LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(18 points)

a) (3 points)
Define graphically the principal strategy for realization of a output feedback control for a MIMO system and denote the elements and matrices. Write down the equation which defines the control low.

b) (3 points)
What are the differences between eigenvalues and poles of a given dynamical system. Is a pole also an eigenvalue or is a eigenvalue also a pole? Does a system have more eigenvalues or more poles?

c) (5 points)
Calculate the eigenvalues of the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} .$$

Is the system stable? If not, can the system be stabilized (state reason)? What about the Lyapunov stability of the system?

d) (5 points)
Calculate the transfer function for the system given in c) and calculate the poles of the system. Is the calculated pole observable and/or controllable? What about the (other) eigenvalues?

e) (2 points)
Define state stability of a dynamical system with the eigenvalues $\lambda_1 = -2.4$, $\lambda_2 = 2 + 3j$ and $\lambda_3 = 2 - 3j$.

Problem 2

(22 points)

a) (4 points)
Compare Hautus, Kalman, and Gilbert criteria related to the usability of the theorems. For which purpose you will use which criteria?

b) (5 points)
Analyze the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -1 & a & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & c & 0 \\ 0 & 0 & 2c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} .$$

Calculate the eigenvalues of the system. Analyze the stability of the system for the parameter $a = 1$. Please use the terms stable, unstable, boundary stable, stable in the sense of Lyapunov, asymptotic stable in the sense of Lyapunov for the classification of each eigenvalue and for the whole system. For which values of a is the system asymptotic stable in the sense of Lyapunov?

c) (5 points)
Analyse the observability and controllability of the system given with 2b). Under which conditions is the system full observable and controllable? (c=?, b=?, $a = 1$)

d) (5 points)
Design a state feedback control for the system given with 2b) for the parameters a=1 and b=1. The eigenvalues of the controlled system should be $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -3$.

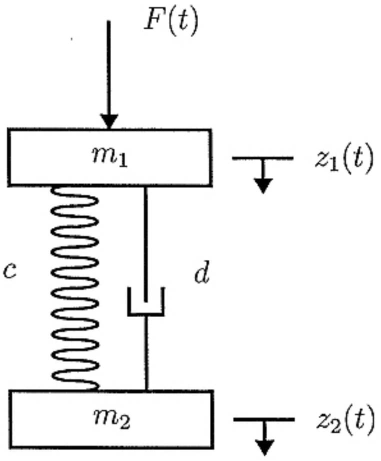
e) (3 points)
Explain (maybe with a short figure/sketch) the practical realization of observers and define in detail the necessary technical and functional equipment for practical realizations.

Problem 3

(60 points)

a) (11 points)

Consider the system given with the following figure (no gravity):



Set up the state space description. The state variable should be the difference between $z_1(t)$ and $z_2(t)$ and the derivative of the difference. The input of the system is the force $F(t)$. The difference between the position of the two masses and the derivative of the difference are measured.

b) (13 points)
Consider a system given by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \boldsymbol{x}(t) \quad .\end{aligned}$$

Calculate the eigenvalues and the transformation matrix that transforms the system into modal/canonical coordinates. The modal input matrix is $\boldsymbol{B}_{\text{mod}} = [0.25 \ \ 0.5 \ \ 0.5]^{\mathrm{T}}$, $b = 1$.

c) (7 points)
Is the system given in b) observable and controllable? Explain your answers.

d) (10 points)
The system given in b) is controlled using an output feedback $u(t) = -ay(t)$, calculate the characteristic equation of the controlled system. For which parameters a and b is the controlled system stable?

e) (14 points)
The system given in b) ($b = 1$) should be controlled using a state feedback control. The controlled system should have the damping $D = \frac{\sqrt{2}}{2}$ and the eigenfrequency w_0 of the undamped system should be $\sqrt{8}$ Hz. One eigenvalue should be $\lambda = -1$. Calculate the state feedback matrix.

f) (5 points)
The system should be excited by an harmonic signal. How would you choose the frequency f to avoid large oscillation amplitudes (state reason).

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**