..electrons) at certain wavelengths λ - characteristic for the absorbing material. The corresponding frequencies, e.g., ν_K for K-electron excitation, depend on the atomic number of the absorbing element similar to that given by eq. (2)

$$\nu_K = (Z - \sigma)^2 Ry, \quad \sigma \approx 1. \quad (3)$$

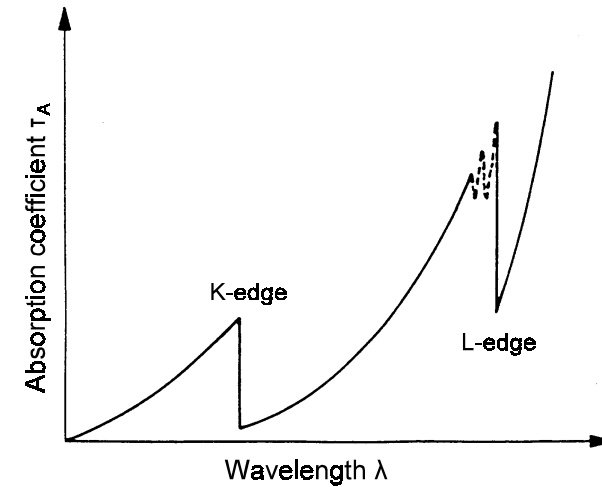


Fig. 4: X-ray absorption spectrum (schematic)

A suitable combination of anode material (in the present experiment copper, $Z=29$) and absorbing filter material (here nickel, $Z=28$) placed into the emitted x-ray beam, allows to obtain an x-ray emission spectrum (continuous and characteristic Cu spectrum), which is weakened very much at short wavelengths so that essentially only the K_α line remains, i.e., monochromatic x-rays with $\lambda_{K_\alpha} = 1.54 \text{ \AA} = 154 \text{ pm}$.

2.4 Detection of X-Rays

X-rays can be detected through the use of their ionizing effect. The intensity of the radiation can be measured, e.g., by measurement of the current pulses through ionization chambers or counting tubes, or by the determination of the blackening of photo plates. The functioning of a simple counting tube, the *Geiger-Müller-tube* (used in the present experiment), is illustrated schematically in Fig. 5:

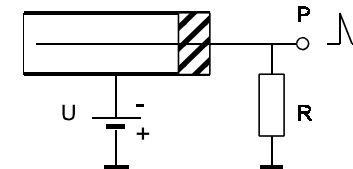


Fig. 5: Geiger-Müller tube

The tube consisting of a metal housing with a thin foil at one end is filled with inert gas (e.g. Argon) and some additional alcohol vapor. A thin steel wire (insulated from the housing) is stretched along the center of the tube and connected through a resistor $R (>1M\Omega)$ to ground. The housing is negatively charged against the center wire by a high voltage, $U \approx 500V$. If the x-rays traverse the tube, they ionize single gas atoms, and the electrons are accelerated toward the center wire causing more ionization on their way resulting finally into a complete discharge of the tube (limited in time by the alcohol vapor). The discharge current across the resistor generates a voltage pulse at P which is counted.

2.5 Bragg Reflection of X-Rays

The wave nature of x-rays was first established in 1906 by Barkla, who was able to exhibit their polarization. In 1912 a method was devised to actually measure the wavelengths. Max v. Laue recognized that the wavelengths hypothesized for x-rays were about on the same order of magnitude as the spacing between adjacent atoms in crystals. He, therefore, used crystals to diffract x-rays with their regular lattices acting as a kind of a three dimensional grating. The diffraction of x-rays in crystals and their subsequent interference, characteristic for their wave nature, results in a diffraction pattern dependent on the x-ray wavelength and the spacing between the atoms. The regular arrays of atoms in crystals (in the present experiment NaCl, Sodium chloride with simple cubic symmetry) "reflect" x-rays with a single wavelength λ only at certain angles. Since the radiation penetrates the crystals, the "reflection" occurs at many atomic planes (selected families of atomic arrays) and actually depends on the distance d between the planes.

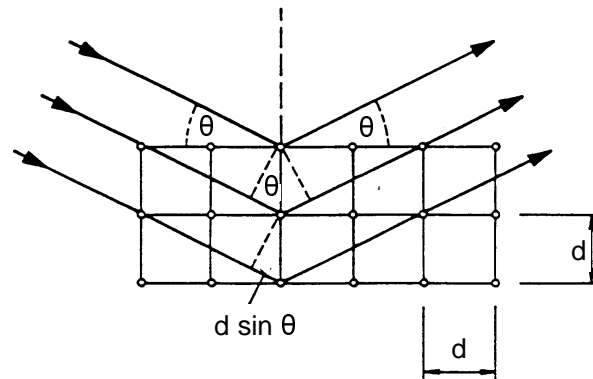


Fig. 6: Bragg reflection at a single crystal

This is illustrated schematically in Fig. 6. A monochromatic beam falls onto the parallel p planes of atoms (separated by distance d) with the angle θ and is "reflected" at the same angle. Constructive interference of the reflected beam, i.e., the so called *Bragg reflection*, only takes place if the path difference between x-rays reflected at adjacent planes is $\lambda, 2\lambda, 3\lambda, \dots$, i.e., $n\lambda$, where n is an integer. According to the geometric situation sketched in Fig. 6, the condition for Bragg reflection is simply

$$n\lambda = 2d \sin \theta_n \quad (\text{Bragg's Law}) \quad (4)$$

The "reflection" of radiation at atoms is actually a scattering and an interference process. The electronic shells ($-Ze$) of every atom of the crystal subjected to the field of the electromagnetic radiation are forced to vibrate around the nuclei ($+Ze$) and thereby emit radiation with the same frequency as the incoming radiation in almost all directions. This radiation is intensified by constructive interference at angles given according eq. (4) and annihilated at other angles. Since the interference maxima, if plotted vs. reflection angle, are very sharp, one gets the impression that the radiation is reflected at certain angles $\theta_1, \theta_2, \dots$.

Using Bragg's law one can determine the spacing d of an unknown lattice if the wavelength λ of the monochromatic x-ray beam is known, or vice versa λ , if the lattice spacing d of the crystal is known. The first condition is important for the determination of unknown crystal structures. Generally, the application of x-rays is of high technological importance in studying materials.

3. Experimental Task

The Bragg angles θ of a NaCl single crystal are to be determined using an x-ray apparatus (with Cu-anode and Ni-filter: $\lambda_{K\alpha} = 154 \text{ pm}$). The distance d of the reflecting atomic planes (here the lattice constant) is to be calculated from the results and compared with the value known from literature, $d_{\text{NaCl}} = 282.0 \text{ pm}$.

4. Experimental Setup

The experimental setup is shown in Fig. 7. The x-ray tube T is mounted under a lead glass screening dome within a round experimental chamber. (Why lead glass?). Having passed through the aperture A, the x-ray beam is incident on the NaCl crystal Cr (in the center of the apparatus) at an angle θ with the crystal

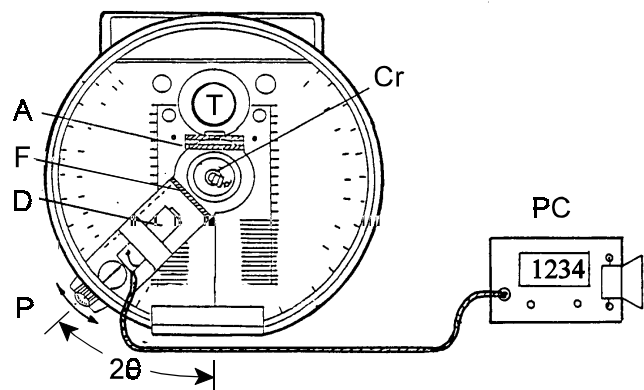


Fig. 7: X-ray apparatus with x-ray tube T, aperture A, crystal Cr, filter F, detector D, pointer P(2θ) on movable arm, pulse counter PC

surface (parallel to the Bragg planes). The beam reflected at the angle 2θ passes a nickel filter F and enters the detector D. Both θ and 2θ are selected by a movable arm on which the detector is mounted. The intensity of the reflected x-ray beam is monitored by a pulse counter PC with start and stop switches and a reset button. For acoustic monitoring of the pulses, a small loudspeaker is attached. Since the intensity of the x-ray beam is very low, the screening of the experimental chamber by a lead/aluminum plate and the plexiglass cover is sufficient to avoid any personal hazard due to ionizing effects of the primary and scattered beams. The apparatus can be operated only if the cover is closed. A red lamp indicates the status of operation (high voltage switched on).

5. Carrying out the Experiment and Evaluation of Results

At first, the pulse rate N_0 (s^{-1}) corresponding to the intensity of the background radiation is to be determined using a counting time of 100 s while the apparatus is switched off. This rate has to be subtracted from the rates determined later. Then, the apparatus is to be switched on *only by the assisting student*.

In the following, the rate $N(2\theta)$ is to be measured in the range $20^\circ < 2\theta < 120^\circ$, starting at small angles and increasing 2θ using intervals of 4° at a time. The counting times depend on the adjustment of the individual x-ray apparatus used and are evaluated by the assisting student. For a more accurate determination of the Bragg angles $\theta_1, \theta_2, \theta_3$, $N(2\theta)$ is determined within a second run at increasing angles in the close vicinity of the rate maxima observed before, but now, using smaller intervals of angles ($2\theta = 2^\circ; 1^\circ; \dots$). For the purpose of control during the

experiment, the rates determined are to be plotted immediately on a graph $N(2\theta)$.

Hint: Use the acoustic monitoring to determine the Bragg angles while simultaneously changing 2θ slowly.

For the evaluation of the results, the rate $N - N_0$ is plotted vs. θ using millimeter scale paper. From the maxima at θ_n , ($n = 1, 2, 3$), the Bragg plane distance d_n is calculated using eq. (4) with $\lambda = 154$ pm for the Cu K_α radiation. The average value of d_n , d (here the lattice constant of NaCl) is to be calculated and compared with the value d_{NaCl} known from the literature. The relative deviation

$$\Delta = \frac{|d - d_{\text{NaCl}}|}{d_{\text{NaCl}}}$$

is to be compared with the relative error $\Delta d/d$ resulting from a typical error $\Delta\theta = 0.25^\circ$ in determining θ_n .

6. Questions for Self-checking

- 1) How does an x-ray tube function?
- 2) What are the various types of x-ray spectra (sketch)?
- 3) Why does the continuous spectrum have a short wavelength cutoff?
- 4) What causes the characteristic spectrum?
- 5) What is stated in Moseley's law ?
- 6) What does the x-ray absorption spectrum look like? How does it occur?
- 7) How can one prove that x-rays are electromagnetic waves?
- 8) What does Bragg's law state?
- 9) How are x-rays detected?
- 10) How does a counting tube function?

Exp. D 5: Spectral Analysis

1. Literature

Keywords: Prism spectrometer, grating spectrometer, light emission from gases, diffraction, law of refraction, dispersion

2. Basics

2.1 Line Spectrum

The most simple and best known model to describe the microscopic structure of atoms is *Bohr's atomic model*. This model, postulated in 1913 by N. Bohr for the description of the Hydrogen atom, connects the laws of classical and quantum mechanical physics. As in a microscopic planet system, the atomic electrons circle around the nucleus on certain fixed trajectories, i.e., the electrons can move only in certain energetically allowed states (quantum states). The energy of these states increases with increasing distance between electron and nucleus. The energetic difference of neighboring trajectories, however, decreases.

To transport an electron from its most inner trajectory around the nucleus, the energetically lowest possible state with energy E_1 (ground state), to the neighboring trajectory, the energetically next higher state with energy E_2 (excited state), a discrete energy $\Delta E = E_2 - E_1$ is necessary. On the other hand, this energy difference ΔE is released if the electron returns from the second to the first trajectory. Under certain conditions, the released energy can be emitted as electromagnetic radiation with a fixed frequency ν which is given by the relation $\Delta E = E_2 - E_1 = h\nu$. The constant h is Planck's constant ($h = 6.625 \times 10^{-34} \text{ Ws}^2$). With the relation $\lambda = c/\nu$, the wavelength λ of the light is fixed as well, because the velocity of light (in free space) c is fixed (What is the value of c in free space?).

Atoms heavier than the Hydrogen atom consist of $Z > 1$ positive nucleus charges Ze , i.e., Z protons (Z =atomic number), additional neutrons (without any electrical charge), and because of electric neutrality, Z electrons as well. The electrons are distributed around the atomic nucleus on trajectories according to certain rules. Their energies vary from element to element. This means that the transition of an electron from the excited to the energetically lower energy states releases a radiation energy $\Delta E = h\nu$ specific for the considered element. If many transitions between the different states of an element are occurring simultaneously, an *emission line spectrum* (light intensities at discrete frequencies) characteristic for

the considered element is observed. One should notice that transitions between states of shells fully occupied with electrons are not possible. The optical observable line spectra are thus caused by transitions of electrons between the outermost unoccupied or partially occupied states of the atoms. The wavelengths of the emitted light characteristic for the considered element are in the visible range of the electromagnetic spectrum (400-800 nm) or within the neighboring infrared or ultraviolet.

How gases (or vapors) are forced to emit light, i.e., to emit characteristic lines, can be seen in the example of neon advertising lights. Gases, vapors of liquids or evaporated solid materials are immersed into an evacuated tube (at reduced pressure) and brought to a high voltage. The gas atoms are excited due to collisions with ions and free electrons (accelerated by the electric field), i.e., the atomic electrons are lifted into energetically higher states. At subsequent backward transitions, light with characteristic spectral lines is emitted.

To analyse the light emitted by the gas atoms, the different spectral colours have to be selected. This can be done by using an optical *grating spectrometer*, where the light is diffracted and emitted into different directions selected into different colours by constructive interference. The other more preferred method for the analysis of light is to use a *prism spectrometer* where the *dispersion* of light by a prism is exploited.

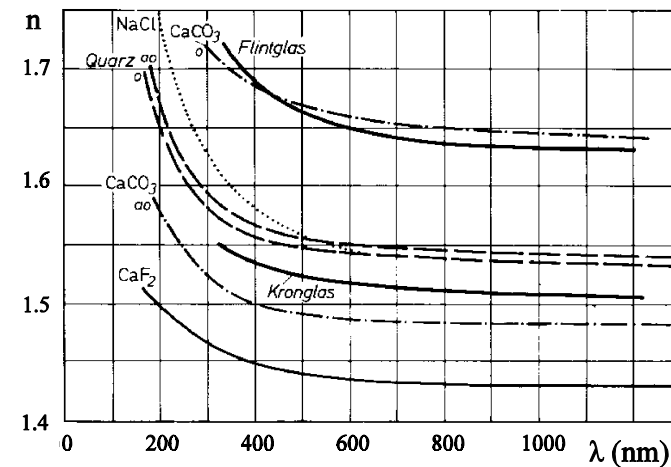


Fig. 1: Dispersions curves $n(\lambda)$ for different prism materials

Light of different colour (frequency) is refracted differently by the prism. The dependence of the index of refraction n on the frequency ν or, correspondingly, on the wavelength λ is characteristic for the prism used (see dispersion curves, Fig. 1). Once the dependence $n(\lambda)$ for a certain prism has been determined using a known spectrum (calibration curve), the frequencies, i.e., the spectrum of another unknown light source can be determined by measuring their index of refraction.

The index of refraction $n(\lambda)$ exhibits for many prism glasses in the visible range of the spectrum (400-800nm) the so called *normal dispersion*. In this case n decreases with increasing wavelength λ (see Fig. 1). This dependence results from the (frequency dependent) electronic polarizability of the atomic shells of the prism glass, which are forced to vibrate in the electromagnetic field of the incoming light.

2.2 Prism Spectrometer

Fig. 2 shows schematically the elements of the prism spectrometer. The light to be investigated is an illuminated slit. The divergent light ray emerging from the slit traverses the slit tube, is parallelized by a lens at the tube end, and is dispersed by refraction at the prism surfaces. The rays emerging from the prism enter a telescope and are focused by the objective lens to form differently coloured slit images (spectral lines, characteristic for the light source) side by side in the focal plane of the telescope where they are viewed with the aid of the ocular lens. The positions s of the slit images on a horizontal scale are a measure for the refraction angle at the prism and are dependent on the wavelength λ due to the dispersion $n(\lambda)$. Instead of fixing a scale directly in the focal plane of the telescope, the image of a scale is projected on this plane using a lighted scale tube.

The prism spectroscope is not suitable for absolute measurements of wavelengths. Therefore, the scale has to be calibrated with spectral lines of known wavelengths. For this purpose, e.g., a helium lamp is used. A graph of the determined scale values s_i against a suitable function of the known wavelengths λ_i of the helium light results in a calibration curve (angle dispersion curve). With this curve the wavelengths of light from unknown elements can be determined, and, thus, the elements can be identified. The angle dispersion curve $s(\lambda)$ depends both on the dispersion $n(\lambda)$ as on the shape of the prism and the adjustment of the spectrometer. The detailed knowledge of this context is not necessary for carrying out the present experiment.

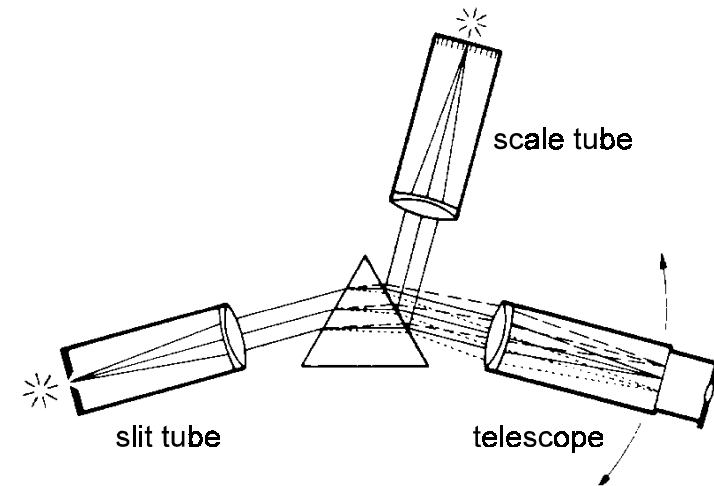


Fig. 2: Prism spectrometer

3 Experimental Task

- 1. Task:** With the aid of the scale image in the telescope, the positions s of the visible spectral lines of helium are to be determined and referred to the wavelengths λ using the spectral table (Tab. 1).
- 2. Task:** The observed scale positions s are to be plotted against $1/\lambda^2$ for the corresponding wavelengths λ of helium, and the data points are to be connected by a smoothed curve (angle dispersion curve).
- 3. Task:** The emission spectra of two other lamps containing unknown elements are to be determined and analysed using the dispersion curve obtained in Task 2. The unknown elements have to be identified using the spectral table (Tab. 1).

4. Carrying out the Experiment and Analysis

First, the scale illumination is switched on and adjusted. The scale image is focused in the telescope and the slit of the adjusted spectrometer is illuminated by a helium lamp. After a short warm up period, the slit images (spectral lines) become visible in the telescope.

To calibrate the spectrometer, the scale positions s of the spectral lines of helium, visible in the telescope, are precisely determined (by interpolation, if necessary). The very weak lines are neglected. The scale positions determined are plotted in a graph $s(1/\lambda^2)$ on millimeter scale paper, having taken the corresponding wavelengths λ from the spectral table. Connection of the data points by a smoothed curve should result in a line with only little curvature. The connection of the points using a ruler should, therefore, not introduce serious errors.

Thereafter, the lamps with unknown elements are positioned in front of the slit one after the other and - as done with the helium lamp - the scale positions of their spectral lines carefully determined. With the aid of the calibration curve, the wavelengths are determined. Comparing the results with the various wavelengths listed in the spectral table, the elements (or combinations of elements) are identified.

5. Discussion of Errors

Estimate the error $\Delta\lambda$ in the determination of wavelengths of the unknown substances using the calibration curve $s(\lambda)$ resulting from an error $\Delta s = \pm 0.01$ scale div. of the line positions in the telescope. Compare this value with the average difference between λ resulting from comparison of the value determined from the calibration curve and that taken from the spectral table, Tab. 1. How can the precision of the wavelength determination be improved?

6. Questions for Self-checking

- 1) Which excitation processes cause the generation of light?
- 2) Which physical quantities characterize the properties and propagation of light?
- 3) What determines the colour of light, frequency or wavelength?
- 4) What is the law of refraction?
- 5) What is diffraction and refraction?
- 6) What is dispersion?
- 7) How does a telescope function?

Tab. 1: Spectral Lines of some Elements

Element (Symbol)			Element (Symbol)		
λ (nm)	Colour	Brightness	λ (nm)	Colour	Brightness
Hydrogen (H)			also Sodium (Na)		
656.28	red	strong	567.57	yellowgreen	weak
486.13	bluegreen	medium	567.02	yellowgreen	weak
434.05	violet	medium	Potassium (K)		
410.17	violet	weak	769.90	darkred	strong
Helium (He)			766.49	darkred	strong
706.54	darkred	weak	404.72	violet	medium
667.82	red	strong	404.41	violet	medium
587.56	yellow	very strong	Cadmium (Cd)		
504.77	green	weak	643.85	red	strong
501.57	green	medium	635.99	yellowred	weak
492.19	bluegreen	medium	508.58	green	strong
471.31	blue	weak	479.99	bluegreen	strong
447.15	blue	strong	467.82	blue	strong
438.79	violet	weak	441.46	blue	medium
Lithium (Li)			Mercury (Hg)		
670.78	red	strong	579.07	yellow	very strong
610.36	yellowred	medium	576.96	yellow	very strong
460.29	blue	weak	546.07	green	strong
Sodium (Na)			491.60	bluegreen	medium
616.08	yellowred	medium	435.84	blue	strong
615.42	yellowred	medium	407.78	violet	medium
589.59	yellow	strong	404.66	violet	medium
589.00	yellow	medium			

Exp. D 8: Radiation from Heated Solids

1. Literature

Keywords: heat radiation, black body (cavity) and gray body radiation, absolute (Kelvin) temperature, spectral exitance, absorption, Kirchhoff's law, Planck's radiation formula, Wien's law of radiation, Wien's displacement law, Stefan-Boltzmann law, pyrometer, thermopile

2. Basics

2.1 Radiation of Heated Solids (Kirchhoff's law)

Every substance with (absolute) temperature T above zero (measured in *Kelvin degrees*) emits radiation (electromagnetic waves) with total power and spectral intensity (spectral width) depending essentially on its temperature. The spectral width becomes obvious if the sun is compared with a radiator. The sun (surface temperature $T \approx 5700$ K) emits radiation in the ultraviolet and visible range (wavelength $\lambda = 400 - 800$ nm) and also in the infrared range (heat). A radiator with $T \approx 350$ K, however, emits radiation only in the infrared range and this, empirically, with very much less intensity. While 1 m^2 surface of the sun emits into the forward hemisphere a total (over all wavelengths integrated) radiation flux $\Phi = 6.1 \times 10^7$ W, 1 m^2 surface of the radiator with $T = 350$ K emits a flux, which is by 5 orders of magnitude (10^5) smaller than that of the sun.

The spectral radiation flux $d\Phi$, emerging from the surface area element dA of a body with temperature T in the wavelength interval between λ and $\lambda + d\lambda$ into the forward hemisphere is given by:

$$d\Phi = E(\lambda, T) dA d\lambda \quad (1)$$

The quantity $E(\lambda, T)$ is called *spectral exitance* (or *spectral emittance* or *spectral flux density*). The sun and the radiator are therefore distinguished, as demonstrated above, by their exitances as well in their spectral distribution as in their total radiation power.

Radiation is not only emitted from solids but also absorbed and reflected (transmission is not considered here). Correspondingly, the *absorption* A of a body is defined as the ratio of absorbed to incident radiation flux, the *reflectivity* R as the ratio of reflected to incident radiation flux. A and R are by definition

dimensionless quantities and obey the relation

$$R = 1 - A, \quad (2)$$

if the transmission D is zero. The quantities A and R depend on the wavelength of the incident radiation, on the temperature of the body and - this is a disadvantage - on physical properties of the body as, e.g., on its surface quality ($A(\lambda, T, \dots)$, $R(\lambda, T, \dots)$).

To find relations which are independent of material properties, two bodies 1 and 2 are considered placed face to face at a close distance (compared to their size). When the bodies have reached thermal equilibrium (i.e. they have the same temperature), there is an equilibrium of the exchanged radiation energy as well. If body 1 emits the flux Φ_1 toward body 2 and reflects the unabsorbed fraction $(1 - A_1)$ of the flux Φ_2 emitted from body 2, the same holds vice versa for the radiation emitted and reflected from body 2. In equilibrium, both quantities are equal, i.e.,

$$\begin{aligned} \Phi_1 + (1 - A_1)\Phi_2 &= \Phi_2 + (1 - A_2)\Phi_1 \\ \text{or} \quad \frac{\Phi_1}{A_1} &= \frac{\Phi_2}{A_2} \end{aligned} \quad (3)$$

If between the two bodies 1 and 2 a spectral filter is placed, the condition within thermal equilibrium are not altered, and the same relation (3) must hold also for the spectral flux densities with eq. (1)

$$\frac{E_1(\lambda, T)}{A_1(\lambda, T)} = \frac{E_2(\lambda, T)}{A_2(\lambda, T)} = f(\lambda, T) \quad \text{Kirchhoff's law} \quad (4)$$

This relation is valid for any two bodies. The ratio $E(\lambda, T)/A(\lambda, T)$ must be, therefore, independent of all material properties and does only depend on the wavelength λ and on the temperature T . This is known as Kirchhoff's law.

A particular position within the description of radiation processes has the so called *black body*. A black body absorbs all the incident radiation, i.e., it has the absorption $A_s = 1$ for all wavelengths and temperatures. If this is inserted into eq. (4), one realizes that the function $f(\lambda, T)$ equals the spectral exitance $E_s(\lambda, T)$ of the black body, and Kirchhoff's law is formulated as

$$\frac{E(\lambda, T)}{A(\lambda, T)} = E_s(\lambda, T) \quad (5)$$

There is no real black body in nature but it can be realized experimentally within good approximation by a hollow body with a small opening. The radiation entering the hole suffers multiple reflections inside the body and has, therefore, little chance to be reflected out again so that the enclosure acts as a perfect absorber.

A *gray body* is by definition a body which has an absorption $A < 1$ independent of wavelength and temperature. In contrast to the black body, many radiating materials behave like gray bodies in nature (as well the lamp used in the present experiment), however, often only for a restricted spectral range.

2.2 Black Body Radiation Law

In 1900 Max Planck proposed a formula for the spectral exitance $E_s(\lambda, T)$ of the black body, which was based on the assumption that the atoms emitting the radiation behave like tiny oscillators exchanging energy with the radiation only in finite quantities. This means that the oscillators absorb and emit radiation energy not in arbitrarily small amounts but only within multiples of the *photon energy* $\Delta E = h\nu = hc/\lambda$ (ν = radiation frequency, h = Planck's constant).

The oscillators interacting in the cavity radiator are at rest at zero temperature T (zero point energy neglected) and are increasingly excited with the rise of temperature T or thermal energy $k_B T$, thus exchanging increasingly energy with the radiation (k_B : Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K).

The spectral exitance $E_s(\lambda, T)$ of the black body is given by *Planck's law*

$$E_s(\lambda, T) = 2\pi \frac{hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} \quad (6)$$

or

$$E_s(\lambda, T) = 2\pi \frac{c_1}{\lambda^5} \frac{1}{e^{\frac{c_2}{\lambda T}} - 1} \quad (7)$$

with the constants $c_1 = hc^2 = 5.96 \times 10^{-17}$ W m²
 $c_2 = hc/k_B = 1.439 \times 10^{-2}$ m K.

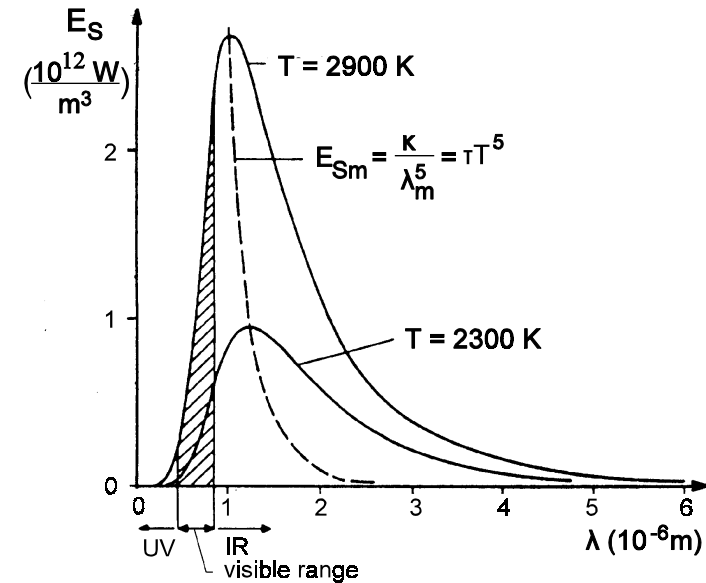


Fig. 1: Black body radiation curves $E_s(\lambda, T_i)$ acc. to Planck's law (—) for 2 different temperatures, maxima $E_{Sm}(\lambda_m, T)$ (---)

Fig. 1 shows two curves of spectral exitances $E_s(\lambda, T)$ of a black body for different temperatures according to Planck's law. (visible spectral range is hatched). Note the units of E_s !

In the limit $hc/\lambda \gg k_B T$, e.g., for the visible range and temperatures $T \leq 3000$ K, the 1 in the denominator of eq. (7) can be neglected, and Planck's formula reduces to *Wien's radiation law* valid for the short wave range of the spectrum so that

$$E_s(\lambda, T) = 2\pi \frac{c_1}{\lambda^5} e^{-\frac{c_2}{\lambda T}} \quad (8)$$

This law is useful for pyrometric applications (see below), because the spectral exitance of a black body radiator is a definite function of its temperature (and vice versa).

Every curve given by Planck's formula exhibits a maximum $E_{Sm}(\lambda_m, T)$ at a certain

wavelength λ_m as shown in Fig. 1. With the condition $dE_s(\lambda, T)/d\lambda = 0$ at the maximum from eqs. (7,8), *Wien's displacement law* is found as

$$\lambda_m T = \text{const.} = w = 2.897 \times 10^{-3} \text{ mK} . \quad (9)$$

The maximum E_{Sm} of the exitance spectrum of the sun is observed at the wavelength $\lambda_m = 480 \text{ nm}$ (green). This corresponds to a black body with $T = 6000\text{K}$. With rising temperature, the maximum shifts to shorter wavelengths (and vice versa with falling temperatures). The maxima of the radiation isothermes, $E_s(\lambda, T=\text{const.})$, lie on the hyperbola $E_{Sm}(\lambda_m)$ (shown in Fig. 1 as dashed curve), resulting from eq.(7) when T is replaced by w/λ_m of eq. (9) so that

$$E_{Sm}(\lambda_m) = \frac{\kappa}{\lambda_m^5} \quad \text{with } \kappa = 2.63 \times 10^{-18} \text{ W m}^2 . \quad (10)$$

Correspondingly, the exitance maximum is given as a function of temperature by

$$E_{Sm}(T) = \tau T^5 \quad \text{with } \tau = 1.29 \times 10^{-5} \text{ W m}^{-3} \text{ K}^{-5} . \quad (11)$$

This means that at the temperature $T_2 = T_1/2$ of a given temperature T_1 , the exitance maximum has only 1/32 of the value at T_1 .

The whole flux (radiation power) emitted from the unit area of the black body into the forward hemisphere Φ/A results from eqs. (1) and (6) by integrating over all wavelengths so that

$$\frac{\Phi}{A} = \int_{\lambda=0}^{\infty} E_s(\lambda, T) d\lambda = \sigma T^4 \quad \text{Stefan-Boltzmann law} \quad (12)$$

with $\sigma = 2\pi^5 k_B^4 / 15 c^2 h^3 = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

2.3 Pyrometer

Pyrometers are instruments for measuring high temperatures ($T > 1000\text{K}$) without coming in contact with the body itself, e.g. metallic melts. They are predominantly used in cases where the application of thermoelements is not

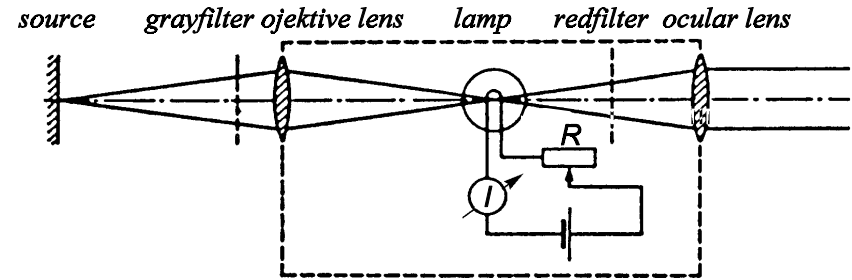


Fig. 2: Pyrometer, schematic

possible. A pyrometer (see Fig. 2) consists of a telescope with an electrically heated tungsten wire (lamp) positioned in the focal (image) plane. The current through the lamp is varied by the resistor R until the brightness (as viewed with the aid of the ocular lens of the telescope) equals that of the image of the radiation source (melt, glowing body, lamp). This brightness corresponds to a certain current I flowing through the tungsten wire.

Since a red filter is positioned into the light ray, the spectral range is limited to a small interval $\Delta\lambda$ around 660 nm. An additional gray filter (light attenuator) expands the brightness (temperature) range of the instrument.

The brightness of radiation emitted by a heated body and perceived by the human eye is proportional to the exitance of the body in the considered spectral range of emission. But even if both the radiation source investigated and the pyrometer lamp emit radiation of equal spectral exitance (i.e. brightness), they may have different temperatures, since - as the discussion within the previous chapter has shown - the exitance of the radiation source depends, even within a narrow spectral range, not only on the temperature but also on its absorption. To calibrate a pyrometer so that the lamp current is described as a function of the temperature of the measured radiation source, the radiation of a black body is used. The registered current, i.e. temperature, is called *black temperature* T_s . The temperature T_s is equal to the true temperature of the investigated object only if the object emits radiation like a black body. In practice, this is not the case - as mentioned earlier-, since (for a narrow spectral range) one deals in general with gray radiation sources ($A = \text{const} < 1$).

If a black and a gray body have the same temperature, the spectral exitance of the

gray body is diminished by its absorption $A < 1$ (see eq. (5) when compared to that of the black body. If a black and a gray body exhibit the same brightness within a certain spectral range, the true temperature of the gray body must be, therefore, always higher than T_s (being the true one of the black body).

To establish a relationship between T and T_s , it is sufficient for practical applications to use Wien's radiation law, eq. (8), since λT is sufficiently small. According to Kirchhoff's law, eq. (5), for equal brightnesses (spectral exitances) of both the investigated object (E) and the tungsten wire of the pyrometer (E_s), the following relation holds:

$$E(\lambda, T) = A E_s(\lambda, T) = E_s(\lambda, T_s) \quad (13)$$

By using eq. (8) to express the ratio $A = E_s(T_s)/E_s(T)$, it follows that

$$\frac{1}{T} - \frac{1}{T_s} = \frac{\lambda}{c_2} \ln A \quad (14)$$

This equation is exactly valid for a gray radiation source. It can be applied, however, with sufficient accuracy to metals having only little temperature dependence of their absorption A . Since metals normally have a high reflectivity and, therefore, a small absorption, the deviation between T and T_s may be considerable.

Resolving eq. (14) with respect to T , we finally get

$$T = T_s \frac{1}{1 + T_s \frac{\lambda}{c_2} \ln A} = T_s k(T_s) \quad (15)$$

The correction factor $k(T_s)$ for the tungsten lamp used in the present experiment is given in chapter 5.

2.4 Thermopile

A thermopile is shown schematically in Fig. 3. It consists of a hollow metal cylinder with an aperture at one end into which the radiation R enters. Having been focussed by reflection at the conical inside walls of the cylinder, the light ray is absorbed in a (blackened) sensing element S . The temperature of S increases by a (small) amount ΔT above the temperature of the surrounding housing H . The

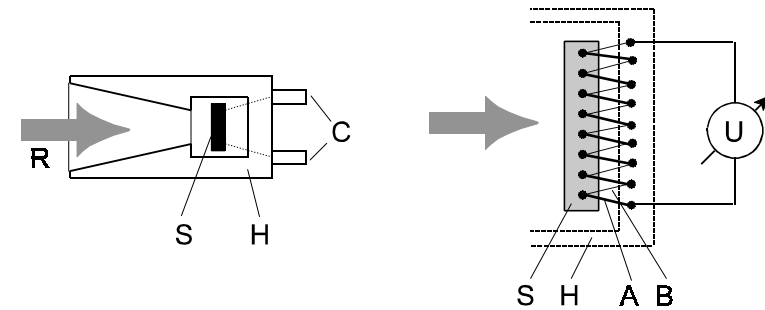


Fig. 3: Thermopile schematic, left: profile indicating radiation R , sensing element S , housing H , electrical connector C ; right: electrical details with array of thermocouples A, B and voltmeter U

temperature difference ΔT between S and H is sensitively registered by an array of

thermocouples A, B , generating a thermovoltage U_{Th} which is measured with a millivoltmeter. Since for small temperature differences U_{Th} is proportional to ΔT , the measured thermovoltage U_{Th} is also proportional to the absorbed radiation power (radiation flux Φ) such that

$$U_{Th} = c \Phi, \quad c = 0.16 \frac{\text{mV}}{\text{mW}} \quad (\text{calibration by manufacturer}).$$

3. Experimental Task

The radiation flux emitted from a tungsten wire incandescent lamp and registered by a thermopile is to be determined as a function of the wire temperature measured with a pyrometer. Using the data obtained, the validity of the Stefan-Boltzmann law has to be verified.

4. Carrying out the Experiment

The experimental setup shown in Fig. 4 consists essentially of a tungsten wire incandescent lamp Q , two apertures B_1 and B_2 , the thermopile TS , and the pyrometer P . The lamp, the apertures, and the thermopile are mounted on an optical bench separated by certain distances. The lamp wire represents the gray

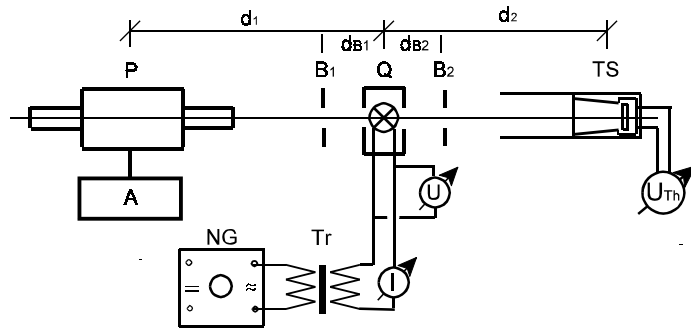


Fig. 4: Experimental setup to measure the temperature and the radiation flux from a tungsten wire incandescent lamp Q: P pyrometer with meter unit A; TS thermopile with voltmeter U_{Th} ; NG AC-voltage source with transformer Tr; $B_{1,2}$ apertures to limit the radiation to the pyrometer and the thermopile

body radiation source and the thermopile the radiation receiver. The aperture B_2 has two functions. On one hand, it limits the solid angle from which radiation is incident on to the receiver, on the other hand, it shields the receiver from background radiation.

The temperature of the tungsten wire is measured using a pyrometer. The pyrometer is oriented toward the lamp, and the wire image is focussed in the plane of the pyrometer lamp (focal plane). Having actuated the redfilter, the current through the pyrometer lamp is adjusted (coarse and fine) until the lamp wire has the same brightness as the tungsten wire. To increase the range of measured temperatures T_s , a grayfilter can be actuated. The temperature range of the pyrometer can be varied depending on the positions of switches, both on the pyrometer and the display unit as indicated in the following table:

Temperature range t_s (°C)	Switch positions	
	Display unit	Pyrometer
650 - 1450	I	1 (without grayfilter)
1450 - 3000	II	2 (with grayfilter)

The tungsten incandescent lamp is connected to an AC voltage source via a transformer and operated at 5V maximum. By varying the voltage, at least 8 different temperatures of the lamp wire are to be adjusted in sequence of increasing and decreasing voltages. Three pyrometer adjustments are to be made for every voltage (temperature) and the average value for the black temperature t_s (in °C) is to be determined.

The thermovoltage U_{Th} created in the thermopile by the absorption of radiation is to be measured as a function of t_s for the fixed distance $d_2 = 50$ cm between lamp and thermopile. During the measurement, one should take care that no convection of air or heat radiation (e.g. from the experimenter) interferes the operation of the thermopile.

5. Evaluation of Results

The temperature t_s (in °C) indicated by the pyrometer display corresponds to the temperature of a black body emitting the same spectral exitance as the investigated hotter gray body. To determine the true temperature T of the gray body (tungsten), the value T_s (in K) has to be corrected with a temperature dependent factor $k(T_s)$ (compare e.g. eq. (15)) according to

$$T = k(T_s)T_s \quad \text{with} \quad T_s = t_s + 273 \text{ K}$$

The values for $k(T_s)$ in the spectral range considered (redfilter) are to be taken from the following table by interpolation.

T_s (K)	k	T_s (K)	k
800	1.028	1600	1.063
1000	1.036	1800	1.073
1200	1.045	2000	1.083
1400	1.054	2200	1.092

Table: Correcting factor $k(T_s)$ for tungsten

From each dataset (U_{Th} , t_s) determined for a fixed distance d , the dataset (Φ , T (in K)) is to be calculated. Φ is to be plotted vs. T^4 in a diagram, and straight lines are to be drawn through the data points separately for each d . The results and possible sources of errors are to be discussed.

5. Questions for Self-checking

- 1) What is heat radiation?
- 2) What does Kirchhoff's law imply?
- 3) How do absorption and reflectivity depend on each other if the transmission is zero?
- 4) What is a (radiating) black body?
- 5) How is Kirchhoff's law modified when the spectral exitance of the black body is taken into account?
- 6) What is a (radiating) gray body?
- 7) What is Planck's radiation formula?
- 8) At which limiting conditions can Wien's radiation law be derived from Planck's formula?
- 9) What is Wien's law of displacement?
- 10) What is the radiation law of Stefan-Boltzmann and how can it be obtained from Planck's law?
- 11) How does a pyrometer function?
- 12) What is the black and the true temperature of a radiating gray body?
- 13) Is the true temperature higher or lower than the black temperature?
- 14) How does a radiation receiver containing a thermopile function?

Appendix: Analysis of Errors

1. Systematic and Statistical Errors

Every result of a measured physical quantity inevitably contains an error. To evaluate the experimental result obtained, it is, therefore, necessary to give an estimate of the numerical error(s) inherent to the experimental quantity.

There are different kinds of errors: systematic and random, i.e., statistical. Systematic errors are caused by the measurement system and can be recognized from the fact that the measured numerical value is strictly too large or too small as compared to those obtained when using other methods of measurement or theory. To minimize systematic errors, one has to change the experimental setup, i.e., the apparatus or the measuring procedure. Alternatively, the numerical result has to be corrected properly to account for the systematic errors involved in the measurement.

A statistical error arises due to random positive and negative deviations of the actually measured value from the mean or precise experimental value. If, e.g., the measured length of a distance is not exactly that of a certain number of scale divisions, one has to estimate a more accurate value by interpolation, thus a statistical error may result. On the other hand, the displayed (analog or digital) value of, e.g., a measured voltage can vary with time, i.e., fluctuating around an unknown (average) value. Thus, by evaluation of a time averaged value, or by the choice of the measurement moment, a statistical error may arise as well. Statistical errors are characterized by a probability distribution, which determines the probability of a measured deviation from the precise (true), i.e., most probable value (expectation value). The more a measurement is repeated, the more precisely are the probability distribution and the most probable value determined and the less becomes the statistical (measuring) uncertainty (see below).

2. Average Value, Standard Deviation, Statistical Uncertainty

The best evaluation for the most probable value of a measured quantity x out of n different single measurements x_i is obtained by taking the *arithmetic average value* \bar{x}

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i . \quad (1)$$

Very often a shortened notation is used in the literature containing statistical equations with expressions of sums, where the lower and upper summation limit and the summation index is suppressed:

$$\bar{x} = \frac{1}{n} \sum x . \quad (2)$$

This shortened notation will be used in the following text.

Once the average value is known, the moduli of differences $|x_i - \bar{x}|$, denoting the deviations of single measuring results from the average value, give some estimate for the precision of the measurements. Because of numerical reasons, the moduli of the differences are replaced by the squares $(x_i - \bar{x})^2$ and the *standard deviation* s is defined by

$$s = \sqrt{\frac{\sum (\bar{x} - x)^2}{n - 1}} , \quad n > 1 . \quad (3)$$

The standard deviation gives the statistical average error of a *single* measurement. By introducing the (positive) square root, the quantity s has the same units as the measured quantity and is therefore comparable with it. The division by $n - 1$ instead by n takes into account, that for only a single measurement ($n = 1$) no statistical statement can be given, i.e., s is not defined.

The equation defining the standard deviation is usually not that given by eq. (3). All pocket calculators use instead the fully equivalent equation

$$s = \sqrt{\frac{\sum x^2 - \frac{1}{n} (\sum x)^2}{n - 1}} , \quad n > 1 , \quad (4)$$

because, when using eq. (4), not the single measured values x , but only the sum $\sum x$ and the sum of the square $\sum x^2$ has to be stored.

Besides the calculation of the average value and the standard deviation, it is often interesting to consider the value of the statistical uncertainty of the average value as well. This is because \bar{x} is just a guess of the result x according eq. (2), which for a small number n of single measurements, can be very unprecise. The

statistical measuring uncertainty u is a measure of the (statistical average) error of the average value \bar{x} :

$$u = \frac{s}{\sqrt{n}} = \sqrt{\frac{\sum x^2 - \frac{1}{n} (\sum x)^2}{n(n - 1)}} , \quad n > 1 . \quad (5)$$

While the standard deviation s as a measure of the statistical spread of single measured values x_i approaches a finite value > 0 with increasing n , the statistical measuring uncertainty u of the average value \bar{x} decreases with increasing n and approaches zero at large n .

Very often the measured values x_i are so called "normally" distributed, i.e., their relative probabilities are given by the *Gaussian distribution function* $\varphi(x)$:

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right) . \quad (6)$$

The integral

$$P(x_1, x_2) = \int_{x_1}^{x_2} \varphi(x) dx , \quad (7)$$

gives the probability that the values x_i (for a large number of measurements $n \rightarrow \infty$) lie within the interval (x_1, x_2) . As shown by Fig. 1, the function $\varphi(x)$ is symmetric around the most probable value x_0 (the expectation value) and has the shape of a bell with a full width at half maximum of somewhat more than 2σ .

For very large n , the average value \bar{x} determined from the measuring series approaches the value x_0 of the function $\varphi(x)$, and the standard deviation s approaches the value σ . The probability that the result x_i of a single measurement lies within the interval $\bar{x} \pm s$, i.e., $x_0 \pm \sigma$, amounts according to eq. (7) to about 68 %, for the interval $x_0 \pm 2\sigma$ to about 95 % and for $x_0 \pm 3\sigma$ already 99.7 %. Similar relations are valid for the statistical measuring uncertainty u of the average value: the probability of the true value x_0 lying within the so called (unit) *range of*

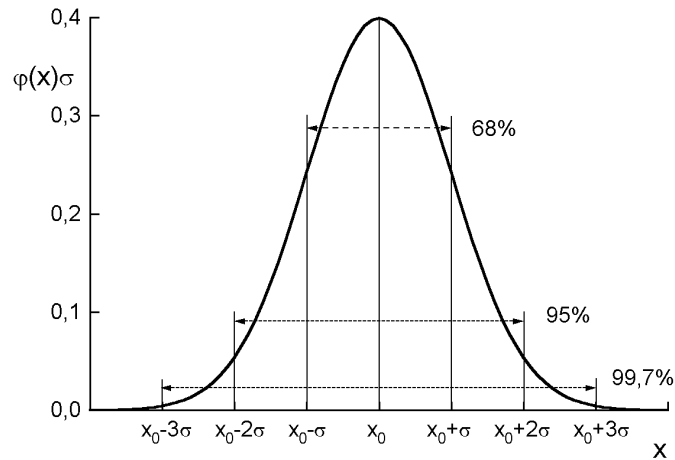


Fig. 1: Gaussian distribution function $\varphi(x)$ with most probable value x_0 and intervals $x_0 \pm \sigma$, $x_0 \pm 2\sigma$, and $x_0 \pm 3\sigma$ for relative frequencies 68%, 95%, and 99,7% of measuring value x_i confidence $\bar{x} \pm u$ is about 68 %, for the twofold and threefold range of confidence $\bar{x} \pm 2u$ and $\bar{x} \pm 3u$ about 95 % and 99,7 %, respectively.

One has to take into consideration, however, that the evaluation of the statistical measuring uncertainty or the range of statistical confidence for the value \bar{x} is physically reasonable only in combination with the evaluation of possible systematic errors. The total error of a result obtained from a measurement is always the sum of moduli for systematic and statistical errors. It is, therefore, not useful to repeat a measurement very often just for minimizing the statistical error if the evaluated systematic error exceeds the former by orders of magnitude.

Example 1: In the experiment B10 (experiment with x-rays) the lattice constant d of NaCl is to be determined from the Bragg reflection using Bragg's law

$$n\lambda = 2d\sin\theta_n ,$$

with the wavelength $\lambda = 154 \text{ pm}$ and the measured Bragg angles θ_n ($n = 1, 2, 3$).

Typical values are given in the table below.

n	θ_n (°)	d_n (pm)
1	16.0	279.4
2	33.2	281.3
3	55.15	281.5

Using these values the following are found:

the average value: $d = 280.73 \text{ pm}$,

the standard deviation: $s = 1.16 \text{ pm}$,

the statistical uncertainty u of the average value d , $u = 0.67 \text{ pm}$,

Finally,

$$d = (280.73 \pm 0.67) \text{ pm} = 280.73 \text{ pm} \pm 0.2 \%$$

if no error (statistical or systematic) in the measured angles θ_n is considered. (Otherwise see example 2 below!)

Notice: Any estimated error should not contain more than 2 or 3 decimal positions. The final result is to be rounded correspondingly. Very often it does not make any sense and only simulates precision to present the many decimal places displayed by a pocket calculator.

Additional notice: Comparison of the result for d given above with the value $d_{\text{NaCl}} = 282.0 \text{ pm}$, known from the literature, and with those obtained using other experimental setups in the laboratory, reveals a systematic deviation of θ_n (of roughly 0.5% - depending on the system used - because of inaccurate adjustments). In this case, the value of the statistical uncertainty presented is not significant at all to estimate the (actual true) error of d . If u is replaced by the standard deviation s , the result

$$d = (280.73 \pm 1.16) \text{ pm} = 280.73 \text{ pm} \pm 0.4 \%$$

can account to a better extent the actual error but this presentation does not solve the problem principally, i.e., how the systematic error present in this case is properly taken into account. One possibility to solve the problem is to take into consideration that an inherent (constant) systematic error $\Delta\theta^s$ (because of

misalignment of both the crystal and the detector) has to be taken into account when using Bragg's law for the evaluation of θ from the experimental data, i.e., $\theta = \theta^m + \Delta\theta^s$, where θ^m is the measured value. Since in the experiment considered the task is not to validate Bragg's law but to determine the lattice constant d , we state that Bragg's law is valid in any order n , i.e., that the value of d is independent of n . Combining Bragg's law, e.g., for $n = 1$ and 2 , $\Delta\theta^s$ can be calculated with the values in the table of example 1 from

$$\tan \Delta\theta^s = \frac{\sin \theta_2^m - 2 \sin \theta_1^m}{2 \cos \theta_1^m - \cos \theta_2^m}, \quad \Delta\theta^s = -0.2^\circ.$$

Using this systematic correction $\Delta\theta^s$ of the angle θ^m , the values $d_1 = 282.8$ pm; $d_2 = 282.8$ pm (i.e., $d_1 = d_2$) and $d_3 = 282.2$ pm are obtained. Thus, the average of the corrected result for d is

$$d = 282.6 \text{ pm},$$

which is closer to the value $d_{\text{NaCl}} = 282.0$ pm known from the literature than the value $d = 280.73$ pm presented above, without taking into consideration the correction of the systematic error.

3. Propagation of Errors

Generally, the determination of a physical quantity y requires the measurement of several single (different) parameters x_1, x_2, \dots . In this context, the question arises how far the single errors Δx_i resulting from the measurement of the individual parameters determine the uncertainty Δy of the quantity y . If the errors Δx_i are small as compared to x_i , the function y expanded into a power series of the errors Δx_i around the values x_i is approximately given by the terms linear in Δx_i . The error Δy resulting, e.g., from a single error Δx_i is given by

$$\Delta y \Big|_{x_j \neq i = \text{const}} = \frac{\partial y}{\partial x_i} \Delta x_i. \quad (8)$$

To estimate the *largest possible absolute error* $(\Delta y)_{\text{max}}$ by taking into account all possible single errors Δx_i , the following relation is defined:

$$(\Delta y)_{\text{max}} = \left| \frac{\partial y}{\partial x_1} \Delta x_1 \right| + \left| \frac{\partial y}{\partial x_2} \Delta x_2 \right| + \dots \quad (9)$$

Example 2: In the experiment B10 (experiment with x-rays, see also example 1) the lattice constant d of NaCl is to be determined from the Bragg reflection using the relationship

$$n\lambda = 2d_n \sin \theta_n, \quad d = \frac{1}{3} \sum d_n = \frac{\lambda}{6} \left(\frac{1}{\sin \theta_1} + \frac{2}{\sin \theta_2} + \frac{3}{\sin \theta_3} \right),$$

with the wavelength $\lambda = 154$ pm and the measured Bragg angles θ_n ($n = 1, 2, 3$). Assuming a statistical error $\Delta\theta = \pm 0.25^\circ = 0.0044$ of every measured angle θ_n , the maximum statistical error Δd_{max} is according to eq. (9)

$$\begin{aligned} (\Delta d)_{\text{max}} &= \left| \frac{\partial d}{\partial \theta_1} \Delta \theta_1 \right| + \left| \frac{\partial d}{\partial \theta_2} \Delta \theta_2 \right| + \left| \frac{\partial d}{\partial \theta_3} \Delta \theta_3 \right| \\ &= \frac{\lambda \Delta \theta}{6} \left(\frac{\cos \theta_1}{\sin^2 \theta_1} + \frac{2 \cos \theta_2}{\sin^2 \theta_2} + \frac{3 \cos \theta_3}{\sin^2 \theta_3} \right). \end{aligned}$$

Taking the numerical θ -values presented in example 1 we obtain

$$(\Delta d)_{\text{max}} = 0.112 (12.65 + 5.58 + 2.55) \text{ pm} = 2.33 \text{ pm}; \quad (\Delta d)_{\text{max}} / d = 0.83 \%$$

It is noted that the error contribution from the first Bragg angle θ_1 is the largest one, because the error $\Delta\theta/\theta_1$ is the largest one, consistent with the observation in the table of example 1, where the deviation of $(d_1 - d_{\text{NaCl}})$ is the largest one too.

Example 3: In the experiment B8 (determination of the specific charge e/m of the electron) the ratio e/m is determined from measurements of the acceleration voltage U and of the current I through the coils with radius R for an electron beam with circular radius r .

$$\frac{e}{m} = 2 \left(\frac{R}{0.715 \mu_0 n} \right)^2 \frac{U}{(rI)^2}$$

Since e/m is a product of powers of the variables R , U , r , and I , the expression of the *largest possible relative error* of e/m is rather simple. Taking into account the errors ΔR , ΔU , Δr , and ΔI , the largest possible relative error of e/m is given by

$$\left[\frac{\Delta(e/m)}{(e/m)} \right]_{\max} = \frac{1}{e/m} \left(\left| \frac{\partial(e/m)}{\partial R} \Delta R \right| + \left| \frac{\partial(e/m)}{\partial U} \Delta U \right| + \left| \frac{\partial(e/m)}{\partial r} \Delta r \right| + \left| \frac{\partial(e/m)}{\partial I} \Delta I \right| \right) \quad (10)$$

$$= \left| \frac{2\Delta R}{R} \right| + \left| \frac{\Delta U}{U} \right| + \left| \frac{2\Delta r}{r} \right| + \left| \frac{2\Delta I}{I} \right| .$$

Typical estimates of errors and values of measured quantities are

$\Delta R = \pm 2 \text{ mm}$	$R = 20 \text{ cm}$	$\Delta R/R = 0.01$
$\Delta U = \pm 6 \text{ V}$	$U = 200 \text{ V}$	$\Delta U/U = 0.03$
$\Delta r = \pm 0.5 \text{ mm}$	$r = 3 \text{ cm}$	$\Delta r/r = 0.017$
$\Delta I = \pm 90 \text{ mA}$	$I = 2 \text{ A}$	$\Delta I/I = 0.045$

resulting in

$$\left[\frac{\Delta(e/m)}{(e/m)} \right]_{\max} = 0.02 + 0.03 + 0.034 + 0.09 = 0.174 .$$

4. Graphs and Linear Regression

In experimental physics, the aim is often to validate a theoretically predicted functional dependence of two quantities x and y by a measurement. In simple cases, the quantities x and y are linearly connected, i.e.,

$$y = a + bx . \quad (11)$$

Since every measurement is inherent with an error, the data (x_i, y_i) in a graph y_i vs. x_i will scatter more or less around a straight line drawn through the data points. It is the task to find an optimum straight line balancing the deviations due to errors (i.e. that line which would represent the data if the errors were absent). This is the *regression line*. It can be simply done by a *visual estimate*, i.e., by drawing a suitable straight line with a ruler. A more objective way to construct the regression

line is the *least mean square fit*; the calculation of the parameters a and b of eq. (11) (a : intercept on y -axis, b : slope of line) with the condition

$$\sum (a + bx - y)^2 ,$$

being an absolute minimum. If this is the case, a and b fulfil the condition

$$\frac{\partial}{\partial a} \sum (a + bx - y)^2 = 0 , \quad \frac{\partial}{\partial b} \sum (a + bx - y)^2 = 0 . \quad (12)$$

Differentiation yields the equation system

$$an + b \sum x = \sum y , \quad a \sum x + b \sum x^2 = \sum xy , \quad (13)$$

with the solution

$$a = \frac{\sum y \sum x^2 - \sum x \sum xy}{n \sum x^2 - (\sum x)^2} , \quad b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} . \quad (14)$$

Note: For a regression line through the origin of the coordinate system, i.e. for the line $y = b x$ ($a = 0$) the corresponding solution is

$$b = \frac{\sum xy}{\sum x^2} \quad (15)$$

The linear regression algorithm using the method of the least mean squares is almost always applied to find an optimum fit to the data. It is (besides the arithmetic average) the most frequently used algorithm and, therefore, is included in many pocket calculators.

When using the (critical) regression coefficient R^2 , one has to be cautious. With R^2 the quality of fit functions with several independent measuring series can be compared. The coefficient of a single measuring series, however, has no physical meaning.

In cases where the quantity $Y(X)$ does not linearly depend on X , it may be possible to find a proper transformation $Y(X) \rightarrow y(x)$ so that a linear relationship

$y = a + bx$ holds.

Some examples are given in the following table:

function Y(X)	transformation		$y = a + b x$	
	y(X,Y)	x(X,Y)	a	b
$A X^B$	$\ln(Y)$	$\ln(X)$	$\ln(A)$	B
$A \exp(BX)$	$\ln(Y)$	X	$\ln(A)$	B
$A X + B X^3$	Y / X	X^2	A	B