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ADAPTIVE NEURAL NETWORK BASED PREDICTIVE CONTROL OF NONLINEAR SYSTEMS WITH SLOW DYNAMICS

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ABSTRACT

In this paper a data-driven approach for model-free control of nonlinear systems with slow dynamics is proposed. The system behavior is described using a local model respectively a neural network. The network is updated online based on a Kalman filter. By predicting the system behavior two control approaches are discussed. One is obtained by calculating a control input from the one step ahead prediction equation using least squares, the other is obtained by solving a standard linear model predictive control problem. The approaches are tested on a constrained nonlinear MIMO system with slow dynamics.

Keywords: Model Predictive Control, Neural Networks, Nonlinear Systems, Constrained Control

1. INTRODUCTION

In traditional model-based control a system model is derived based on the physical understanding of the underlying process. The obtained model may be imprecise due to parameter uncertainties or variations, unmodeled dynamics, and changes of the operating conditions [1], [2]. Additionally the modeling process may be complex, time consuming, and expensive [3], [4].

Due to increased computational capabilities data-driven controllers (DDC) are getting more and more popular. According to [5] DDC approaches can be splitted up into two main categories. Approaches in which a priori knowledge about the controller structure is assumed to be known are assigned to the first class. Typically in this class controller parameters are determined beforehand like for the PID controller [6]. Virtual reference control introduced in [7] is another tuning method that can be applied to linear SISO systems. Iterative tuning of linear controllers for reference tracking of nonlinear systems is considered in [8]. In case of repetitive control tasks application of iterative learning controllers may be taken into consideration [9]. The second class of DDC approaches follows the strategy to first obtain an approximate description of the system behavior and then calculate a suitable control input based on that. Classical subspace identification approaches which determine a linear prediction equation based on orthogonal projections belong to this group [10]. Another common practice is the usage of neural networks [11] or support vector machines [12], [13] for the identification process. Dynamic linearization methods which continuously update a linear prediction model have been proposed in [14]. The resulting control approach is applicable to nonlinear MIMO systems. An intelligent PID (iPID) controller for nonlinear systems using a local system description is introduced in [15]. In comparison to the original PID the proposed iPID shows better tracking performance and increased robustness [16].

Model predictive control (MPC) is a well-established control approach in industry. It can be applied to complex constrained systems with various in- and outputs and offers an optimal control solution [17]. However, solving the MPC problem can be computationally demanding and has limited the application to systems with slow dynamics [18]. Regarding robustness disturbance effects can be considered during the online optimization process of MPC. The control input is determined by considering worst case disturbance effects on the optimization criteria (minimum effect of maximum disturbance). The resulting methods can be divided into open loop and closed loop min-max optimization approaches [19]. Both of the approaches suffer from high computational burden [19].

In this paper a data-driven predictive control approach is proposed. Instead of designing a robust controller for a possible uncertain system the proposed approach focuses on online adaptation of a local linear model which describes the system behavior in the near future. The local linear model can be interpreted as a linear neural network whose weights are updated at each time step. Using the linear model the control input can be determined

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by solving a standard linear MPC problem. Alternatively the control input can be determined from a one step ahead prediction using least squares (LS). The adaptation of the linear prediction model can be achieved in short time by applying a Kalman filter.

The paper is organized as follows. In Section 2 the neural network is introduced and the training process is explained. Using the network a one step ahead predcition and a state space prediction equation are obtained. Based on the prediction models two control approaches are formulated in Section 3. One is based on a LS estimation strategy the other one is achieved by solving the standard MPC problem. The controllers are applied to a nonlinear system with slow dynamics in Section 4.

2. SYSTEM IDENTIFICATION BASED ON ADAPTIVE NEURAL NETWORKS

Consider the class of nonlinear discrete-time MIMO systems whose input-output behavior can be described by the NARX model

$$\mathbf{y}_{k+1} = f_k(\mathbf{y}_k, \dots, \mathbf{y}_{k-n_y+1}, \mathbf{u}_k, \dots, \mathbf{u}_{k-n_u+1}) = f_k(\mathbf{s}_k), \tag{1}$$

with $\mathbf{y}_k \in \mathbb{R}^r$, $\mathbf{u}_k \in \mathbb{R}^m$, and $\mathbf{s}_k \in \mathbb{R}^{rn_y+mn_u}$ being defined as

$$\mathbf{s}_k = \begin{bmatrix} \mathbf{y}_k^T & \dots & \mathbf{y}_{k-n_y+1}^T & \mathbf{u}_k^T & \dots & \mathbf{u}_{k-n_u+1}^T \end{bmatrix}^T. \tag{2}$$

Using Taylor series expansion the i-th component of $\mathbf{y}_{k+1}$ can be written as

$$\begin{aligned} \mathbf{y}_{k+1}^{(i)} &= \mathbf{y}_k^{(i)} + (\mathbf{s}_k - \mathbf{s}_{k-1})^T \mathbf{D}\mathbf{f}_k^{(i)}(\mathbf{s}_{k-1}) \\ &+ \frac{1}{2}(\mathbf{s}_k - \mathbf{s}_{k-1})^T \mathbf{D}^2\mathbf{f}_k^{(i)}(\mathbf{s}_{k-1})(\mathbf{s}_k - \mathbf{s}_{k-1}) + \dots, \end{aligned} \tag{3}$$

where $\mathbf{D}\mathbf{f}_k^{(i)}$ and $\mathbf{D}^2\mathbf{f}_k^{(i)}$ denote the gradient and Hessian of the i-th component of f_k. Considering only the linear parts in (3) a linearization of (1) is obtained as

$$\begin{aligned} \mathbf{y}_{k+1} \approx \mathbf{A}_k^{(1)}\mathbf{y}_k + \dots + \mathbf{A}_k^{(n_y)}\mathbf{y}_{k-n_y+1} + \mathbf{N}_k\mathbf{u}_k \\ + \mathbf{B}_k^{(1)}\mathbf{u}_{k-1} + \dots + \mathbf{B}_k^{(n_u-1)}\mathbf{u}_{k-n_u+1}. \end{aligned} \tag{4}$$

The matrices $\mathbf{A}_k, \mathbf{B}_k, \mathbf{N}_k$ in (4) define the transfer function matrix of a linear MIMO system [20]. The linear system (4) can be rewritten as a neural network

$$\begin{aligned} \mathbf{y}_{k+1} &\approx \mathbf{A}_k\bar{\mathbf{y}}_k + \mathbf{N}_k\mathbf{u}_k + \mathbf{B}_k\bar{\mathbf{u}}_{k-1} + \mathbf{b}_k, \\ &= \underbrace{\begin{bmatrix} \mathbf{A}_k & \mathbf{N}_k & \mathbf{B}_k & \mathbf{b}_k \end{bmatrix}}_{\mathbf{X}_k} \underbrace{\begin{bmatrix} \bar{\mathbf{y}}_k \\ \mathbf{u}_k \\ \bar{\mathbf{u}}_{k-1} \\ 1 \end{bmatrix}}_{\mathbf{p}_k}, \end{aligned} \tag{5}$$

with inputs

$$\bar{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k \\ \vdots \\ \mathbf{y}_{k-n_y+1} \end{bmatrix}, \quad \bar{\mathbf{u}}_{k-1} = \begin{bmatrix} \mathbf{u}_{k-1} \\ \vdots \\ \mathbf{u}_{k-n_u+1} \end{bmatrix}, \tag{6}$$

$\mathbf{u}_k$, weighting matrices $\mathbf{A}_k, \mathbf{B}_k, \mathbf{N}_k$, and bias vector $\mathbf{b}_k$. Input vector $\mathbf{p}_k \in \mathbb{R}^n$ is of dimension $n = rn_y + mn_u + 1$.

The parameters

$$\mathbf{x}_k = \mathrm{vec}(\mathbf{X}_k), \tag{7}$$

of the network (5) can be estimated and adapted by means of a Kalman filter so that an updated approximation of the nonlinear system (1) for time step k is available. According to [21] a well-known Kalman filter based estimation of the network parameters

$$\hat{\mathbf{x}}_k = \mathrm{vec}(\hat{\mathbf{X}}_k), \tag{8}$$

is given as

$$\hat{\mathbf{x}}_{k+1|k} = \hat{\mathbf{x}}_{k|k}, \tag{9}$$

$$\mathbf{P}_{k+1|k} = \mathbf{P}_{k|k} + \mathbf{Q}, \tag{10}$$

$$\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_{k+1}(\mathbf{y}_{k+1} - \mathbf{H}_{k+1}\hat{\mathbf{x}}_{k+1|k}), \tag{11}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1|k}\mathbf{H}_{k+1}^T(\mathbf{H}_{k+1}\mathbf{P}_{k+1|k}\mathbf{H}_{k+1}^T + \mathbf{R})^{-1}, \tag{12}$$

$$\mathbf{P}_{k+1|k+1} = \mathbf{K}_{k+1}\mathbf{R}\mathbf{K}_{k+1}^T \tag{13}$$

$$+ (\mathbf{I}_{nr} - \mathbf{K}_{k+1}\mathbf{H}_{k+1})\mathbf{P}_{k+1|k}(\mathbf{I}_{nr} - \mathbf{K}_{k+1}\mathbf{H}_{k+1})^T. \tag{14}$$

The output matrix

$$\mathbf{H}_{k+1} = \mathbf{p}_k^T \otimes \mathbf{I}_r, \tag{15}$$

is obtained by applying the vector operator on (5). The input-output data

$$\{\mathbf{y}_{k+1}, \bar{\mathbf{y}}_k, \mathbf{u}_k, \bar{\mathbf{u}}_{k-1}\}, \tag{16}$$

is assumed to be noise-free, as measurement noise would affect the output matrix in (15). As Kalman filtering is related to weighted least squares (WLS) estimation [22] algorithm (9-14) finally minimizes

$$\begin{aligned} a = \arg\min_{(\mathbf{x}_i^\star)_{i=0}^k} &\left\|\mathbf{x}_0 - \mathbf{x}_0^\star\right\|^2_{\mathbf{P}_0^{-1}} \\ &+ \sum_{i=0}^{k}\left\|\mathbf{y}_k - \mathbf{H}_k\mathbf{x}_k^\star\right\|^2_{\mathbf{R}^{-1}} + \sum_{i=0}^{k-1}\left\|\mathbf{x}_{k+1}^\star - \mathbf{x}_k^\star\right\|^2_{\mathbf{Q}^{-1}}, \end{aligned} \tag{17}$$

where $a = (\hat{\mathbf{x}}_{i|i})_{i=0}^k$ are the Kalman filter estimations. The weighting matrices $\mathbf{Q} = \alpha\mathbf{I}_{nr}$, $\mathbf{R} = \beta\mathbf{I}_r, \alpha \geq 0, \beta > 0$ are design variables. Regarding WLS problem (17) it can be seen that matrix $\mathbf{R}$ determines how exact the estimated network parameters are fitted to the input-output data, and $\mathbf{Q}$ influences the learning rate of the network. Using the estimated network parameters the linear one step ahead prediction equation

$$\hat{\mathbf{y}}_{k+1} \approx \hat{\mathbf{A}}_k\bar{\mathbf{y}}_k + \hat{\mathbf{N}}_k\mathbf{u}_k + \hat{\mathbf{B}}_k\bar{\mathbf{u}}_{k-1} + \hat{\mathbf{b}}_k, \tag{18}$$

of (1) is obtained. A linear state space realization is obtained as

$$\begin{aligned} \underbrace{\begin{bmatrix} \bar{\mathbf{y}}_{k+1} \\ \bar{\mathbf{u}}_k \\ \hat{\mathbf{b}}_{k+1} \end{bmatrix}}_{\bar{\mathbf{x}}_{k+1}} &= \underbrace{\begin{bmatrix} \bar{\mathbf{A}}_{11} & \bar{\mathbf{A}}_{12} & \bar{\mathbf{A}}_{13} \\ 0 & \bar{\mathbf{A}}_{22} & 0 \\ 0 & 0 & \mathbf{I}_r \end{bmatrix}}_{\bar{\mathbf{A}}} \underbrace{\begin{bmatrix} \bar{\mathbf{y}}_k \\ \bar{\mathbf{u}}_{k-1} \\ \hat{\mathbf{b}}_k \end{bmatrix}}_{\bar{\mathbf{x}}_k} + \underbrace{\begin{bmatrix} \bar{\mathbf{N}}_1 \\ \bar{\mathbf{N}}_2 \\ 0 \end{bmatrix}}_{\bar{\mathbf{N}}} \mathbf{u}_k, \\ \hat{\mathbf{y}}_{k+1} &\approx \begin{bmatrix} \mathbf{I}_r & 0 & \dots & 0 \end{bmatrix} \bar{\mathbf{y}}_{k+1}, \end{aligned} \tag{19}$$

with

$$\bar{\mathbf{A}}_{11} = \begin{bmatrix}\hat{\mathbf{A}}_k \\ \mathbf{T}\end{bmatrix}, \quad \bar{\mathbf{A}}_{12} = \begin{bmatrix}\hat{\mathbf{B}}_k \\ 0\end{bmatrix}, \quad \bar{\mathbf{A}}_{13} = \begin{bmatrix}\mathbf{I}_r \\ 0\end{bmatrix},$$
$$\bar{\mathbf{N}}_1 = \begin{bmatrix}\hat{\mathbf{N}}_k \\ 0\end{bmatrix}, \quad \bar{\mathbf{A}}_{22} = \begin{bmatrix}0 \\ \mathbf{S}\end{bmatrix}, \quad \bar{\mathbf{N}}_2 = \begin{bmatrix}\mathbf{I}_m \\ 0\end{bmatrix},$$

where $\mathbf{T}$ is a $r(n_y-1)\times rn_y$ matrix of the form

$$\mathbf{T} = \begin{bmatrix}\mathbf{I}_r & 0 & \dots & 0 & 0\\ 0 & \mathbf{I}_r & \dots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \dots & \mathbf{I}_r & 0\end{bmatrix}, \tag{20}$$

and $\mathbf{S}$ is a $m(n_u-2)\times m(n_u-1)$ matrix of the form

$$\mathbf{S} = \begin{bmatrix}\mathbf{I}_m & 0 & \dots & 0 & 0\\ 0 & \mathbf{I}_m & \dots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \dots & \mathbf{I}_m & 0\end{bmatrix}. \tag{21}$$

3. MODEL FREE PREDICTIVE CONTROL

In this Section two model-free control strategies are considered. First, a minimal norm LS solution based on the one step ahead prediction of the system is determined. Then a model-free predictive control approach based on the identified linear state space description is considered.

Consider $\mathbf{y}_k$ to be measured and

$$\mathbf{z}_k = \mathbf{L}\mathbf{y}_k, \tag{22}$$

with $\mathbf{z}_k \in \mathbb{R}^l$, to be controlled. For setpoint tracking the reference value is denoted by $\mathbf{z}_k^{ref}$. Based on (18) the one step ahead prediction

$$\mathbf{z}_{k+1}^{ref} = \mathbf{L}\hat{\mathbf{A}}_k\bar{\mathbf{y}}_k + \mathbf{L}\hat{\mathbf{N}}_k\mathbf{u}_k + \mathbf{L}\hat{\mathbf{B}}_k\bar{\mathbf{u}}_{k-1} + \mathbf{L}\hat{\mathbf{b}}_k, \tag{23}$$

can be considered, where $\mathbf{u}_k$ needs to be determined to achieve $\mathbf{z}_k^{ref}$. It cannot be guaranteed that

$$\mathbf{c}_k = \mathbf{L}\hat{\mathbf{N}}_k\mathbf{u}_k, \tag{24}$$

with

$$\mathbf{c}_k = \mathbf{z}_{k+1}^{ref} - \mathbf{L}\hat{\mathbf{A}}_k\bar{\mathbf{y}}_k - \mathbf{L}\hat{\mathbf{B}}_k\bar{\mathbf{u}}_{k-1} - \mathbf{L}\hat{\mathbf{b}}_k, \tag{25}$$

has a solution. The rank of $\mathbf{L}\hat{\mathbf{N}}_k$ is unknown as it depends on the network weights which are estimated online during the identification process. However, it can be guaranteed that at least one LS solution $\mathbf{u}_k^*$ of (24) exists as the normal equation

$$(\mathbf{L}\hat{\mathbf{N}}_k)^T\mathbf{c}_k = (\mathbf{L}\hat{\mathbf{N}}_k)^T\mathbf{L}\hat{\mathbf{N}}_k\mathbf{u}_k^*, \tag{26}$$

always has at least one solution (see Lemma 2.A.2 in [23]). If $\mathbf{L}\hat{\mathbf{N}}_k$ does not have full column rank an infinity amount of LS solutions exists [23]. As in control minimization of the signal energy of the control input is desirable it is suggest to choose the LS solution with minimal norm $\left\|\mathbf{u}_k^*\right\|^2$. Let

$$\mathbf{L}\hat{\mathbf{N}}_k = \begin{bmatrix}\mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{\Sigma}_1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{V}_1 & \mathbf{V}_2\end{bmatrix}^T, \tag{27}$$

be the singular value decomposition of $\mathbf{L}\hat{\mathbf{N}}_k$, then according to [24] Proposition 3.3 the min norm LS solution is given as

$$\mathbf{u}_k^* = \mathbf{U}_1\boldsymbol{\Sigma}_1^{-1}\mathbf{V}_1^T\mathbf{c}_k. \tag{28}$$

In the following predictive control i. e. minimization of

$$\begin{aligned}&\arg\min_{\vec{\mathbf{u}}} \frac{1}{2}\left(\sum_{i=k}^{k+n_p-1}\mathbf{e}_i^T\mathbf{Q}_i^{PC}\mathbf{e}_i + \sum_{i=k}^{k+n_c-1}\mathbf{u}_i^T\mathbf{R}_i^{PC}\mathbf{u}_i\right),\\ &\text{s.t.}\quad \mathbf{A}_c\vec{\mathbf{u}} \le \mathbf{b}_c,\quad \vec{\mathbf{u}} = \begin{bmatrix}\mathbf{u}_k & \dots & \mathbf{u}_{k+n_c-2}\end{bmatrix}^T,\end{aligned} \tag{29}$$

with tracking error $\mathbf{e}_k$, symmetric weighting matrices $\mathbf{Q}^{PC} \ge 0$, $\mathbf{R}^{PC} > 0$, $n_p > n_c$, and constraints $(\mathbf{A}_c, \mathbf{b}_c)$, is considered. As the linear state space description (19) is available the problem reduces to a well-known linear model predictive control (MPC) problem. According to [25] a brief solution of this MPC problem is given as follows.

The state space model (19) is augemented by the reference variable leading to

$$\begin{aligned}\underbrace{\begin{bmatrix}\bar{\mathbf{x}}_{k+1}\\ \mathbf{z}_{k+1}^{ref}\end{bmatrix}}_{\tilde{\mathbf{x}}_{k+1}} &= \underbrace{\begin{bmatrix}\bar{\mathbf{A}} & 0\\ 0 & \mathbf{I}_l\end{bmatrix}}_{\tilde{\mathbf{A}}}\underbrace{\begin{bmatrix}\bar{\mathbf{x}}_k\\ \mathbf{z}_k^{ref}\end{bmatrix}}_{\tilde{\mathbf{x}}_k} + \underbrace{\begin{bmatrix}\bar{\mathbf{N}}\\ 0\end{bmatrix}}_{\tilde{\mathbf{N}}}\mathbf{u}_k,\\ \mathbf{e}_k &= \underbrace{\begin{bmatrix}\mathbf{L} & 0 & \dots & 0 & -\mathbf{I}_l\end{bmatrix}}_{\mathbf{C}}\tilde{\mathbf{x}}_k.\end{aligned} \tag{30}$$

Based on (30) the prediction equation of the tracking error

$$\begin{aligned}\underbrace{\begin{bmatrix}\mathbf{e}_k\\ \mathbf{e}_{k+1}\\ \vdots\\ \mathbf{e}_{k+n_p-1}\end{bmatrix}}_{\vec{\mathbf{e}}} &= \underbrace{\begin{bmatrix}\mathbf{C}\\ \mathbf{C}\tilde{\mathbf{A}}\\ \vdots\\ \mathbf{C}\tilde{\mathbf{A}}^{n_p-1}\end{bmatrix}}_{\mathbf{P}}\tilde{\mathbf{x}}_k\\ &+ \underbrace{\begin{bmatrix}0 & 0 & \dots\\ \mathbf{C}\tilde{\mathbf{N}} & 0 & \dots\\ \vdots & \vdots & \ddots\\ \mathbf{C}\tilde{\mathbf{A}}^{n_p-2}\tilde{\mathbf{N}} & \mathbf{C}\tilde{\mathbf{A}}^{n_p-3}\tilde{\mathbf{N}} & \dots\end{bmatrix}}_{\mathbf{H}}\underbrace{\begin{bmatrix}\mathbf{u}_k\\ \vdots\\ \mathbf{u}_{k+n_p-2}\end{bmatrix}}_{\vec{\mathbf{u}}^{n_p}},\end{aligned} \tag{31}$$

can be obtained. Using (31) problem (29) can be written as a quadratic programm

$$\arg\min_{\vec{\mathbf{u}}} \frac{1}{2}\vec{\mathbf{u}}^T\mathbf{G}\vec{\mathbf{u}} + \mathbf{f}^T\vec{\mathbf{u}}, \quad \text{s.t.}\quad \mathbf{A}_c\vec{\mathbf{u}} \le \mathbf{b}_c, \tag{32}$$

TABLE 1: PARAMETERS OF THE THREE TANK SYSTEM ([14])

Parameter	Symbol	Value
Section of cylinders	S_A	$0.0154\ \mathrm{m}^2$
Section of connections	S_n	$5\times 10^{-5}\ \mathrm{m}^2$
Maximum liquid levels	$H_{\max}$	0.6 m
Maximum supply flow rates	$Q_{\max}$	$0.0001\ \mathrm{m}^3/\mathrm{s}$
Outflow coefficient	γ_1	0.22
Outflow coefficient	γ_2	0.28
Outflow coefficient	γ_3	0.27

with

$$\mathbf{G} = \mathbf{M}^T\mathbf{H}^T\tilde{\mathbf{Q}}^{PC}\mathbf{H}\mathbf{M} + \tilde{\mathbf{R}}^{PC}, \quad \mathbf{f}^T = \tilde{\mathbf{x}}_k^T\mathbf{P}^T\tilde{\mathbf{Q}}\mathbf{H}\mathbf{M},$$
$$\tilde{\mathbf{Q}}^{PC} = \mathbf{I}_{n_p}\otimes\mathbf{Q}^{PC}, \quad \tilde{\mathbf{R}}^{PC} = \mathbf{I}_{n_c}\otimes\mathbf{R}^{PC}, \quad \vec{\mathbf{u}}^{,\mathbf{n_p}} = \mathbf{M}\vec{\mathbf{u}},$$

where $\mathbf{M}$ is the move blocking matrix keeping $\mathbf{u}_{k+k^*} = \mathbf{u}_{k+n_c}$ fixed for all predictions $k^* > n_c$.

Based on $\mathbf{Q}^{PC}\geq 0$, $\mathbf{R}^{PC} > 0$, leading to $\tilde{\mathbf{Q}}^{PC}\geq 0$, $\tilde{\mathbf{R}}^{PC} > 0$, it follows $\mathbf{G} > 0$, so problem (32) is convex [26].

4. EXAMPLE

In this section the min norm LS (min-LS) approach of (28) and the predictive control (PC) approach of (32) are both applied to a nonlinear MIMO sytem.

Consider a nonlinear three tank water system ([14], [27])

$$S_A\dot{h}_1 = Q_1 - Q_{13} - Q_{10}, \tag{33}$$
$$S_A\dot{h}_3 = Q_{13} - Q_{32}, \tag{34}$$
$$S_A\dot{h}_2 = Q_2 + Q_{32} - Q_{20}, \tag{35}$$

with

$$Q_{13} = \gamma_1 S_n \mathrm{sgn}(h_1 - h_3)\sqrt{2g|h_1-h_3|}, \tag{36}$$
$$Q_{32} = \gamma_3 S_n \mathrm{sgn}(h_3 - h_2)\sqrt{2g|h_3-h_2|}, \tag{37}$$
$$Q_{20} = \gamma_2 S_n \mathrm{sgn}(h_2)\sqrt{2g|h_2|}, \tag{38}$$
$$Q_{10} = \gamma_2 S_n \mathrm{sgn}(h_1)\sqrt{2g|h_1|}, \tag{39}$$

where h_1, h_2, h_3 are the water levels of the three tanks, Q_1, Q_2 are the incoming water flows from pump 1 and 2, Q_{10}, Q_{20} are the flows in the outflow valves of tank 1 and 2, and Q_{13}, Q_{32} are flows in the connecting pipes of tank 1, 2 and 3. The parameters of the system are shown in Table 1. The water levels h_1, h_2 should be controlled based on the inputs Q_1, Q_2. The levels h_1, h_2, h_3 are measured. The system is discretized based on Euler method with sample time 5 s, and a simulation duration of 1500 s is considered.

The input signals are restricted to

$$0 \leq Q_1 \leq Q_{\max}, \quad 0 \leq Q_2 \leq Q_{\max}. \tag{40}$$

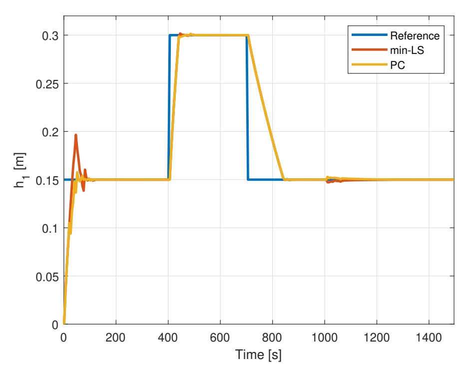
The initial values of the system are $h_1 = h_2 = h_3 = 0$ m. For the delayed inputs of the network $n_y = n_u = 5$ is considered. The network weights are initialized with $\hat{\mathbf{x}}_0 = \mathbf{I}_{nr\times 1}$, $\mathbf{P}_{0|0} = \mathbf{I}_{nr\times nr}\times 10^{10}$. The learning rate is considered to be $\alpha = 0.01$, and β is chosen as $\beta = 0.001$. The network is initially trained from $t^* = 0\ldots100\ s$ based on the system outputs generated by the input

TABLE 2: PERFORMANCE EVALUATION THREE TANK SYSTEM W/O CONSTRAINTS

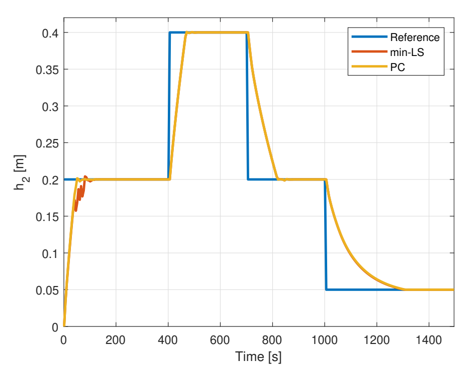
	$\sum_{i=1}^N \mathbf{u}_i^2/N$	$\sum_{i=1}^N \mathbf{e}_i^2/N$
min-LS	$2.810578e^{-9}$	$3.472702e^{-3}$
PC ($n_p = 5, n_c = n_p - 1$)	$2.731188e^{-9}$	$3.465359e^{-3}$
PC ($n_p = 10, n_c = n_p - 1$)	$2.731148e^{-9}$	$3.465435e^{-3}$
PC ($n_p = 15, n_c = n_p - 1$)	$2.732690e^{-9}$	$3.462436e^{-3}$
PC ($n_p = 20, n_c = n_p - 1$)	$2.737842e^{-9}$	$3.454841e^{-3}$
PC ($n_p = 25, n_c = n_p - 1$)	$2.738486e^{-9}$	$3.458818e^{-3}$

$$Q_1 = Q_2 = \begin{cases} u_t & \text{if } t^* \bmod 10 \text{ is even,} \\ 0 & \text{if } t^* \bmod 10 \text{ is odd,}\end{cases} \tag{41}$$
$$u_t = 0.00002\times\cos\left(\frac{2\pi}{100}t^*\right) + 0.00008.$$

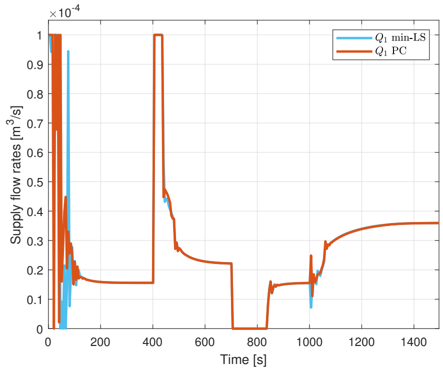
The weighting matrices considered for the model free predictive approach are $\mathbf{Q}^{PC} = \mathbf{I}_{l\times l}, \mathbf{R}^{PC} = \mathbf{I}_{m\times m}$.



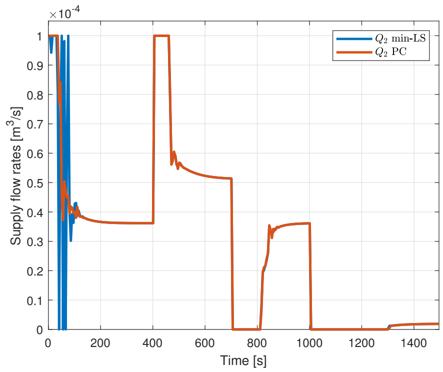
(a) Reference Tracking Water Level h_1



(b) Reference Tracking Water Level h_2

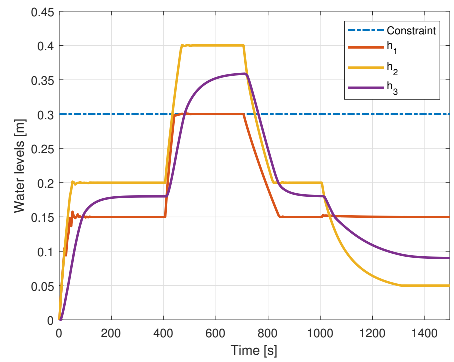


(a) Supply flow Q_1

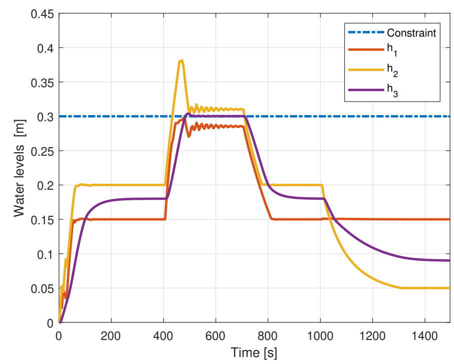


(b) Supply flow Q_2

FIGURE 2: PERFORMANCE EVALUATION OF MIN NORM LS (MIN-LS) AND PREDICTIVE CONTROL (PC) APPROACHES ($n_p = 20, n_c = 19$)



(a) Without Consideration of Constraint



(b) With Consideration of Constraint

FIGURE 3: PERFORMANCE OF PREDICTIVE CONTROL (PC) APPROACH IN CASE OF CONSTRAINTS ($n_p = 20, n_c = 19$)

4.1 Unconstrained Problem

For the unconstrained problem the reference values of the control variables h_1, h_2 are

$$h_1^{ref}(t) = \begin{cases} 0.15\,\text{m} & \text{if } t \leq 400\,\text{s}, \\ 0.3\,\text{m} & \text{if } 400\,\text{s} < t \leq 700\,\text{s}, \\ 0.15\,\text{m} & \text{if } 700\,\text{s} < t \leq 1500\,\text{s}, \end{cases} \tag{42}$$

$$h_2^{ref}(t) = \begin{cases} 0.2\,\text{m} & \text{if } t \leq 400\,\text{s}, \\ 0.4\,\text{m} & \text{if } 400\,\text{s} < t \leq 700\,\text{s}, \\ 0.2\,\text{m} & \text{if } 700\,\text{s} < t \leq 1000\,\text{s}, \\ 0.05\,\text{m} & \text{if } 1000\,\text{s} < t \leq 1500\,\text{s}. \end{cases} \tag{43}$$

Based on the results shown in Fig. 2 and Table 2 it can be concluded that reference tracking can be achieved by both min-LS and PC approach. According to Table 2 the predictive control approach has lower tracking error and lower input energy in comparison to the min-LS approach.

4.2 Constrained Problem

For the constrained problem the same reference values as for the unconstrained problem are considered (42, 43), in addition the constraint

$$h_3 \leq 0.3\,\text{m}, \tag{44}$$

should be achieved. The constraint can be formulated as $\mathbf{A}_c \vec{\mathbf{u}} \leq \mathbf{b}_c$ and is implemented as a soft-constraint, for details see e. g. [25].

From Fig. 3 it can be seen that constrained optimization can be achieved by PC approach.

5. CONCLUSION

Model-free control of nonlinear systems with slow dynamics has been considered. Constrained and reference control can be achieved based on a local system approximation which is updated online. The proposed method is easy to implement and has low computational costs.

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