Observer-Based Control and Disturbance Compensation of Elastic Mechanical 2D-/3D-Structures

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Abstract. In this paper, recent experimental results of observer-based vibration control and disturbance compensation of elastic structures are presented. The used method is applied to a very thin elastic plate. Bonded piezoelectric patches are used as actuators of the loop to affect the plate. Core of the method is an extended high-gain observer-approach, the Proportional-Integral-Observer (PIO), which is used for the estimation of the system states as well as for the estimation of occurring disturbances. The disturbances and most of the system states are assumed to be unmeasured resp. immeasurable. So the estimations are needed to be able to set up the vibration control and the active suppression of external effects acting on the elastic structure. Due to the observer approach only a small number of measurements is necessary. This and the use of only few sensors and actuators makes the realization of active structures very cost-effective.

1 Introduction

Modeling elastic structures yields a large number of elastic coordinates while the number of available measurements remains small. Observers can be applied to estimate non-measured states. The PI-Observer (PIO) [1] can be used to estimate the non-measured states as well as unknown inputs acting on the elastic structure using a small number of measurements. The estimated states and disturbances can be used for diagnosis algorithms and/or for vibration control of the structures.

2 Control

The time invariant state space representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Nn(t)\,, \\ y(t) &= Cx(t)\,, \\ z(t) &= Fx(t) \end{aligned} \tag{1}$$

with the state vector $x(t)$ of order n_d, the measurement vector $y(t)$ of order r_1, and the input vector $u(t)$ of order m describes the system to be controlled. The system matrix A, the input matrix B and the output matrix C are of appropriate dimensions. The matrix N locates the unknown inputs given by the vector $n(t)$ of order r_2 to the system and the matrix F defines the output variables $z(t)$ to be controlled. To stabilize the system given by (A, B, C) and to reject the influence of unknown inputs $n(t)$ to the coordinates $z(t)$ the feedback signal is calculated as ([2])

$$u(t) = -K_S x(t) - K_{v1} n(t) - K_{v2} \dot{n}(t) + Vw(t) . \tag{2}$$

The term $K_S \hat{x}(t)$ realizes a state feedback whereas $K_{v1} n(t)$ and $K_{v2} \dot{n}(t)$ form the disturbance rejection parts. The reference signal is given by $Vw(t)$. The state feedback matrix K_S used for vibration control can be calculated by classical approaches like the pole placement method. The unknown inputs $n(t)$ and $\dot{n}(t)$ can be estimated using the PIO, cf. chapter 3. Using the input introduced in Eqn. (2) the controlled system is given by

$$\dot{x}(t) = (A - BK_S)x(t) + (N - BK_{v1})n(t) - BK_{v2}\dot{n}(t) , \tag{3}$$

and with the new state vector

$$v(t) = x(t) + BK_{v2} n(t) \tag{4}$$

the relation between the disturbance $n(t)$ and the variables to be controlled is illustrated by the system description

$$\begin{aligned} \dot{v}(t) &= (A - BK_S)v(t) + [(A - BK_S)BK_{v2} - (N - BK_{v1})]n(t) , \\ z(t) &= Fv(t) - FBK_{v2} n(t) . \end{aligned} \tag{5}$$

The goal is to minimize the transfer function between the disturbances $n(t)$ and the controlled output $z(t)$

$$G_{nz}(s) = F[[Is - (A - BK_S)]^{-1}[(N - BK_{v1}) - (A - BK_S)BK_{v2}] - BK_{v2}] . \tag{6}$$

For the stationary case ($\dot{v}(t) = 0$) the transfer function $G_{nz}(s)$ is zero if

$$[F(A - BK_S)^{-1} B]K_{v1} = F(A - BK_S)^{-1} N . \tag{7}$$

This condition is also given by Müller in [3]. The feedback matrix K_{v1} is necessary for the stationary case and can be calculated from Eqn. (7). By means of the matrix K_{v2} the dynamical behavior of the disturbance accommodation is improved. By minimizing $\|G_{nz}(s)\|_\infty$ suitable values for the feedback matrix K_{v2} can be evaluated. More details are given in [2, 4].

3 Design of a Proportional-Integral-Observer

For the previously introduced control approach the states $x(t)$, the unknown inputs $n(t)$ and their derivatives $\dot{n}(t)$ are needed. In the case of the control of elastic structures there are typically much more states than measurements. Anyway, the disturbances are assumed to be immeasurable. Considering the time-invariant system given in (1) the PIO is derived as

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{n}} \end{bmatrix} &= \begin{bmatrix} A & N \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{n} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y(t) - \hat{y}(t)), \\ \hat{y}(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{n}(t) \end{bmatrix} \end{aligned} \tag{8}$$

with the error dynamics

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}_e(t) \end{bmatrix} = \begin{bmatrix} A - L_1 C & N \\ -L_2 C & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ f_e(t) \end{bmatrix} - \begin{bmatrix} 0 \\ \dot{n}(t) \end{bmatrix} \quad \text{with} \quad e(t) = \hat{x}(t) - x(t),\ f_e(t) = \hat{n}(t) - n(t) . \tag{9}$$

The PIO is realized digitally so the signal $\dot{\hat{n}} = L_2(y(t) - \hat{y}(t))$ (see Eqn. (8)) is at call, so no derivation of $\hat{n}(t)$ is necessary. As can be seen from Eqn. (9), the time derivative of the unknown inputs acts on the estimation errors. To minimize this effect in order to achieve good estimation results, the transfer function from $\dot{n}(t)$ to the states $e(t)$ and $f_e(t)$ given by

$$M = [Is - (A_e - LC_e)]^{-1} N_e \quad \text{with}$$
$$A_e = \begin{bmatrix} A & N \\ 0 & 0 \end{bmatrix}, C_e = \begin{bmatrix} C & 0 \end{bmatrix}, L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \text{and } N_e = \begin{bmatrix} 0_{n_d x r_2} \\ I_{r_2 x r_2} \end{bmatrix} \tag{10}$$

has to be minimized (cf. [4]).

Assuming the extended system given by Eqn. (8) is observable and the feedback matrix L is calculated solving the Riccati equation

$$A_e P + P A_e^T + Q - P C_e^T R^{-1} C_e P = 0\,, \tag{11}$$

then the observer feedback matrix is given by

$$L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}^T = P C_e^T R^{-1}\,. \tag{12}$$

If the weighting matrices are chosen as $Q = I_Q + q N_e N_e^T$ and $R = I_{r_1 x r_1}$, then for $q \to \infty$ the matrix $\left(A_e - LC_e\right)$ (see Eqn. (9)) will be asymptotically stable and $\left\| [Is - (A_e - LC_e)]^{-1} N_e \right\|_\infty$ will be minimized. In practice the gain factor q is limited due to measurement noise $d(t)$ and model uncertainties $h(t)$. Assuming $d(t)$ forms an additive part in the output equation of Eqn. (1) and $Hh(t)$ an additive part in the state equation, the estimation error depending on the design parameter q can be illustrated qualitatively, Fig. 1. More details are given in [2, 3, 4].

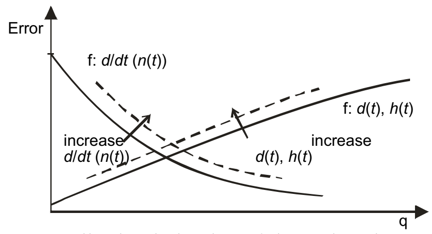
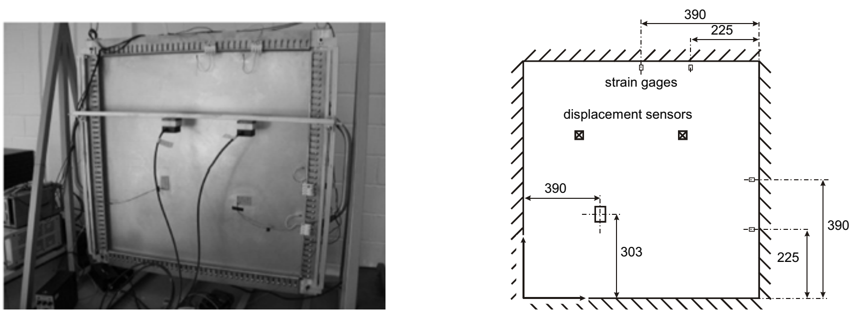


Fig. 1. Qualitative behavior of the estimation error

4 Experimental Results



(a) Figure of the test rig (b) Sketch of the test rig

Fig. 2. All side clamped plate

The control concept and the PIO have been combined and applied on a thin elastic all side clamped plate, Fig. 2 (a). The plate measures 780×780 mm and has a thickness of 0.7 mm. For control, a piezo actuator (PZT patch) is bonded on the plate and two laser-sensor-systems are used for displacement measurements. Instead of the displacement measurements, strain measurements can be used [4]. Strain gages are bonded at different positions on the plate. A scheme of the test rig with the positions of the actuators and the sensors is shown in Fig. 2 (b). The plate is excited by a hammer, which directly measures the contact force. More detailed information about the test rig is given in [4].

The system is modeled using the finite element method. The actuating forces of the PZT patch are regarded as torques acting in the nodes of the FE-mesh closest to the corners of the patch. Furthermore the PZT torques are assumed to be proportional to the applied voltage.

The displacements are taken in the coordinates (x = 260 mm, y = 540 mm) and (x = 600 mm, y = 540 mm) in which the former is considered for control instead of the output $z(t)$. For this experiment the disturbance compensation part is not applied since the PZT patch has not enough power for a static deformation of the plate, so only the vibration control is realized. The PIO is used to estimate the states. Unknown inputs are assumed to be present in the inputs of the system ($N = B$), thus the estimation of the states is sufficient in spite of the simple linear PZT patch model. The control is tuned to reduce the first two eigenmodes. The Fourier transformed result is shown in Fig. 3: The first eigenmode is reduced clearly while the second is less affected. This is due to model uncertainties caused by the temperature-sensitive plate behavior. Because of the plate's clamping, its thermal enlargement is lessened. Thus, the tension of the plate and thereby its dynamical behavior changes with the temperature. So the eigenfrequencies and modeshapes are not constant. From Fig. 4 it can be seen that the first three modeshapes are totally unsymmetrical relative to the middle of the plate du to dent effects. Accordingly, Fig. 3 shows two strong separated eigenfrequencies for the modeshapes 2a and 2b (second and third peak) which are congruent for an ideal plate.

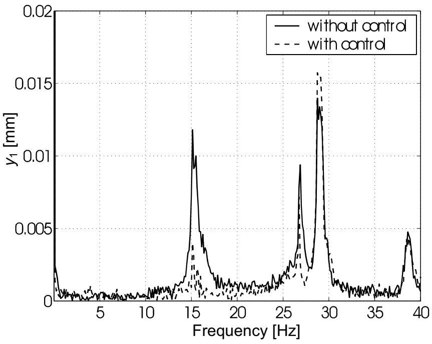


Fig. 3. Vibration control result

(a) Modeshape 1 (b) Modeshape 2a (c) Modeshape 2b

Fig. 4. First three unsymmetrical modeshapes of the plate due to temperature-dependent dent effects

References

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