

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

**Problem 1**

(each 2 points)

- a) For what purpose is control technique, that means the feedback of technical values to the input of a technical process, needed?
- b) What is the basic idea of technical control?
- c) For what purpose is the theoretical (or analytical) modeling needed?
- d) Set up the graphical representations of three typical signals in system dynamics and state their names. Additionally, name the corresponding outputs of two of them, in case the names are standardized like the input signals.
- e) What is a transfer function and how is it typically represented?

**Problem 2**

(each 2 points)

- a) What is the time-domain, what is the frequency-domain, and how are the respective variables plotted (in dependence of which variables)?
- b) What is the stability of a transfer system? Name two methods in which you can mathematically define the stability.
- c) What is the stability of a closed loop system? Describe the difference to the stability of a transfer system (see 2b). Name two methods to proof the stability graphically/mathematically.
- d) Consider a system with  $PIDT_1$ -behavior. Give the differential equation describing the transfer behavior and denote the detailed parameters.
- e) Consider a controller with  $PIDT_1$ -behavior. Give the transfer function describing the transfer behavior and denote the detailed parameters.

**Problem 3**

(each 2 points)

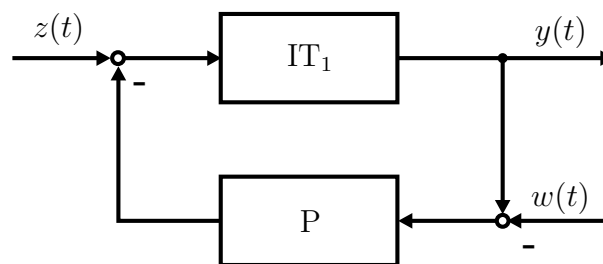
- a) The transfer behavior of a human-vehicle-interaction can be described with a frequency model

$$G_{\text{Human}}G_{\text{System}} = \frac{\omega_c}{j\omega} e^{-\omega\tau_e}$$

with  $\omega_c$  describing the gain cross-over frequency. Which meaning has the parameter  $\tau_e$ ?

- b) A transfer element with pure proportional transfer behavior is connected with a transfer element with pure integral transfer behavior by negative feedback. Which character have the transfer functions for the desired and for the disturbance behavior?

- c) Consider the control system (Figure 3.1):



**Figure 3.1:** Control system

Calculate the permanent difference for step response reference as well as for step response disturbance-magnitudes by using the final-value theorem.

- d) A transfer system with  $PT_2$ -behavior (parameters  $K_p, T_1, T_2$ ), and a transfer system with  $IT_1$ -behavior (parameters  $T_1, T_3$ ) are arranged in serial. Each system on its own is stable. Which conclusion can be made for the stability of both systems together, or which methods must be used for the parameters ( $K_p = 1, T_1 = 2\text{ s}, T_2 = 3\text{ s}, T_3 = 4\text{ s},$  and  $T_1 = 123.45\text{ s}$ )?
- e) The following state space representation of a system is given. Give the associated input notation (one scalar equation).

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k+k_R}{m} & \frac{-d_1+d_2}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \dot{u} \quad (3.1)$$

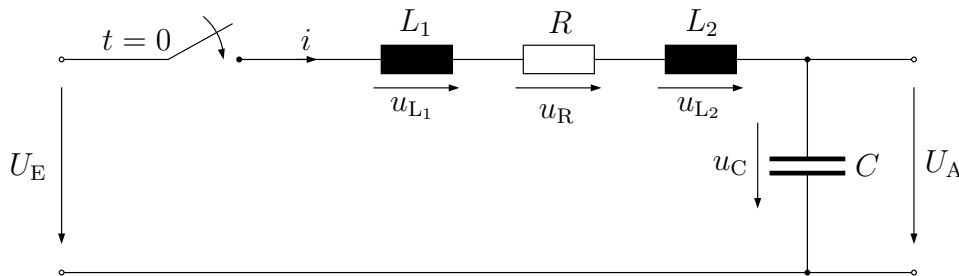
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (3.2)$$

**Problem 4**

(15 points)

In Figure 4.1 an energy free network is represented which at time  $t = 0$  is connected through a switch to a direct current source. The output is the voltage over the capacitor. Generally known is

$$u_R = Ri, \quad u_C = \frac{1}{C} \int i dt, \quad u_L = L \frac{di}{dt} \quad \text{and} \quad U_E - u_{L_1} - u_R - u_{L_2} - u_C = 0 .$$

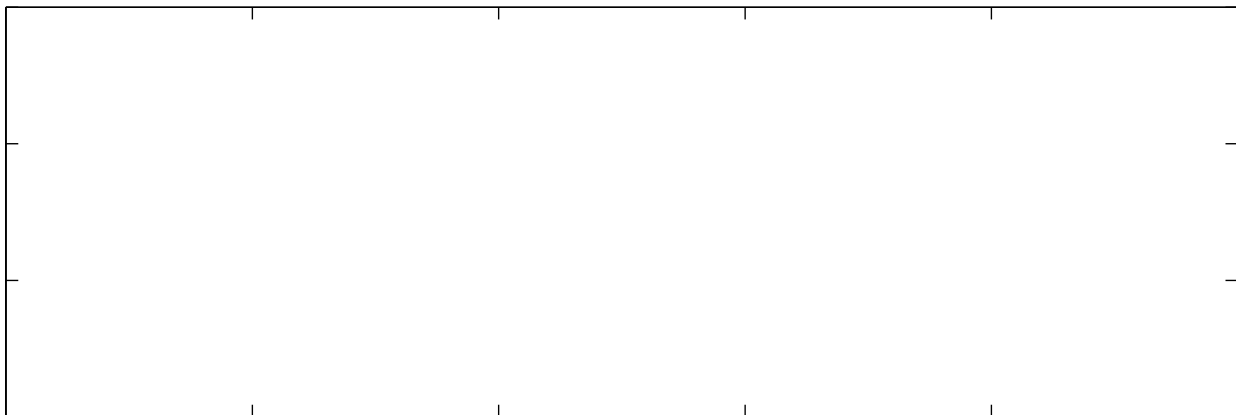
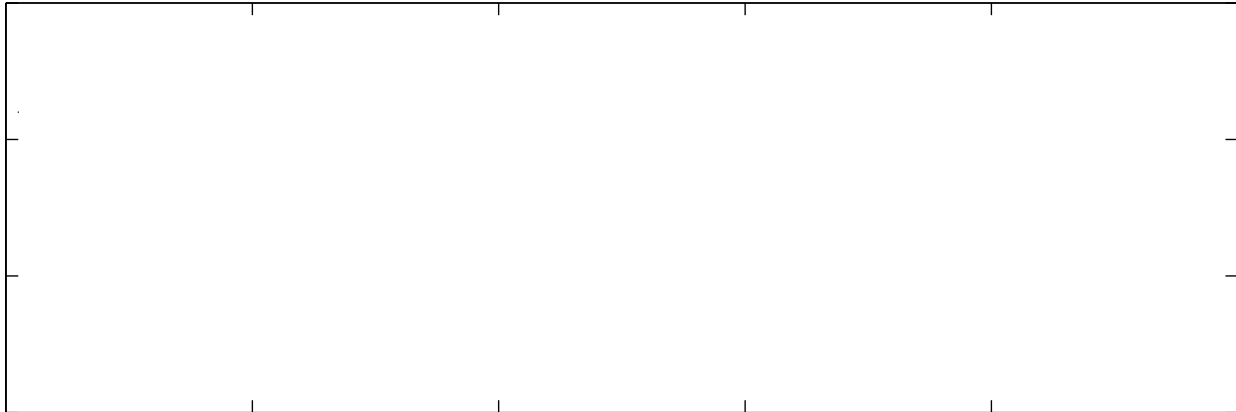
**Figure 4.1:** Electrical network

- Set up the differential equation for the electrical network in Figure 4.1, with  $U_A = f(U_E, C, L_1, L_2, R)$ .
- Assume a  $PT_2$ -system behavior for the network. Give the cut-off frequency  $\omega_0$  and the damping  $D$  for the transfer system as a function of the component parameters  $C$ ,  $L_1$ ,  $L_2$ , and  $R$ , and set up the frequency response  $G(j\omega) = \text{Re}\{G(j\omega)\} + j \text{Im}\{G(j\omega)\}$ .

The following values are given for the calculation:

$$D = 0.3, \quad \omega_0 = 30 \frac{\text{rad}}{\text{s}} \quad \text{and} \quad K = 1.$$

- Plot the polar plot of the system and denote the gain, the axes descriptions, the characteristics of the angular frequency and the eigenfrequency of the undamped system.
- Add in Figure 4.2 the axis values and draw qualitatively the corresponding Bode diagram (asymptotes and cut-off frequency).

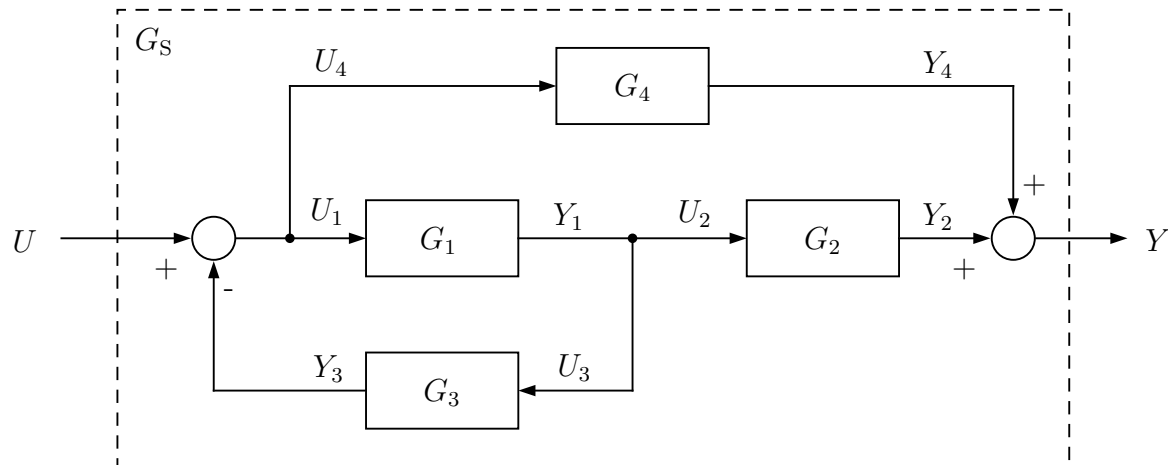


**Figure 4.2:** Bode diagram for a PT<sub>2</sub>-system

**Problem 5**

(15 points)

The following block diagram is given.

**Figure 5.1:** Block diagram of the system

a) Calculate the transfer function  $G_S = \frac{Y}{U}$  as a function of the blocks  $G_i$ .

b) The blocks are now given with

$$G_1(s) = K, \quad G_2(s) = \frac{1}{s-3}, \quad G_3(s) = s(s+4)^2, \quad \text{and} \quad G_4(s) = 0.$$

Set up the transfer function  $G_S(s)$ . For the control circuit with negative feedback a PD-controller is used  $G_R(s) = s-3$ . Calculate the transfer function of the open-loop and the closed-loop system.

c) The following system is given

$$\frac{d^4 y}{dt^4} + 11 \frac{d^3 y}{dt^3} + 29 \frac{d^2 y}{dt^2} + \frac{dy}{dt} + Ky = \frac{d^2 u}{dt^2} + 4 \frac{du}{dt} - 5u.$$

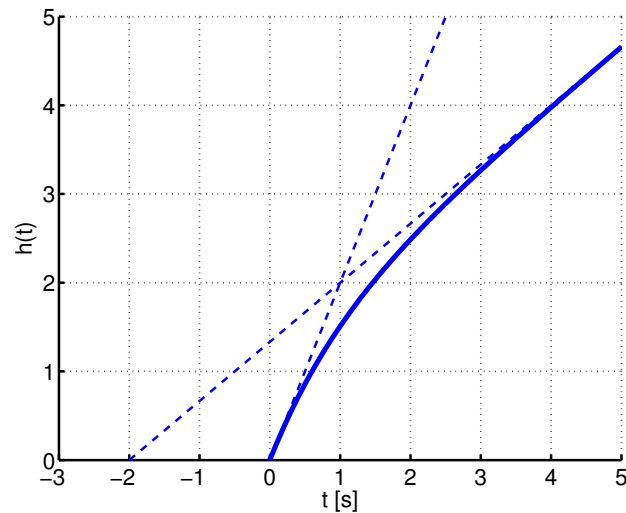
Set up the transfer function  $G(s)$  of the system.

For which  $K$  is the system steady state stable? Is the system also I/O-stable (I/O-stable = BIBO-stable)? State the reason of your answer for each case.

**Problem 6**

(40 points)

A novel drive of a remotely controlled vehicle has to be investigated. From the measured step response (given in Figure 6.1), a mathematical model has to be determined and further examined.

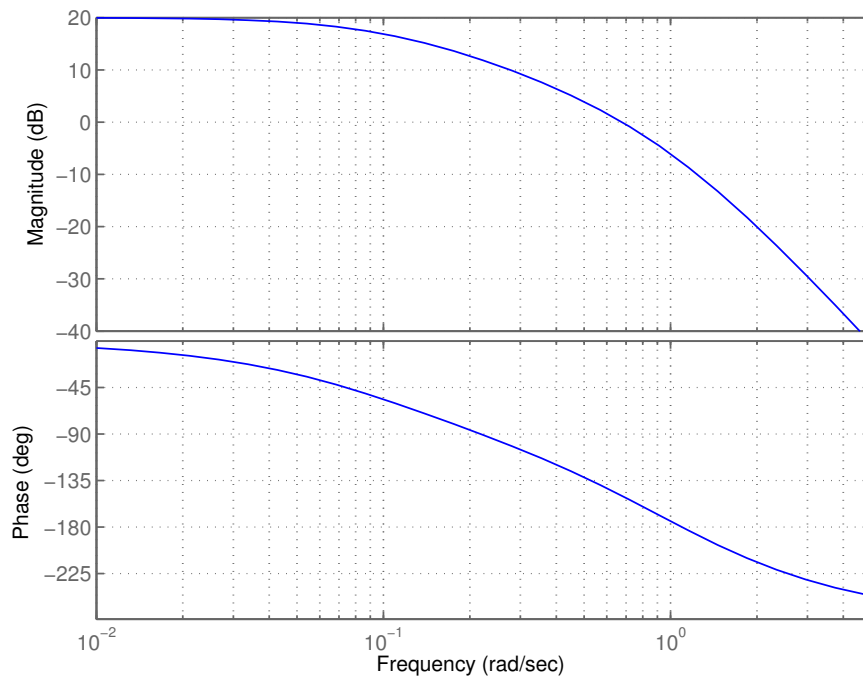
**Figure 6.1:** Step response

- From the step response (see Figure 6.1), the describing differential equation and the transfer function have to be calculated. Describe additionally the behavior of the system.
- Set up the state space representation of the system.
- Is the system BIBO stable? (State reason).

The drive is controlled with negative feedback. The controlled system is described by the transfer function

$$G(s) = \frac{10}{(1 + 10s)(s^2 + 2s + 1)}.$$

- Classify the system's behavior and draw the step response qualitatively as well as the polar plot.
- The measured Bode diagram is shown in Figure 6.2. Determine the phase and amplitude margins of the controlled system. Is the system stable? (State reason).
- The input applied to the system is chosen with  $u(t) = 2 \sin(3t)$ . Calculate the frequency, phase, and amplitude of the output in a steady state. Check your results by drawing the corresponding points into the Bode diagram in Figure 6.2.



**Figure 6.2:** Bode diagram

The controlled system  $G(s)$  is controlled remotely, and can be considered as a superposed control circuit. The behavior of the human operator is modeled as a dead-time system with the transfer function

$$G_H(s) = K_H e^{-T_t s}.$$

- g) Assuming a gain  $K_H = 1$ , which dead-time  $T_t$  is allowed so that the remotely controlled system is stable (state reason)? The cross-over frequency  $\omega_S$  can be obtained from Figure 6.2.

The controlled system  $G(s)$  is no longer controlled remotely, a controller with the transfer function

$$G_R(s) = K_D(1 - s) \quad \text{with} \quad K_D > 0$$

on board of the vehicle is used.

- h) Is the closed loop system with
- 1.) negative and
  - 2.) positive feedback
- stable for large  $K_D$ ? Use the root locus approach to illustrate the answers for case 1. and 2.

Maximum achievable points:	<b>100</b>
Minimum points for the grade 1,0:	<b>95</b>
Minimum points for the grade 4,0:	<b>50</b>