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Prof. Dr. Hans Niels Jahnke, Essen  
Prof. Dr. Helmut Pulte, Bochum

International Conference

***Explanation and Proof in Mathematics: Philosophical  
and Educational Perspectives***

University of Duisburg-Essen, Campus Essen, Nov. 1 through Nov. 4,  
2006

Conference Program (preliminary)

Conference Place

Bildungszentrum für die Entsorgungs-  
und Wasserwirtschaft GmbH

Wimberstr. 1  
D-45239 Essen-Heidhausen

[http://www.bew.de/bew/bew\\_essen/](http://www.bew.de/bew/bew_essen/)

## **Background**

Developments in the *practice of mathematics* over the last three decades have led to new types of proof and argumentation, challenging the established norms in this area. These developments originated from the use of computers (both as a heuristic device and as a means of verification), from a new quality in the relations of mathematics to its applications in the empirical sciences and technology (see the Jaffe-Quinn controversy, for example), and from a stronger awareness of the social nature of the processes leading to the acceptance of a proof.

These developments are reflected in the philosophy of mathematics partly *ex post*, and partly even *anticipated*. Philosophers have long sought to define the nature of mathematics, notably by focusing upon its *logical foundations* and its formal structure. Over the past forty years, however, the search has shifted direction markedly. This shift was brought about in part by the crucial problems encountered in the programs of logicism and formalism, and in part by the developments within mathematics mentioned above.

As a consequence, in the *philosophy and history of mathematics* the approach to understanding mathematics has changed dramatically. This change was first highlighted in the late sixties by the work of Imre Lakatos, who pronounced mathematics a “quasi-empirical science”. His work continues to be of high relevance for the philosophy of mathematics as well as for its educational aspects.

The work of Lakatos and others gave rise to conceptions of mathematics in general, and of proof in particular, that are based on detailed studies of mathematical practice. In recent times, these studies have been frequently combined with the epistemological points of view and cognitive approaches that are commonly subsumed under the heading of “naturalism”. In this context, philosophers have come to a greater recognition of the central importance of mathematical understanding, and so have looked more closely at how understanding is conveyed and at what counts as explanation in mathematics. As might be expected with these two changes in focus, philosophers of mathematics have turned their attention more and more from the *justificatory* to the *explanatory* role of proof. Their central questions are no longer only why and how a proof makes a proposition true, but also how it contributes to an adequate understanding of the proposition in question and what role is played in this process by factors that go beyond logic.

The computer has caused a radical change in *educational practices* as well. In algebra, analysis, geometry and statistics, for example, computer software already provides revolutionary capabilities for visualisation and *experimentation*, and holds the promise of further change. At the same time, there is a stronger emphasis on *communication* in the classroom, an emphasis reflected in the didactics of mathematics by the idea of *constructivism*. Thus the role of traditional notions of proof in education has changed, and new forms and types of explanation and argumentation have evolved.

In sum, trends in the philosophy and history of mathematics as well as in mathematics education have lead to a diversity of notions of proof and explanation in both fields. These trends interact, as people in one field are sensitive to developments in the other. The tendencies in the different fields are not identical, however, and each field retains its own peculiarities.

## **Aim**

The conference we propose intends to bring together a group of 25 people from the philosophy and history of mathematics and from mathematics education in order to better understand these developments as they affect our notions of proof and explanation. We would like to strengthen, in particular, the mutual awareness in the philosophy of mathematics and in mathematics education of the new types of proof and explanation, and to contribute to a ‘converging’ theoretical framework based upon recent advances in both fields. This seems quite possible (and one might even say necessary) in light of the strong empirical and realistic tendencies now shared by philosophy of mathematics and mathematics education.

Both aims are all the more important for the fact that philosophers of mathematics and researchers in mathematics education, though they share a strong interest in a new understanding of mathematical explanation and proof, usually work in different institutional settings and in different research programs.

In order to reach these aims it will be important that both groups (representatives of philosophy/history of mathematics and of mathematics education) take interest in the problems and questions of the respective other one. So, philosophers and historians are invited to reflect on which dimensions of mathematical proof and explanation could be relevant to the general culture and to broadly educated adults while people from didactics are supposed to specifically elicit the epistemological and methodological aspects of their ideas.

## **Structure**

To orient the work of the conference we have identified specific sub-themes which will help to focus the discussions. They concern issues, that have proved influential and fruitful both in the philosophy of mathematics and in mathematics education. These are:

### ***(1) The Legacy of Lakatos***

Lakatos’ conception of mathematics as a “quasi-empirical science” has proved influential for the philosophy of mathematics as well as for the educational context. Though the naïve idea that Lakatos’ concepts could be transferred directly into the classroom, in the hope that insights into the need for proof would arise immediately from classroom discussions, has been proven untenable, Lakatos’ work is still an inspiration for philosophers and educators.

### ***(2) Diagrammatic Thinking***

The term “diagrammatic thinking” was coined by C. S. Peirce to designate the fact that thinking cannot be explained by purely logical means, but is deeply dependent upon the systems of symbols and representations that are used. Independently of Peirce and philosophical discourse, this idea plays a key role in the didactics of mathematics, and can be applied in particular to mathematical argumentation and proof.

### ***(3) Mathematical Proof and the Empirical Sciences***

There are quite a number of authors who view *mathematics in its connection with the empirical sciences*, especially with physics. One can designate this approach as a form of physicalism – albeit in the broad meaning of that term. This does not at all mean that

mathematics itself is considered to be an empirical science in a strict epistemological sense. This position stresses, rather, that the contents, methods and meaning of mathematics are to be discussed under the point of view that mathematics contributes via the empirical sciences to our understanding of the world around us. Theoretical concepts of mathematics, such as group and vector space, are to be set on a par with theoretical concepts of physics such as electron and electromagnetic wave.

This approach goes back at least to the twenties of the last century, when in 1928 Hermann Weyl remarked in regard to Hilbert's formalism that one finds the Beyond to which the symbols of mathematics are related by allowing mathematics to fuse completely with physics. Russell and others also showed some inclination to this type of physicalism, and the realist philosophy of mathematics developed after the Second World War by Quine and Putnam reveals strong structural similarities to the earlier approach.

#### ***(4) Different Types of Reasoning and Proof***

In the practice and teaching of mathematics different forms of mathematical argumentation have evolved, some of which are considered proofs proper and some as heuristic devices. We mention, besides formal proofs, the various forms of induction, enumeration, algebraic manipulation, visualisation, computer experimentation, computer proof and modelling. The conference should try to understand these modes of argumentation better and in greater depth, and to analyse the different views of their acceptability and fruitfulness on the part of mathematicians, philosophers and mathematics educators.

## Programme

**October 31**  
20.00

**Travel day**  
Opening dinner

**November 1**

### **Morning session:**

08.30 – 09.00

Welcome and introduction

### **Section 1: The role of representations and diagrams in proof**

09.00 – 10.00

Marcus Giaquinto (University College London, London), *A false dichotomy: algebraic v. geometric thinking in mathematics*

10.00 – 11.00

Mary Catherine Leng (Cambridge university), *Mathematical Proof: An Algebraic Perspective*

11.00 - 11.30

coffee break

11.30 – 12.30

Evelyne Barbin (Université de Nantes): *Proofs of the main proposition on geometrical proportion : from icons to symbols*

### **Afternoon session**

14.30 – 15.30

Willibald Dörfler (Universität Klagenfurt): *Verbales Argumentieren als Sprechen über Diagramme*

### **Section 2: Proofs as experiments and their role in the empirical sciences**

15.30 – 16.30

Alfred Nordmann (Technische Universität Darmstadt): *Proof as Experiment in Wittgenstein*

16.30 – 17.00

coffee break

17.00 – 18.00

Moritz Epple (Universität Frankfurt): *Vage Anschauung vs. strenges Beweisen? Topologische Argumentationsformen im späten 19. und frühen 20. Jahrhundert.*

18.00 – 19.00

Teun Koetsier (Vrije Universiteit Amsterdam), *Motion and geometry in antiquity*

**November 2**

**Morning session:**

**Section 2: Proofs as experiments and their role in the empirical sciences (continued)**

- 08.30 – 09.30 Michael Stöltzner (Universität Bielefeld): *The principle of least action as a Mathematical thought experiment*
- 09.30 – 10.30 James Robert Brown (University of Toronto): *Thought experiments and the continuum hypothesis*
- 10.30 – 11.00 coffee break
- 11.00 – 12.00 Kazuhiko Nunokawa: *Explanations in mathematical problem solving*

**Afternoon session**

- 14.00 – 15.00 Michael D. de Villiers (University of Durban Westville): *Baking a mathematical pudding: what's the role of proof and experimentation?*

15.00 – 16.00 **Round Table 1: Proofs, diagrammatic thinking and empirical contexts** (Moderator Helmut Pulte)

- 16.00 – 16.30 coffee break

**Section 3: Genesis, epistemological functions and social practices of proof**

- 16.30 – 17.30 Kenneth Ruthven (University of Cambridge): *What needs explaining in classroom mathematics? What functions (h)as proof?*
- 17.30 – 18.30 Aiso Heinze Universität Augsburg): *On the acceptance of mathematical proofs: Observations about social processes in the mathematical community and possible implications for the mathematics classroom*

**November 3**

**Morning session**

**Section 3: Genesis, epistemological functions and social practices of proof (continued)**

- 08.30 – 09.30 Jean Paul van Bendegem (Vrije Universiteit Brussel), *What Turns an Argument into a Proof?*
- 09.30 – 10.30 Thomas Mormann (University of the Basque Country, San Sebastian): *Proof and Idealization in Mathematics*
- 10.30 – 11.00 coffee break
- 11.00 – 12.00 Nicolas Balacheff (Laboratoire Leibniz Grenoble): *Bridging knowing and proving: the complexity of the epistemological genesis of mathematical proof*

**Afternoon session**

- 14.00 – 15.00 Brendan Larvor (University of Hertfordshire de Havilland Campus): *What can Lakatos teach about teaching ?*
- 15.00 – 16.00 **Round Table 2: The cultural meaning of proof** (Moderator Hans Niels Jahnke)

**Evening** **Visit of the Abbey of Werden and walk along the Ruhr river**

**November 4**

**Morning session**

**Section 4: Proof and mathematical understanding. Different types of argumentation and proof**

- 08.30 – 09.30 Karine Chemla (Understanding, proving and the description of algorithms in the Book of mathematical procedures from China (ca 186 BCE))
- 09.30 – 10.30 Mariolina Bartolini-Bussi (Università di Modena): *Contexts for Approaching at Validation: The Function of Artefacts of Ancient Technologies*
- 10.30 – 11.00 coffee break
- 11.00 – 12.00 Maria Alessandra Mariotti (Università di Siena): *Contexts for Approaching at Validation: The Function of Artefacts of Information Technologies*

**Afternoon session**

- 14.00 – 15.00 Michael Neubrand (Universität Oldenburg): *Proving as Part of Mathematical Achievement: Concepts and Results from the PISA Study*
- 15.00 – 16.00 David Tall (University of Warwick): *The Cognitive Development of Different Types of Reasoning and Proof*
- 16.00 – 16.30 coffee break
- 16.30 – 17.30 Erich Christian Wittmann (University of Dortmund): *Operative Proofs*
- 17.30 – 18.30 **Round Table 3: Proof and explanation** (Moderator Gila Hanna)

**November 5** **Travel day**

## **List of contributions**

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