Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your writing utensils. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.
Durch die Teilnahme versichere ich, dass ich prüfungsfähig bin. Bei Krankheit werde ich die Klausur vorzeitig beenden und unmittelbar eine Ärztin/einen Arzt aufsuchen.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ____________ ______________________________
(Date) (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ____________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Die Bewertung gem. PO in Ziffern ist der xls-Tabelle bzw. dem Papierausdruck zu entnehmen.	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Prof. Dr.-Ing. Mohieddine Jelali, Priv.-Doz.)

____

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung: (alternativ: siehe xls-Tabelle bzw. beigefügter Papierausdruck)

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ______________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequisite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**72**
Minimum points for the grade 1.0:	**95%**
Minimum points for the grade 4.0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For tasks with an individual evaluation of partial tasks, the following applies: Only correct partial task answers are evaluated with the given sub-point number.
   ii) The positive points of the partial tasks will be summarized.
   iii) If there are partial tasks with more than two answer options: The marking of more than one answer will lead to the evaluation not correct, because of the not clear volition. In this case zero point will be given.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc.: take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (31 Points)

1a) (4 × 5 × 1 Point, 20 Points)
Which of the following statements are true and which are false?

The system considered is described by

$$0 = f(\dot{x}, x, u, t), \quad x(t = 0) = x_0, \tag{1.1}$$
$$y = g(x, u). \tag{1.2}$$

No.	Task/Question/Judgement	True	False
A.1)	Depending on the dimensions of the input and output vectors the system is a SISO, MISO, SIMO, or MIMO system.	☒	○
A.2)	The given system description is a typical description to be solved with an initial value problem solver.	☒	○
A.3)	The description $\dot{x} = A(t)x + B(t)u, \quad x(t = 0) = x_0,$ $y = C(t)x + D(t)u$ may be a linearized special case valid for specific working points.	☒	○
A.4)	A linearized version is assumed as $\dot{x} = Ax + bu, \quad x(t = 0) = x_0,$ $y = cx.$ The linearized description above can be categorized as a linear time-invariant vector ODE.	☒	○
A.5)	These kinds of equations ((1.1) and (1.2)) can only be solved symbolically (due to the function $f(\cdot)$).	○	☒

□

Fundamentals of stability

No.	Task/Question/Judgement	True	False
B.1)	Poles of technical systems can be real or conjugate complex numbers.	⊗	○
B.2)	The Hurwitz criterion is used to define criteria/conditions for asymptotically stable system behavior.	⊗	○
B.3)	A system with unstable poles can not be stabilized.	○	⊗
B.4)	Observers can only be applied to observable and stable systems.	○	⊗
B.5)	Using the Ljapunov equation (Sylvester equation) the stability of the system described by the system matrix A can be validated numerically.	⊗	○

Fundamentals of control

No.	Task/Question/Judgement	True	False
C.1)	The feedback rule $u = -Ky$ is typical for reference control.	○	⊗
C.2)	Pole placement design is a typical design approach for MIMO system control.	⊗	○
C.3)	Output control is a possible feedback technique for MIMO as well as for SISO systems.	⊗	○
C.4)	Non-controllable or non-observable eigenvalues can never be unstable.	○	⊗
C.5)	A system can be state-unstable and I/O-stable.	⊗	○

Fundamentals of filters/observers

No.	Task/Question/Judgement	True	False
D.1)	Finite differences are used to describe systems' dynamical Input-Output behavior within a time-discrete framework.	⊗	○
D.2)	In discrete-time consideration the matrices A, B, H are identical to those of the continuous one.	○	⊗
D.3)	Kalman Filter design is based on a predictor-corrector scheme, improving the differences between estimation and real state. Therefore no feedback gain (matrix) is needed.	○	⊗
D.4)	Beside state estimation Kalman Filter is also used for denoising and/or fusion of sensor signals.	⊗	○
D.5)	Kalman Filter design is based on covariance matrices unlike Luenberger observer design is using weighting matrices. Both are denoted as Q und R.	⊗	○

For the following tasks, a system description is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 & 0 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} d & 0 \end{bmatrix}$$

with $a_{1,2,3} \neq 0$, $b_{1,2} \neq 0$, and $c_{1,2} \neq 0$.

1b) (1.5 Points)
State the equation for calculating the transfer function matrix of the system depending on the parameters $a_{1,2,3}$, $b_{1,2}$, $c_{1,2}$, and d.

Hint: The calculation of the inverse matrix is not required.

$$G(s) = \begin{bmatrix} c_1 & c_2 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ a_1 & a_2 & s+a_3 \end{bmatrix} \begin{bmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix} + \begin{bmatrix} d & 0 \end{bmatrix}$$

1c) (3 Points)
Calculate the characteristic polynomial of the system matrix A assuming $a_1 = 2$, $a_2 = 3$, and $a_3 = 4$. Is the system asymptotically stable? State reason(s).

The system is asymptotically stable.

1d) (2 Points)

Assume $A^* = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ and $B^* = \begin{bmatrix} 0 & 0 \\ b & 0 \\ 0 & 1 \end{bmatrix}$; $b \neq 0$. Is the system controllable depending on b? Use the Kalman criterion. State reason(s).

The system is fully controllable independent of b.

1e) (4.5 Points)

A system described by $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & c_1 & 0 \\ 0 & 0 & c_2 \end{bmatrix}$ is considered. Check the observability of the eigenvalues ($\lambda_1 = -2$, $\lambda_2 = -1$, and $\lambda_3 = 1$) using the original Hautus criterion by calculating first the right eigenvectors.

For $\lambda_1 = -2$: Observable for $c_1 \neq 0$ or $c_2 \neq 0$

For $\lambda_2 = -1$: observable for $c_1 \neq 0$ or $c_2 \neq 0$

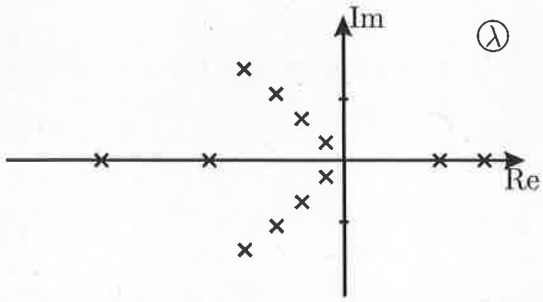
For $\lambda_3 = 1$: observable for $c_1 \neq 0$ or $c_2 \neq 0$

$\sum$

Problem 2 (28 Points)

2a) (5 × 1 Point, 5 Points)
Which of the following statements are true and which are false?

Complex systems and design

No.	Task/Question/Judgement	True	False
1)	A system is described by the following eigenvalue distribution. The system is state stable. 	○	⊗
2)	The LQR control design method can be applied to arbitrary controllable systems, assuming linearity. As a result the controlled system behavior is asymptotically stable even in the case the system to be controlled is unstable.	⊗	○
3)	A system with $A = \begin{bmatrix} 0 & 1 \\ -1 & -a \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -b \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ has to be controlled using output feedback $u = -Cx$. The system is energy free for $t = 0$. In the case $a > 0$ and $b > 0$ the controlled system is always asymptotically stable.	○	⊗
4)	Consider the system given in 2a)3), assuming that derivatives can/should not be realized. In this case, observers are required for state feedback.	⊗	○
5)	The ranks of matrices B and C given in 2a)3) with $b > 0$ are identical and equal to one.	⊗	○

2b) (5 × 1 Point, 5 Points)
Which of the following statements are true and which are false?

Model-based approaches and design			
No.	Task/Question/Judgement	True	False
1)	Regarding the properties of Kalman Filters and observers: Both are suitable to estimate the system state.	⊗	○
2)	The covariance matrices Q and R of the Kalman Filter design steps are used to describe the stochastic nature of process and measurement noise.	⊗	○
3)	Assuming full observability of the system to be observed: Kalman Filter provides full noise estimation.	○	⊗
4)	Due to the discrete-time realization Kalman Filter can be easily realized for online use without large computational load.	⊗	○
5)	Luenberger observers use a predictor-corrector scheme.	○	⊗

2c) (6 × 1 Point, 6 Points)
For the system with the eigenvalues

$$\begin{aligned}\lambda_{1,2} &= -3 \pm j,\\ \lambda_{3,4} &= -1 \pm j,\\ \lambda_{5,6} &= 2 \pm 3j,\\ \lambda_7 &= 4, \text{ and}\\ \lambda_{8,9} &= 0\end{aligned} \tag{2.1}$$

a controller has to be designed. It has to be stated that the non-observable eigenvalues are $\lambda_{1,2} = -3 \pm j$ and $\lambda_{8,9} = 0$, as well as non-controllable eigenvalues with $\lambda_{3,4} = -1 \pm j$.

No.	Task/Question/Judgement	True	False
1)	The system has 3 poles.	⊗	○
2)	The system is stabilizable.	⊗	○
3)	The eigenvalues $\lambda_{8,9} = 0$ are input decoupling zeros.	○	⊗
4)	The eigenvalues $\lambda_{1,2} = -3 \pm j$ are both input and output decoupling zeros.	○	⊗
5)	A full state feedback controller can place the eigenvalues of the closed loop system to $\tilde{\lambda}_{1,2,3,4,5,6,7,8,9} = -1 \pm j$.	⊗	○
6)	The observability of $\lambda_{3,4} = -1 \pm j$ must be considered to check the detectability of the system.	○	⊗

2d) (12 Points)
A system is given by

$$A = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} -bc \\ b-c \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad D = [0].$$

2d) i) (3 Points)
The transfer function matrix $G(s)$ is

○ $\frac{s^2 + (b-c)s - bc}{s^3 - 6s^2 - 11s - 6}$. ○ $\frac{(s-b)(s+c)}{(s+1)(s+2)(s+3)}$.

☒ $\frac{s^2 + (b-c)s - bc}{s^3 + 6s^2 + 11s + 6}$. ○ $\frac{(s-b)(s+c)}{(s-1)(s-2)(s-3)}$.

2d) ii) (2 Points)
The eigenvalues of A are:

- ○ $\lambda_1 = 1$, and $\lambda_2 = 2$, $\lambda_3 = 3$.
- ☒ $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -3$.
- ○ $\lambda_1 = -1$, $\lambda_2 = -1$, and $\lambda_3 = -2$.
- ○ None.

2d) iii) (2 Points)
Is the system fully controllable?

- ○ Yes, always.
- ○ Fully controllable only for $b = 1, 2, 3$.
- ○ No, never.
- ⊗ Not fully controllable for $b = 1, 2, 3$ or $c = -1, -2, -3$.

2d) iv) (2 Points)
Is the system fully observable?

- [x] Yes, always.
- [ ] Fully observable only for $c = 1,\ 2,\ 3$.
- [ ] No, never.
- [ ] Not fully observable for $b = 1,\ 2,\ 3$ or $c = -1,\ -2,\ -3$.

2d) v) (3 Points)

Assume $b = -1$ and $c = 0$. Calculate the elements of the feedback gain matrix for state feedback control of the given system using pole placement. The desired eigenvalues of the controlled system are $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -6$.

- [x] $k_1 = 1/4$, $k_2 = -3/4$, $k_3 = 9/4$.
- [ ] $k_1 = -1/4$, $k_2 = -3/4$, $k_3 = -9/4$.
- [ ] $k_1 = -1/4$, $k_2 = 3/4$, $k_3 = -9/4$.
- [ ] None.

$\sum$

Problem 3 (13 Points)

The dynamics of a quarter car system can approximately be modeled by a spring-mass-damper system as shown in Figure 3.1. The corresponding differential equations are given by

$$m_a\ddot{x}_1 + d_1(\dot{x}_1 - \dot{x}_d) + d_2(\dot{x}_1 - \dot{x}_2) + c_1(x_1 - x_d) + c_2(x_1 - x_2) = -f_u, \quad (3.1)$$

$$m_r\ddot{x}_2 + d_2(\dot{x}_2 - \dot{x}_1) + c_2(x_2 - x_1) = f_u. \quad (3.2)$$

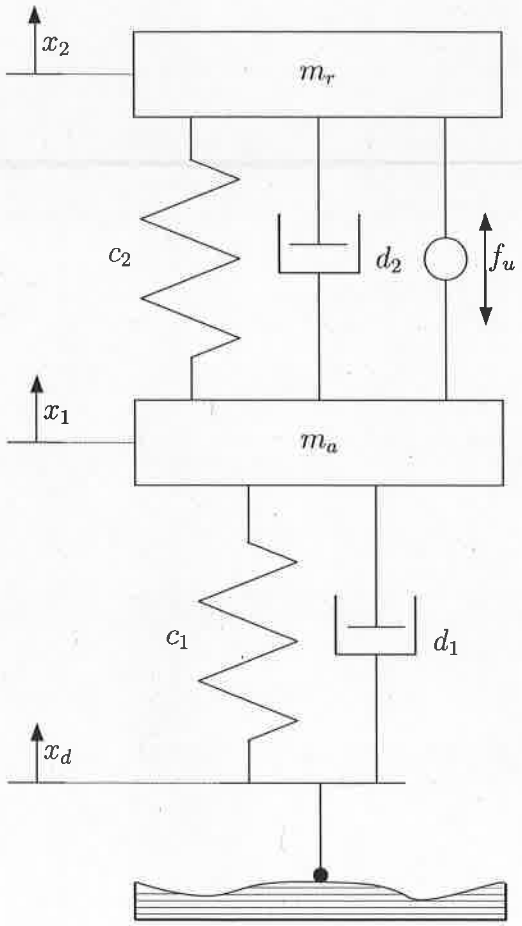


Figure 3.1: Quarter car model

3a) (5 Points)
The states are defined as $[\ x_1 \ \ x_2 \ \ x_3 \ \ x_4\ ]^T = [\ x_1 \ \ x_2 \ \ \dot{x}_1 \ \ \dot{x}_2\ ]^T$. The displacements x_1 and x_2 are measured. Set up the state space model of the system with the parameters $m_a = m_r = 1$, $c_1 = 3$, $c_2 = 1$, $d_1 = d_2 = 2$, $x_d = 0$, and $\dot{x}_d = 0$. Calculate the eigenvalues of the system. Is the system stable? State reason(s).

Hint: Two eigenvalues of the system are given as $\lambda_{1/2} = -0.5 \pm \frac{\sqrt{3}}{2}j$.

The system is asymptotically stable because all $Re\{\lambda_i\} < 0$.

3b) (3 Points)
Using the parameters $m_u = m_r = 2$, $c_1 = c_2 = 4$, $d_1 = 6$, $d_2 = 2$, $x_d = 0$, and $\dot{x}_d = 0$ the matrices A and B result in

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 2 & -4 & 1 \\ 2 & -2 & 1 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}.$$

With the input f_u the system is fully controllable. Calculate the feedback matrix

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$

in such a way that the eigenvalues of the controlled system are $\lambda_{1,des} = -1$, $\lambda_{2,des} = -2$, $\lambda_{3,des} = -3$, and $\lambda_{4,des} = -4$ (let $k_1 = 4$ and $k_3 = 2$).

$k_2 = 20$

$k_4 = 12$

3c) (3 Points)

Based on the differential equations, use the states, parameters, and measurements given in 3b) except for $d_1 = 0$ and $x_d \neq 0$. Set up the new state space model of the system. Denote the transfer function from the road disturbance x_d to the output $y_1 = x_1$.

Hint: The inverse matrix of $[sI - A_{new}]$ is given by

$[sI - A_{new}]^{-1} =$

$$= \frac{1}{s^4 + 2s^3 + 6s^2 + 2s + 4} \begin{pmatrix} s^3 + 2s^2 + 2s & 2s & s^2 + s + 2 & s + 2 \\ 2s - 2 & s^3 + 2s^2 + 4s + 2 & s + 2 & s^2 + s + 4 \\ -4s^2 - 2s - 4 & 2s^2 & s^3 + s^2 + 2s & s^2 + 2s \\ 2s^2 - 2s & -2s^2 - 4 & s^2 + 2s & s^3 + s^2 + 4s \end{pmatrix}.$$

$$G(s) = \frac{2(s^2 + s + 2)}{s^4 + 2s^3 + 6s^2 + 2s + 4}$$

3d) (2 Points)
Assume the system to be controlled, which is given by the equations (3.1) and (3.2), is fully controllable. An LQR-designed controller has to be implemented. Suggest suitable weighting matrices Q and R so that in principle all states and the control input are weighted equal, beside the last state for which more attention has to be paid to in the sense of gained feedback. You can only choose one scalar parameter w_p to express the desired weighting relations. The solution should show the correct matrix dimensions.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w_p \end{bmatrix}$$

$$R = 1$$

$\sum$