problem 1) a) <u>usage in time domain</u>

differential equations => stationary + not stationary

<u>usage in frequency domain</u>

transfer functions => stationary

b) validity of the principle of super-position

c) step function:

$$1(t) = \begin{cases} 0 & \forall \ t < 0 \\ 1 & \forall \ t \geq 0 \end{cases} \qquad F(s) = \frac{1}{s}$$

Dirac (Impulse) function:

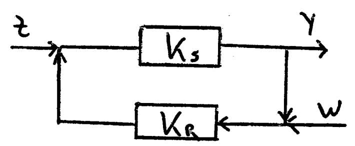
$$u(t) = \left\{ \frac{1}{\alpha} \cdot \left[ 1(t) - 1(t-\alpha) \right] \right. \qquad F(s) = 1$$

ramp function

$$u(t) = \int_{-\infty}^{t} 1(\tau) d\tau = 1(t)\, t \qquad F(s) = \frac{1}{s^2}$$

d) Laplace Transform of the differential equation
graphical description of the transfer behavior by
the phase plot and frequency response

e)




<u>disturbance transfer function</u>

$$G_z(s) = \frac{K_s}{1 + K_s K_R}$$

difference:

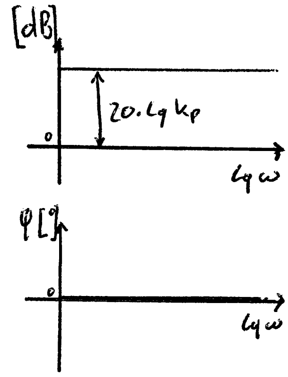
<u>reference transfer function</u>

$$K_R \to \infty \implies \begin{matrix} G_z(s) \to 0 \\ G_w(s) \to 1 \end{matrix}$$

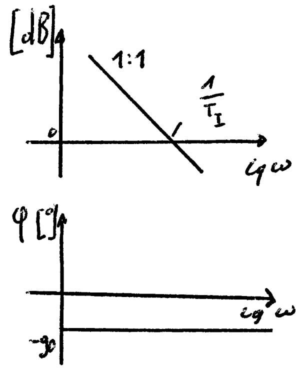
$$G_w(s) = \frac{K_s K_R}{1 + K_s K_R} = \frac{K_s}{K_s + \frac{1}{K_R}}$$

# Problem 2

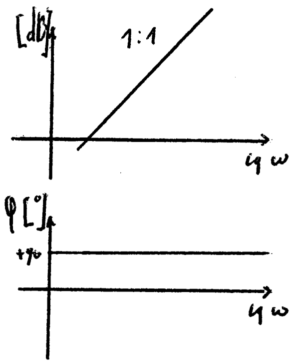
a) proportional




integrating




differentiating

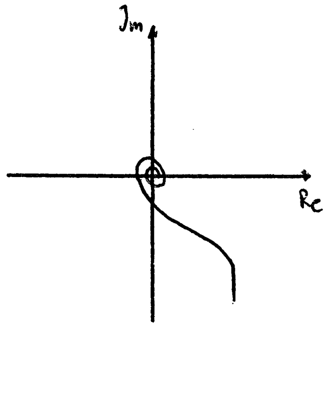



b) checking of the eigen values of the matrix A
checking of the poles of the transfer function

stability: A system is stable, if it returns after a deflection to the equilibrium position or doesn't leave it with increasing amplitude

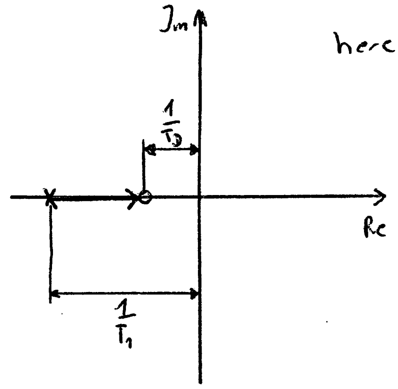
c) $T_2^2 \ddot{x}_a(t) + T_1 \dot{x}_a(t) + x_a(t) = \frac{1}{T_I} \int x_e(t-T_t)\,dt + k_p x_e(t-T_t)$

polar plot:



Problem 2

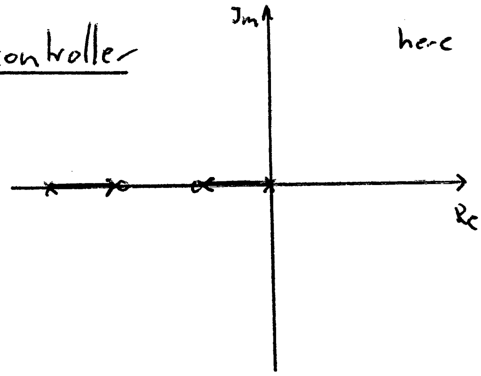
d) <u>with P controller</u>




here $T_1 < T_D$

stable

<u>with PI controller</u>




here $T_D < T_I$

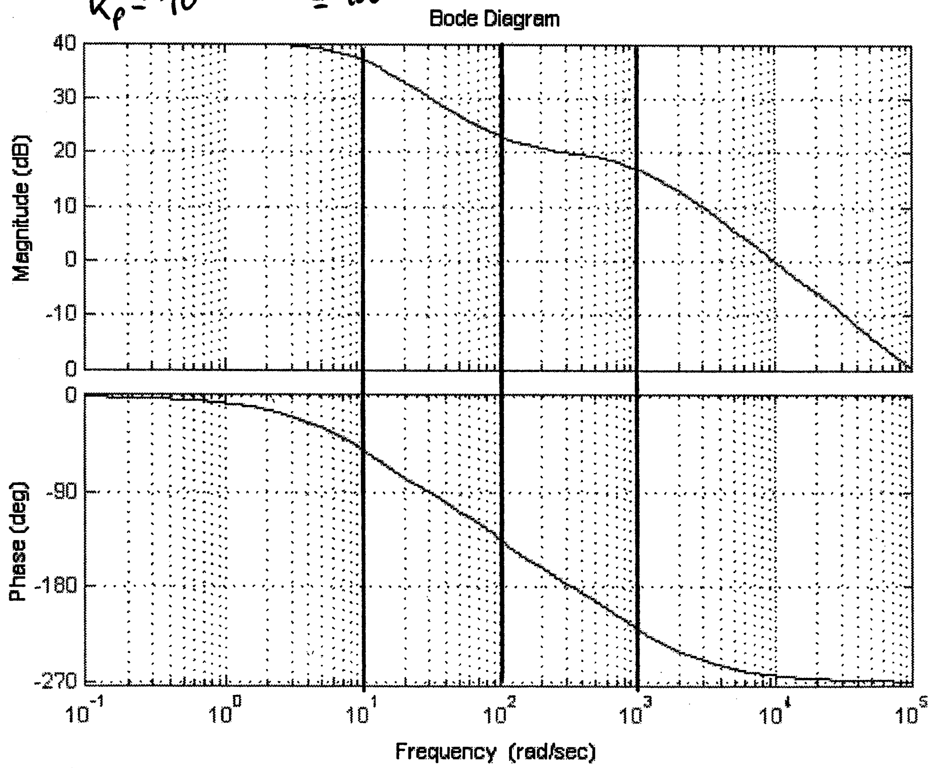
$T_1 < T_D$

semi stable

for $k = 0$

e)

$$k_p = 10^{Lg(\frac{40}{20})} = 100$$




$$\Rightarrow \quad G(s) = 100 \frac{s \cdot 0{,}01 - 1}{(s \cdot 0{,}1 + 1)(s \cdot 0{,}001 + 1)}$$

Problem 3

a) $x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$ $\quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$

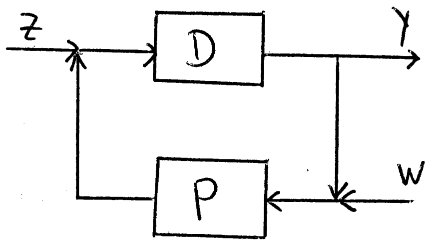
$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$ $\quad C = [0 \;\; 1] \quad , \; d = 0$

$\det(A - \lambda E) \stackrel{!}{=} 0$

$\lambda^2 + \frac{d}{m} \lambda + \frac{k}{m} = 0$

Stodola : $m > 0 \rightarrow d, k > 0$ for stability

b)




disturbance transfer function $\quad DT_1$

$G_z(s) = \frac{s \cdot k_D}{1 - s\, k_D \cdot k_p} \stackrel{\text{stability}}{\Longrightarrow} k_D \cdot k_p < 0$

reference transfer function $\quad DT_1$

$G_w(s) = \frac{s \cdot k_D \cdot k_p}{1 - s \cdot k_D\, k_p} \stackrel{\text{stability}}{\Longrightarrow} k_D \cdot k_p < 0$

Problem 3)

c) characteristic equation

$$(s+1)(s-k_2)(s^2+k_1 s+s+k_1)=0$$

$$s_{1,2} = -\frac{k_1+1}{2} \pm \sqrt{\frac{(k_1+1)^2}{4} - k_1}$$

$$= -\frac{k_1+1}{2} \pm \sqrt{\frac{k_1^2-2k_1+1}{4}}$$

$$= -\frac{k_1+1}{2} \pm \frac{k_1-1}{2}$$

$$s_1 = \cancel{-\frac{k_1}{2}+\frac{k_1}{2}} - \frac{1}{2} - \frac{1}{2} = -1$$

$$s_2 = -\frac{k_1}{2} - \frac{k_1}{2} - \frac{1}{2} + \frac{1}{2} = -k_1 \implies -k_1 < 0$$

$$k_1 > 0$$

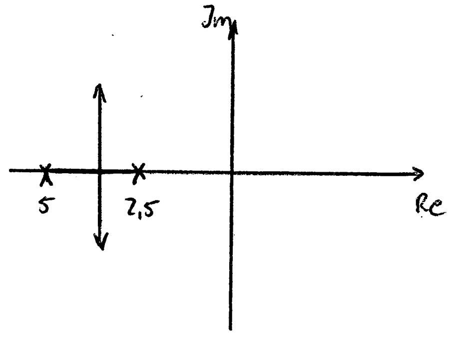
constraints for stability

$k_2 < 0 \ \wedge \ k_1 > 0$

d) assumptions for the special nyquist criterion

- negative feedback / gain $k > 0$
- no pole/zero cancellation
- $G_0$ stable
- max two poles in origin
- order of numerator < order of denominator

e)




The system is always stable

A critical gain $k_{crit}$ doesnt exist

Problem 4

a) PT1 ①: $G_1 = \frac{8}{0.2s+1}$

PT1 ②: $G_2 = \frac{0.5}{s+1}$

I ③: $G_3 = \frac{2}{s}$

$G(s) = \frac{G_1}{1+G_1G_2} \cdot G_3$

$$\Rightarrow G(s) = \frac{\frac{8}{0.2s+1}}{1+\frac{8}{0.2s+1}\frac{0.5}{s+1}} \cdot \frac{2}{s}$$

$$= \frac{16(s+1)}{s(0.2s+1)(s+1)+4s}$$

$$= \frac{16(s+1)}{s(0.2s^2+1.2s+5)}$$

poles: $s_1 = 0$ $\quad s_{2,3} = \frac{-1.2 \pm \sqrt{1.2^2-4}}{0.4} = \frac{-1.2 \pm 1.6j}{0.4} = -3 \pm 4j$

zero: $s_{01} = -1$

$s_1 = 0 \Rightarrow$ at least one of the eigenvalues has not negative real part.

$\Rightarrow$ The system is not asymptotically stable.

b) $G(s) = \frac{Y(s)}{U(s)} = \frac{16(s+1)}{s(0.2s^2+1.2s+5)}$

$\Rightarrow$ Differential equation:

$$0.2\dddot{y} + 1.2\ddot{y} + 5\dot{y} = 16\dot{u} + 16u \qquad \circledast$$

Integrate the equation until there is explicit output $y$.

$$0.2\ddot{y} + 1.2\dot{y} + 5y = 16u + 16\int u$$

$\Rightarrow$ The system is a PIT2 system.

b) Use equation $\circledast$

$$\Rightarrow \quad \dddot{y} + 6\ddot{y} + 25\dot{y} + 0\cdot y = 80\dot{u} + 80u$$

State space representation

$$\dot{x} = Ax + bu \qquad y = cx$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -25 & -6 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = [\,80 \;\; 80 \;\; 0\,]$$

c) Eigenvalues:

$$\det(\lambda I - A) = 0$$

$$\begin{vmatrix} \lambda+0.5 & 0 \\ -1 & \lambda+1 \end{vmatrix} = (\lambda+0.5)(\lambda+1) = 0 \qquad \lambda_1 = -0.5 \quad \lambda_2 = -1$$

for $\lambda_1 = -0.5$

$$Av_1 = \lambda_1 v_1$$

$$\begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = -0.5\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$-0.5v_{11} = -0.5v_{11} \;;\quad v_{11} - 0.5v_{12} = 0 \qquad v_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} v_{11} \\ 2v_{11} \end{bmatrix}$$

let $v_{11} = 1$ $\quad v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

for $\lambda_2 = -1$ $\quad Av_2 = \lambda_2 v_2$

$$\begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -v_{21} \\ -v_{22} \end{bmatrix} \qquad \begin{matrix} v_{21} = 0 \\ -v_{22} = -v_{22} \end{matrix} \qquad v_2 = \begin{bmatrix} 0 \\ v_{22} \end{bmatrix}$$

let $v_{22} = 1$ $\quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

d) Transformation Matrix $T = \mathcal{V} = [v_1 \; v_2]$

$\tilde{x} = T^{-1}x \quad T = \mathcal{V} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad T^{-1} = \mathcal{V}^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$$\tilde{A} = \mathcal{V}^{-1} A \mathcal{V} = T^{-1} A T = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\tilde{b} = \mathcal{V}^{-1} b = T^{-1} b = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

$$\tilde{c} = c\mathcal{V} = cT = [-1 \quad -1]$$

Canonical form:

$$\dot{\tilde{x}} = \begin{bmatrix} -0.5 & 0 \\ 0 & -1 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} u \quad , \quad y = [-1 \quad -1]\tilde{x}$$

e) Equations of motion

$$\tilde{x} = e^{\tilde{A}t}\tilde{x}_0 + \int_0^t e^{\tilde{A}(t-\tau)} \tilde{b}\, u(\tau) d\tau$$

$$= \begin{bmatrix} e^{-0.5t} & 0 \\ 0 & e^{-t} \end{bmatrix} \tilde{x}_0 + \int_0^t \begin{bmatrix} e^{-0.5(t-\tau)} & 0 \\ 0 & e^{-(t-\tau)} \end{bmatrix} \begin{bmatrix} 0.5 \\ -1 \end{bmatrix} u(\tau) d\tau$$

$$\ldots = \underbrace{\begin{bmatrix} e^{-0.5t} \cdot \tilde{x}_{10} \\ e^{-t} \cdot \tilde{x}_{20} \end{bmatrix}}_{\text{eigen motion}} + \underbrace{\int_0^t \begin{bmatrix} 0.5 \cdot e^{-0.5(t-\tau)} \\ -1 \cdot e^{-(t-\tau)} \end{bmatrix} u(\tau) d\tau}_{\text{excited motion}}$$

Problem 5

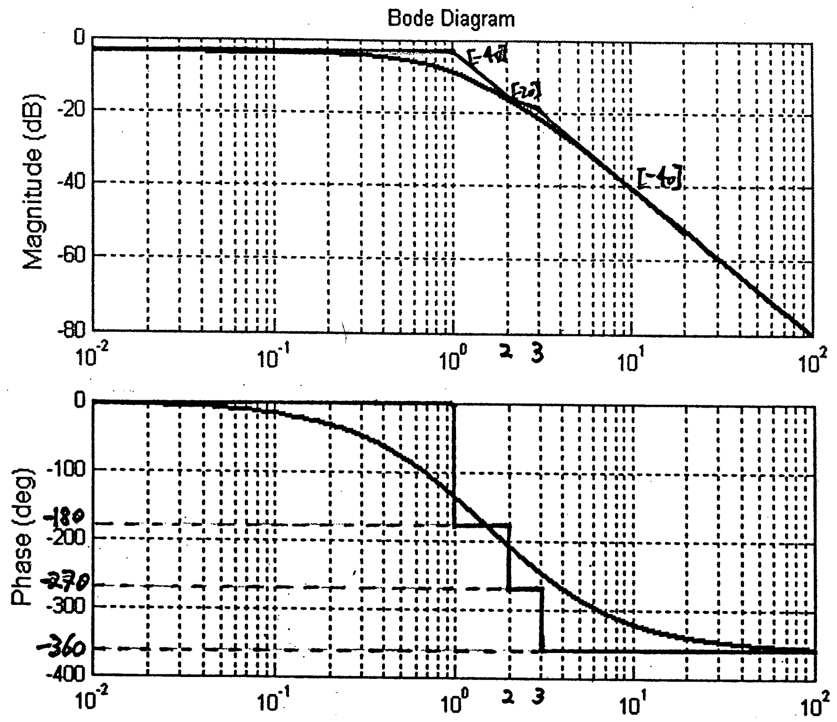
a) poles:

$s_1 = -3$

$s_{2,3} = -1$

zeros:

$s_{01} = 2$



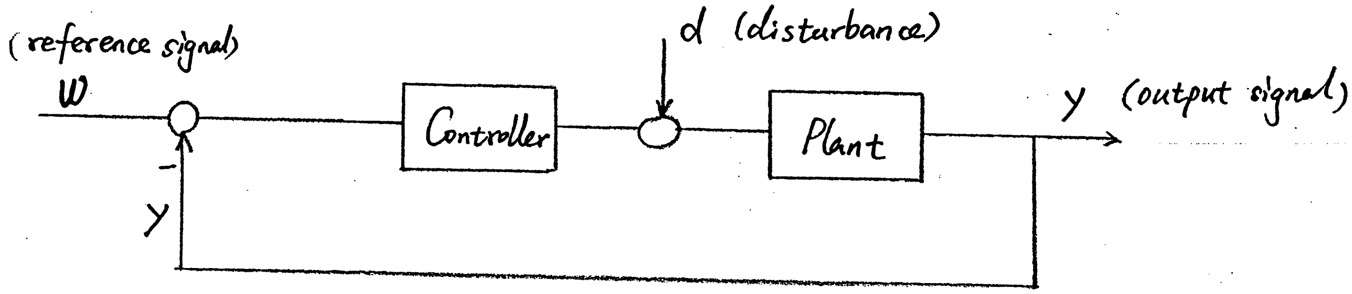

b) $s_1 = -3$, $s_{2,3} = -1$

$\mathrm{Re}\{s_i\} < 0$ for $i = 1, \dots, 3$

$\Rightarrow$ The system is I/O stable.

c) $$G_0 = K_1 \cdot \frac{s-2}{(s+3)(s^2+2s+1)}$$

$$G = \frac{G_0}{1+G_0} = \frac{K_1(s-2)}{(s+3)(s^2+2s+1) + K_1(s-2)} = \frac{K_1(s-2)}{s^3+5s^2+(7+K_1)s+3-2K_1}$$



c) Hurwitz-criterion:

$$\left.\begin{array}{l} a_3 = 1 > 0 \\ a_2 = 5 > 0 \\ a_1 = 7+K_1 > 0 \Rightarrow K_1 > -7 \\ a_0 = 3-2K_1 > 0 \Rightarrow K_1 < \frac{3}{2} \end{array}\right\} \Rightarrow -7 < K_1 < \frac{3}{2} \qquad ①$$

$$\begin{vmatrix} 5 & 3-2K_1 & 0 \\ 1 & 7+K_1 & 0 \\ 0 & 5 & 3-2K_1 \end{vmatrix}$$

$$|5| > 0$$

$$\begin{vmatrix} 5 & 3-2K_1 \\ 1 & 7+K_1 \end{vmatrix} = 35+5K_1-3+2K_1 = 7K_1+32 > 0$$

$$\begin{vmatrix} 5 & 3-2K_1 & 0 \\ 1 & 7+K_1 & 0 \\ 0 & 5 & 3-2K_1 \end{vmatrix} = (3-2K_1)\cdot(7K_1+32) > 0$$

$$\Rightarrow \left.\begin{array}{l} (3-2K_1) > 0 \\ (7K_1+32) > 0 \end{array}\right\} \Rightarrow -\frac{32}{7} < K_1 < \frac{3}{2} \qquad ②$$

$$①, ② \Rightarrow -\frac{32}{7} < K_1 < \frac{3}{2}$$

$\Rightarrow$ for $-\frac{32}{7} < K_1 < \frac{3}{2}$, the controlled system is stable.

d) $e(t) = w(t) - y(t) \qquad E(s) = W(s) - Y(s)$

$w(t) = 1(t) \qquad W(s) = \frac{1}{s}$

$$Y(s) = G(s)\cdot U(s) = G(s)\cdot W(s) = \frac{K_1(s-2)}{s^3+5s^2+(7+K_1)s+3-2K_1}\cdot\frac{1}{s}$$

$$E(s) = \frac{1}{s}\left[1 - \frac{s-2}{s^3+5s^2+8s+1}\right]$$

remaining control error: $e(\infty)$

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0}\left[1 - \frac{s-2}{s^3+5s^2+8s+1}\right]$$

$$= 1 - \frac{-2}{1} = 3$$

e) $$G_{new} = \frac{G_{0new}}{1+G_{0new}} = \frac{(K_2+\frac{K_3}{s})G_s}{1+(K_2+\frac{K_3}{s})G_s}$$

$$= \frac{(K_2 s+K_3)(s-2)}{s(s+3)(s^2+2s+1)+(K_2 s+K_3)(s-2)}$$

$$E(s) = W(s) - Y(s)$$

$$= W(s) - G_{new}\cdot W(s)$$

$$= \frac{1}{s}\left[1 - \frac{(K_2 s+K_3)(s-2)}{s(s+3)(s^2+2s+1)+(K_2 s+K_3)(s-2)}\right]$$

remaining control error

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s)$$

$$= \lim_{s\to 0} s\cdot\frac{1}{s}\left[1 - \frac{(K_2 s+K_3)(s-2)}{s(s+3)(s^2+2s+1)+(K_2 s+K_3)(s-2)}\right]$$

$$= \left[1 - \frac{-2K_3}{-2K_3}\right]$$

$$= 1-1 = 0$$

for P_controller $e(\infty) = 3$

for PI-controller $e(\infty) = 0$ , it is because that P_plant with I-controller leads automatically to zero stationary control error.

# A6



a) $\underline{G_W}$ : $Z := 0$

$$Y = G_1 \cdot W - G_2 \cdot \varepsilon$$

$$\varepsilon = W - G_3 \cdot G_2 \cdot \varepsilon \rightarrow \varepsilon = \frac{1}{1 + G_2 G_3} W$$

$$\Rightarrow Y = G_1 \cdot W - \frac{G_2}{1 + G_2 G_3} W$$

$$G_W = \frac{Y}{W} = \underline{\underline{\frac{G_1 + G_1 G_2 G_3 - G_2}{1 + G_2 G_3}}}$$

$G_Z$ : $W := 0$

$$\underline{Y} = -(Z + G_2 \cdot \varepsilon)$$

$$\varepsilon = -G_3 \cdot (Z + G_2 \varepsilon) \rightarrow \varepsilon = \frac{-G_3}{1 + G_2 G_3} Z$$

$$\Rightarrow Y = -Z - \frac{-G_2 G_3}{1 + G_2 G_3} Z$$

$$G_Z = \frac{Y}{Z} = \underline{\underline{-\frac{1}{1 + G_2 G_3}}}$$

b) $y_2 = \log(\varepsilon + 1) - 2\varepsilon \quad \rightarrow \quad y_2 = G_2(\varepsilon)$

Taylor: $\delta y_2 = G_2(\varepsilon_0) + \left.\frac{\partial G_2(\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon_0} \cdot (\varepsilon - \varepsilon_0)$

$\left[(\log_{10} x)' = \frac{1}{x \cdot \ln 10}\right]$

$$\Rightarrow \delta y_2 \approx \underbrace{-1{,}57}_{= G_{2,lin} := G_2} \cdot \varepsilon$$

... b) $Y(s) = G_w(s) \cdot W(s) + G_z(s) \cdot Z(s)$

$$= \frac{G_1 + G_1 G_2 G_3 - G_2}{1 + G_2 G_3} W(s) - \frac{1}{1 + G_2 G_3} Z(s)$$

With given transfer behavior:

$$Y(s) = \frac{k(s+T_3) + 1{,}57(s+T_3-k)}{s + T_3 - 1{,}57} W(s) - \frac{s+T_3}{s+T_3-1{,}57} Z(s)$$

c) $1(t) \circ\!\!-\!\!\bullet \frac{1}{s}$

$$\lim_{t \to \infty} y(t) \longrightarrow \lim_{s \to 0} s \cdot F(s)$$

i) $W(s) = \frac{1}{s}$, $Z(s) = 0$

$$\lim_{s \to 0} s \cdot \left[ G_w(s) \cdot \frac{1}{s} + G_z(s) \cdot 0 \right]$$

$$\lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{(k-1)(s+T)}{s+T+1}$$

$$= \frac{T(k-1)}{T+1}$$

ii) $W(s) = 0$, $Z(s) = \frac{1}{s}$

$$\lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{-(s+T)}{s+T+1} = \frac{-T}{T+1}$$

A6)



... c) iii) $Z(s) = W(s) = \frac{1}{s}$

From linearity:

$$\lim_{s \to 0} s \cdot \left[ G_w(s) \cdot \frac{1}{s} + G_z(s) \cdot \frac{1}{s} \right]$$

$$= \underline{\underline{\frac{T(K-2)}{T+1}}}$$

d) For direct feedthrough: $\lim_{s \to \infty} \left[ s \cdot \frac{1}{s} G(s) \right] \neq 0$

<u>here:</u> $\lim_{s \to \infty} \frac{(k-1)(s+T)}{s+T+1} = \frac{s\left[k + \frac{kT}{s} - 1 - \frac{T}{s}\right]}{s\left[1 + \frac{T}{s} + \frac{1}{s}\right]} = K-1$

If $K \neq +1 \Rightarrow$ system has a direct throughput!

---

e)

$\omega_1 = 0 \ \frac{rad}{s} \to -20 \frac{dB}{dek}, \ -90° \Rightarrow$ neg. pole

$\omega_2 = \frac{1}{10} \ \frac{rad}{s} \to +20 \frac{dB}{dek}, \ +90° \Rightarrow$ neg. zero

$\omega_3 = 50 \ \frac{rad}{s} \to -20 \frac{dB}{dek}, \ -90° \Rightarrow$ neg. pole

$\omega_4 = 10^3 \ \frac{rad}{s} \to +20 \frac{dB}{dek}, \ +90° \Rightarrow$ neg. zero

$K = 1$

$$G(s) = K \cdot \frac{(1+10s)\left(1+\frac{1}{10^3}s\right)}{\left(1+\frac{1}{50}s\right) \cdot s}$$

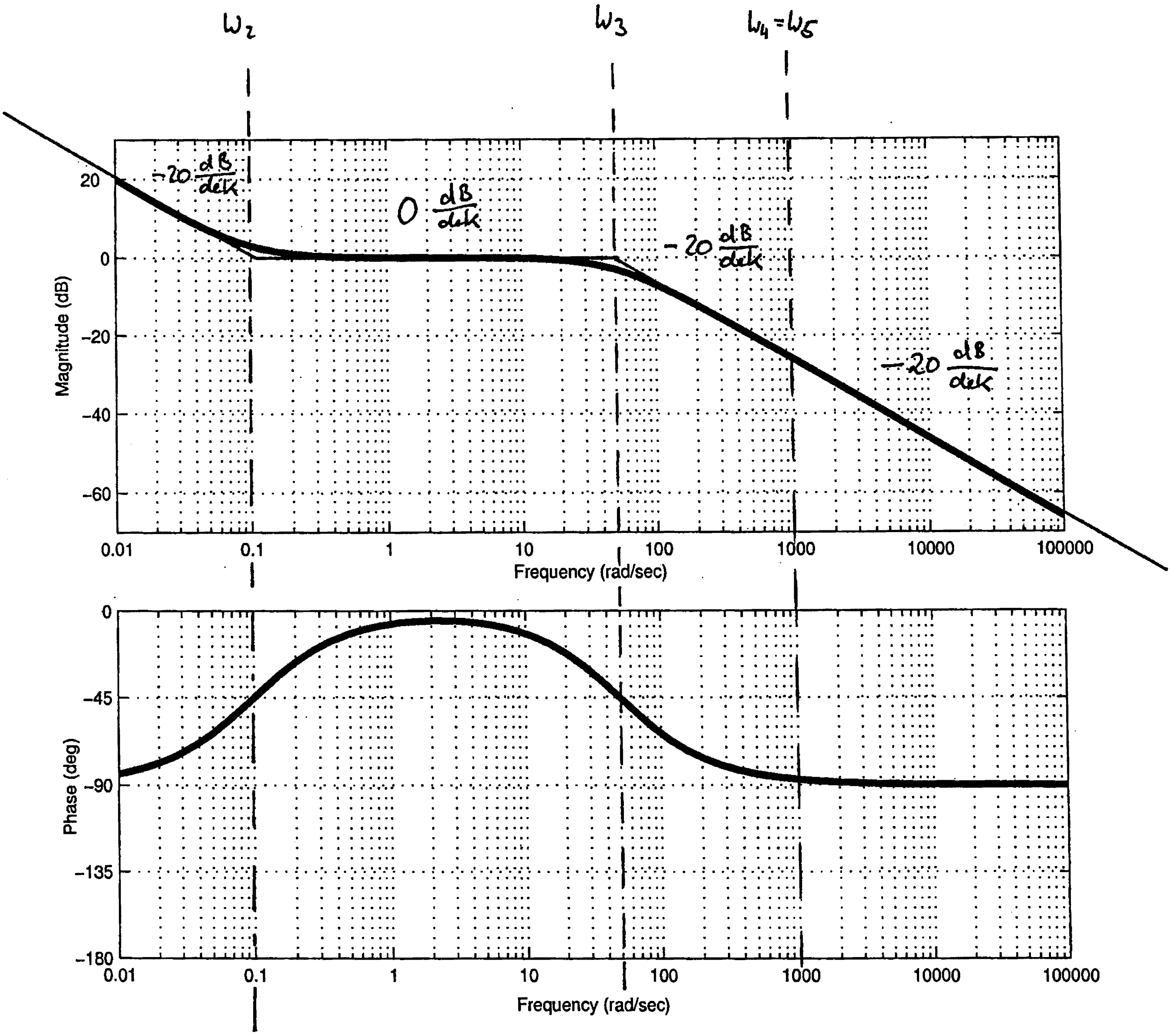
$$\Rightarrow \left(s + T_3 s^2\right) Y(s) = K \cdot \left[1 + (T_2 + T_4)s + T_2 T_4 s^2\right] U(s)$$

$$\Rightarrow y(t) + T_3 \dot{y}(t) = K \cdot \left[\int u(\tau)\, d\tau + (T_2 + T_4) u(t) + T_2 T_4 \frac{du(t)}{dt}\right]$$

$\Rightarrow$ <u>$PIDT_1$ - Transferelement</u>

A6 f)




$\omega_2$
$\omega_3$
$\omega_4 = \omega_5$
$-20\frac{dB}{dek}$
$0\frac{dB}{dek}$
$-20\frac{dB}{dek}$
$-20\frac{dB}{dek}$
Magnitude (dB)
Frequency (rad/sec)
Phase (deg)
Frequency (rad/sec)

A6| 0/9

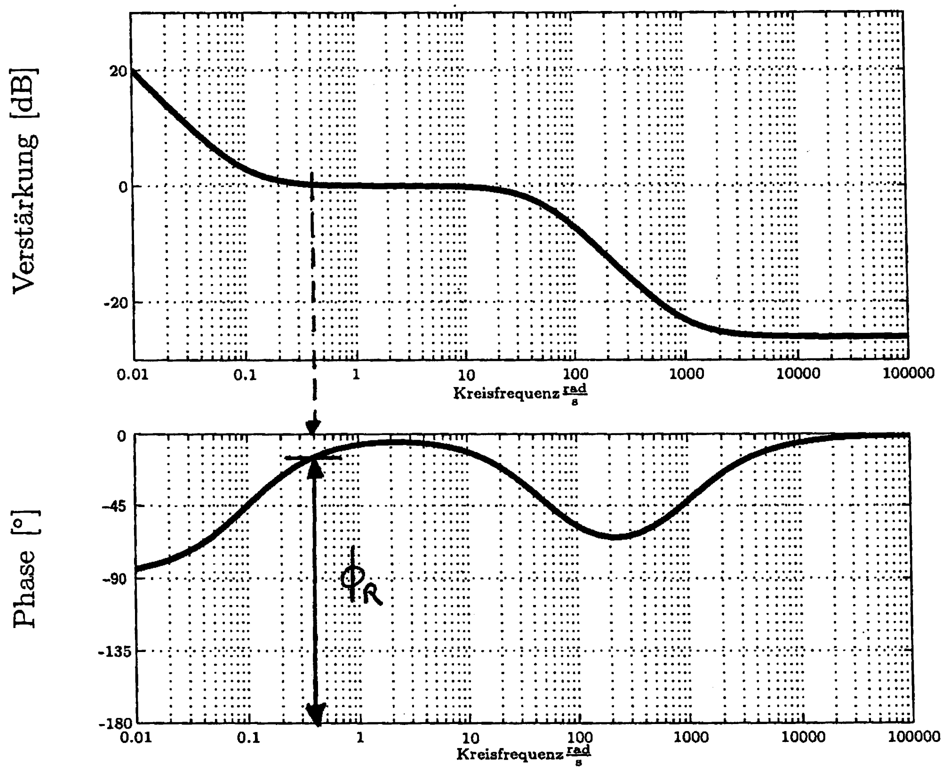
g) Yes, both are phase minimum systems:

e): 2 neg. poles, 2 neg. zeros $\rightarrow$ all real parts $\leq 0$ ✓

f): 3 " " , 2. " . " $\rightarrow$ " " " $\leq 0$ ✓

$\Rightarrow$ phase minimum systems

h)




$\phi_R \approx 165°$

$A_R$ — because no intersection with $-180°$

i) open loop: $$G_0 = \frac{(1+10s)(1+\frac{1}{10^3}s)}{s\cdot(\frac{1}{50}s+1)\cdot(1+\frac{1}{10^3}s)}$$

Integral part, so automatically boundary stable, not asyntotical stable!

Alternative way: Also from Hurwitz criterion:

$$\underbrace{\frac{1}{50\cdot 10^3}}_{a_3} s^3 + \underbrace{\frac{21}{10^3}}_{a_2} s^2 + \underbrace{s}_{a_1} = 0$$

$a_0$ missing

A6|

...i) closed loop: $G = \frac{G_0}{1+G_0}$

$$\Rightarrow G = \frac{50\,(10s+1)}{s^2+550s+50}$$

$\rightarrow$ Hurwitz: 1) all coefficients exist and have the same sign ✓

2) $\begin{bmatrix} \boxed{a_2}^{D_1} & 0 \\ a_0 & a_1 \end{bmatrix}$

$$\det D_1 = |a_2| > 0$$
$$= 1 > 0 \checkmark$$

$$\det D_2 = \begin{vmatrix} 1 & 0 \\ 50 & 550 \end{vmatrix}$$
$$= 550 > 0 \checkmark$$

$\Rightarrow$ closed loop asymptotical stable

---

j) $PT_3$ transfer behavior

k) $\dddot{y} + T_1\ddot{y} + T_2\dot{y} + T_3 y = k \cdot u$

$$G = \frac{k}{s^3 + T_1 s^2 + T_2 s + T_3}$$

l) Canonical normal form:

$n = 3$, $m = 0$

A6



... l)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -T_3 & -T_2 & -T_1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} k & 0 & 0 \end{bmatrix}$$

$$D_1 = 0$$

$$x_1 = \frac{1}{k} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

A: state matrix $(n \times n)$

B: input " $(n \times m)$

C: output " $(r \times n)$

D: direct transmission matrix $(r \times m)$

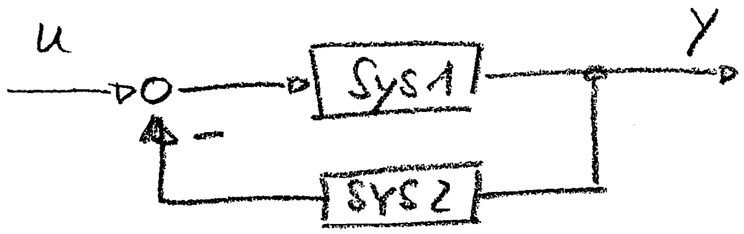
x: state vector $(n \times 1)$

u: input (SISO) $(1 \times 1)$

y: output (SISO) $(1 \times 1)$

A6

m)






$$A_2 = \left[-\frac{1}{T_R}\right], \quad x_2 = [x]$$

$$B_2 = [1]$$

$$C_2 = \left[-\frac{1}{K T_R}\right]$$

$$D_2 = [0]$$

$$\Rightarrow \dot{x}_G = \underbrace{\begin{bmatrix} A_1 & -B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix}}_{A_G} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x_G} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{B_G} u$$

$$y_G = \underbrace{\begin{bmatrix} C_1 & 0 \end{bmatrix}}_{C_G} x + \underbrace{0}_{D_G} \cdot u$$

$$x_G = \begin{bmatrix} \frac{y}{k} \\ \frac{\dot{y}}{k} \\ \frac{\ddot{y}}{k} \\ x \end{bmatrix}, \quad A_G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -3 & -3 & \frac{1}{k T_R} \\ k & 0 & 0 & -\frac{1}{T_R} \end{bmatrix}$$

$$B_G = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad C_G = \begin{bmatrix} k & 0 & 0 & 0 \end{bmatrix}$$

$$D_G = 0$$

A6)

... m) $\det\left| A_G - \lambda I \right| \stackrel{!}{=} 0$

$$\left| \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ -1 & -3 & 3-\lambda & \frac{1}{k T_R} \\ k & 0 & 0 & -\frac{1}{T_R}-\lambda \end{bmatrix} \right| \stackrel{!}{=} 0$$

$$\Rightarrow -\lambda \cdot \begin{vmatrix} -\lambda & 1 & 0 \\ -3 & 3-\lambda & \frac{1}{k T_R} \\ 0 & 0 & -\frac{1}{T_R}-\lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 0 & 1 & 0 \\ -1 & 3-\lambda & \frac{1}{k T_R} \\ k & 0 & -\frac{1}{T_R}-\lambda \end{vmatrix} \stackrel{!}{=} 0$$

$$\Rightarrow \lambda \cdot \left[ \left(\lambda^3 - 3\lambda^2 + 3\lambda + 1\right) + \frac{1}{T_R}\left(\lambda^2 - 3\lambda + 3\right) \right] \stackrel{!}{=} 0$$

There is always one eigenvalue with $\lambda = 0$

$\Rightarrow$ system <u>cannot be</u> asymptotical stable!

Independent from $T_R$: system boundary stable.