Change in Mathematical Views of First-Year-University Students

Günter Törner & Iris Kalesse
University of Duisburg

Ninety years ago the famous Göttinger mathematician Felix Klein started to create a series of books presenting elementary mathematics from a higher viewpoint (Klein 1908). In his preface he mentioned the later on often quoted two discontinuities[4]. Although the mathematical curricula for university students as well as for highschool students have changed much since those days, there are hints that the mentioned discontinuities are still existing, in particular on the level of mathematical beliefs.

Purpose of the study

The aim of the study was to identify and describe changes in the attitude spectra of teacher students within their first year at university in order to answer the question: Is there a real discontinuity? What are the influencing parameters of such a discontinuity. In a separate study a series of interviews were conducted with prospective teachers having left university and entering school on which the first author will report elsewhere. Here, seven students were interviewed within their first four weeks at university. Individual interviews were again arranged with the same group at the beginning of their second semester. Then we compared the 'measured' beliefs that evolved then with those verbalized in the first interview. This topic of prospective teacher freshmen has been focused by few authors (see Reichel 1991, 1992a, 1992b, Doig 1994, Malone 1996, Sander 1996 and others) during the last years.

[4] "Der junge Student sieht sich am Beginn seines Studiums vor Probleme gestellt, die ihn in keinem Punkte mehr an Dinge erinnern, mit denen er sich auf der Schule beschäftigt hat; natürlich vergißt er daher alle diese Sachen rasch und gründlich. Tritt er aber nach Absolvierung des Studiums ins Lehramt über, so soll er plötzlich eben diese herkömmliche Elementarmathematik schulmäßig unterrichten; da er diese Aufgabe kaum selbständig mit seiner Hochschulmathematik in Zusammenhang bringen kann, so wird er in den meisten Fällen recht bald die althergebrachte Unterrichtstradition aufnehmen und das Hochschulstudium bleibt ihm nur eine mehr oder minder angenehme Erinnerung, die auf seinen Unterricht keinen Einfluß hat. *Diese doppelte Kontinuität...*"

Theoretical framework

Since there is no commonly accepted standardized definition of beliefs we assume that beliefs are the compound of a person's subjective (experienced) implicit knowledge (and feelings) concerning mathematics and its teaching / learning (see Pehkonen & Törner 1996). In this sense we regard beliefs as attitudes constituting itselves through at least three components: an affective one, a behavioral one and a cognitive one. Each component (and, as a consequence, the belief as an attitude) is measured by different ways of reacting. In other words, we are in favour of the "three-components-approach" (Rosenberg & Hovland 1966) seeing attitudes as a - scarcely specified in more detail - system of cognition, affection, and behaviour (conation).

The cognitive component can be marked as subjective knowledge or, in general, as information of a person about an object. It is important to say that this knowledge does not have to be proven valid in an objective manner.

The affective component of a belief affects the emotional relationship to an object. It refers to the idea that a certain feeling or emotional state is connected with that social object an attitude is directed at.

The behavioral (conative) component of a belief which is relevant to action is readiness or tendency (probability) to act in a certain manner (a class of actions) regularly provoked by a social object. It should be mentioned that, in this case, a person is only ready to act; it is not necessary for the action to be carried out (cp. Süllwold, p. 476).

Methodology

The study is based on two series of videotaped interviews with the same seven mathematics teacher students, the first of which already has taken place in the first four weeks of winter semester 95 / 96, the second interview series was recorded in April 96. The interviews were held at the location of Gerhard-Mercator-University of Duisburg. The students were chosen by random. We asked an assistant to name seven students attending the main courses (Calculus I) of the first semester.

When working out the interviews we have being lead to a great extent by the questionnaires that Sander (1995) has introduced (see Kalesse 1996 for details), partly and independently used by Reichel.

Data Collection and Analysis

Our investigation covers two sets of interviews. The questions in the interviews covered different topics given by the categories of Ernest (Ernest 1989), namely (a) the student's conception of the nature of mathematics, (b) a student's model for teaching mathematics, (c) a student's model for learning mathematics and (d) a student's general

principle of education. In this note we focuse only some aspects of these dimensions. During both interviews the students were asked to rank fifteen items which are associated with the nature of mathematics, its teaching and learning process. The highest rank was assigned to the natural number 15, the lowest rank to number 1.

Each item was represented by a card and the students were asked to define some ranking of these cards by ordering them. The student's comments throughout this process were recorded.

		Teacher Students Level Secondary I						Teacher Students Level Secondary II							
		Ms. K 1.S. 2.S.		Ms. M 1.S. 2.S.		Mr. S 1.S. 2.S		Ms. A 1.S. 2.S		Ms. C 1.S. 2.S		Ms. E 1.S. 2.S		Mr. L 1.S. 2.S	
motivation	a/vp	11	7	15	15	15	7	14	10	15	15	15	11	15	14
comprehension	c/vp	14	14	13	9	14	15	10	11	12	1	13	5	12	15
duplicating of proofs	c/p	10	12	12	12	9	9	7	12	14	14	11	15	14	13
fun	a/vp	15	15	11	10	13	11	9	2	4	12	14	10	4	3
knowledge	c/p	13	11	7	14	11	14	12	13	9	8	6	14	9	7
good memory	b/p	4	10	8	11	7	13	13	8	13	13	12	6	7	8
own activity	c/vp	5	13	14	13	8	6	8	7	5	4	10	12	13	12
sense of achievement	a/vp	9	9	10	7	3	12	15	3	7	11	7	9	10	9
competence	c/vp	12	8	8	4	10	8	11	1	8	7	5	13	8	5
creativity	c/vp	7	4	3	8	4	1	5	6	6	5	9	7	11	4
imagination	c/vp	8	3	4	6	5	2	6	5	2	8	8	8	5	6
dull rote learning	b/n	3	1	5	2	2	5	2	15	11	9	4	1	2	2
emotions	a/p	8	8	2	3	6	3	3	4	3	3	3	3	6	11
learning by heart	b/n	1	5	9	5	1	4	1	14	10	10	2	2	1	1
fear	a/n	2	2	1	2	12	10	4	9	1	2	1	4	2	10

Further we decided to link these items with primarly the cognitive (c), the affective (a) or the behavioral (conative) (b) dimension of beliefs. Starting with the affective component, which has two outcomes of expectations (positive, negative), we differentiate the outcomes of the other variables into the degrees: very positive (vp), positive (p) and negative (n). Note that only duplicating of proofs involves a primarly mathematics-related dimension.

The results

We now take a closer, however limited view on each student in order to observe some mechanisms of their changes in attitudes towards the subjects within the first semester.

Mrs. K. Sec. I. She visited Hauptschule till grade 10. Her childhood was not pleasant which seemed to influence her career at school. She likes mathematics, but does not love mathematics. Her favorite subject would have been architecture. Not only the importance of motivation decreased, but also creativity, competence, phantasy, and knowledge. She seems to have the opinion that own activity, sense of achievement, good memory as well as dull rote learning are decisive factors.

Mrs. M. Sec. I. At school she was a very talented student in mathematics courses. Mathematics was a favorite subject of her, however dominated by the plan to become a teacher. Although she is very successful at university, she is not in favor of the kind one is learning mathematics at university. Since she was the best in her class, sense of achievement is no longer as important as it was. Instead of that she is favouring the factors: knowledge, creativitiy and seems to have become more demanding.

Mr. S. Sec. I. He visited Hauptschule till grade 10. His motivation towards mathematics depended much on the teacher. His expectation for the university studies was to get deeper insight in school mathematics. In the first interview he is afraid that he will not be capable to reach the exam. His attitude towards mathematics is neutral, on the other hand he admires mathematics. Meanwhile he has left university, however, we were lucky to get a second interview with him. He argued that the only possibility to keep track was to learn by heart which demotivated him. At school the teacher normaly would take care of each student and explains the various topics as long till all the students have acquired the theme. Thus, at university, he felt a need of being successful, however he failed. To him the variables 'creativity' and 'imagination' are no longer linked with mathematics, only the association with the variable 'fear' will remain.

Mrs. A. Secondary Level II. Realschule till grade 10. In Oberstufe she only attended a basic course. Before entering university math was her favorite subject. Her motivation towards mathematics depended much on the teacher. She states a correlation between having success and having fun in mathematics whereby success seems to be the primary factors. There is a dramatic qualitative and quantative change in her estimation of mathematics. She failed the written examinations. So there is a drastic change in her estimations of the relevant variables, e.g. learning by heart as well as fear are highly scored in the second interview. Although she describes her experience as a shock and a horror

trip with respect to affective factors, she decided to continue studying mathematics. Mrs. A. is an significant example how changes in attitudes leading in consequence to an affective self-destruction are caused by a bundle of factors. So she is pleading for a continuous consultation including psychological as well as mathematical aspects. Finally she proposed a separation of the teacher education from the Diploma study.

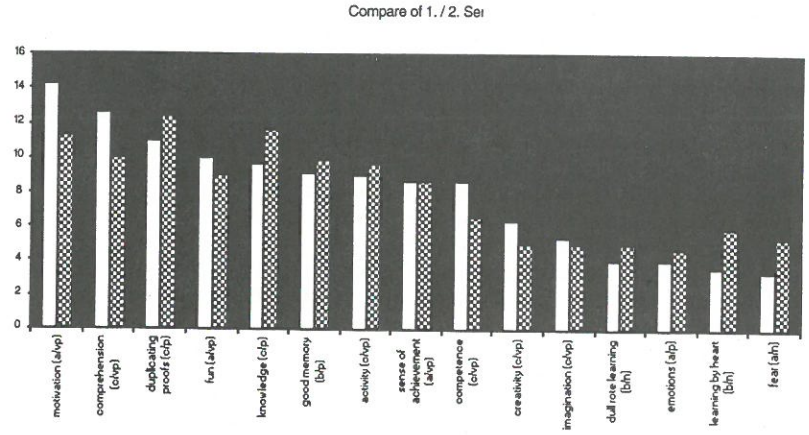
Mrs. C. Sec. II. Primarly she intended to become an ingenieur. She was highly motivated towards mathematics. At school she had experienced excellent math teachers. She successfully passed through both written examinations of the first semester. Although she had no major difficulties, she told that she had to work hard in the first semester. She would never advise students to study mathematics in case they have only attended basic courses at school. Nevertheless she heavily critizes the mode of learning and the quality of teaching. Only due to her unhurt and positive image of mathematics which she acquired at school she was able to 'survive' in the math courses at university and so she will continue her study. She felt a lack of positive stimuli for teacher students towards the intended later profession. So she demands fundamental changes for teacher education.

Mrs. E. Sec. II. At school she passed through advanced courses of mathematics. Her attitude towards mathematics was positive in general. She compare mathematics with an adventure playground. There is a slight, but significant change in her beliefs. She had failed the written examinations. Never before she had thought that mathematics is as hard as she experienced. Further, to her opinion, there are nearly no links to the intended profession. So, she claims for drastic changes.

Mr. L. Sec. II. He attended only basic courses at a school. His positive view towards mathematics was much influenced by teachers at school. To study mathematics was dominated by the fact to become a teacher as it is true for most of the teacher students. He successfully passed through the written examinations. Nevertheless there were moments when he asked himself whether he should quit his study of mathematics. He is complaining heavily on the circumstances of his math education at university. The university courses can be regarded as good counterexamples how students should never be treated. This observations lead to substantial increase in fear.

Some conclusions

The diagram below represents the means of the ranking. It is note suprising that variables with a very positive outcome score lower in the second series. This applies in particular to the variable 'motivation' studying mathematics. Further, affective components with a positive outcome score lower in the second interviews, however fear is increasing. Also the high estimation of cognitive variables suffer losses in general; only the ranking of knowledge, duplicating of proofs as well as own activity and sense of achievement receive better scores. On the other hand, behavioral (conative) terms are ranked higher without exceptions.



All teacher students, the most successful as well as those we have immediately acquired "mathematics anxiety", ask for an integration of a practical training as early as possible. It would help them to prepare themselves for their later role as a teacher on one side and to improve and activate the actual learning processes at university on the other side. We got the impression that nearly each student fears that his undamaged view on mathematics which he has acquired at school is threatened by his experience during the first semester. So, they express their wish that mathematics might remain unhurt as they believed before: characterized by creativity, beauty and utility.

Literature

Doig, B. 1994. Prospective teachers: significant events in their mathematical lifes. In J.P. Ponte & J.F. Matos (Eds.), *Proceedings of the 18 th International Conference for the Psychology of Mathematics Education.* PME XVIII. Volume 2 (pp. 272 - 279). Lisboa (Portugal): University of Lisboa.

Ernest, P. 1989. The knowledge, beliefs, and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching* 15, 13 - 33.

Kalesse, Iris. 1996. Beliefs and Change of Beliefs of First Year Students. In G. Törner (Ed.), Current State of Research on Mathematical Beliefs II; Proceedings of the MAVI 2 Workshop, University of Duisburg, March 8-11 (pp. 19 - 24). *Preprint Nr. 340. Gerhard-Mercator-Universität Duisburg Gesamthochschule: Schriftenreihe des Fachbereichs Mathematik.*

Klein, F. 1908. *Elementarmathematik von einem höheren Standpunkt*. Berlin: Springer-Verlag. Neuauflage.

Malone, J.A. 1996. Preservice secondary mathematics teachers' beliefs: Two case studies of emerging and evolving perceptions. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20 th Conference of the International Group for the Psychology of Mathematics Education.* Volume 3 (pp. 313 - 320). València: Universitat de València. Dept. de Didàctica de la Matemàtica.

Pehkonen, E. & Törner, G. 1996. Mathematical beliefs and different aspects of their meaning. International Reviews on Mathematical Education. 28 (1996) 4, 101 - 108.

Reichel, H.-C. 1991. Teaching student teachers: Integration of mathematics education into 'classical' mathematics courses. Examples and various aspects. *Journal für Mathematikdidaktik* 12, 367 - 377.

Reichel, H.-C. 1992a. Grundhaltungen, Motive und der 'seelische Bodensatz' von Lehramtskandidaten - ein persönlicher Bericht und ein Versuch einer Analyse. *Beiträge zum Mathematikunterricht* 359 - 362.

Reichel, H.-C. 1992b. Das integrierte Schulpraktikum in Wien and was man daraus lernen kann (Affektive Grundhaltungen von Lehramtskandidaten). *Mathematica Didactica* 15, 62 - 79.

Rosenberg, M.J. & C.I.Hovland. Attidude Organisation and Change. New Haven and London. Yale University Press. Fourth printing Feb. 1969.

Sander, H.-J. 1995. What is a good mathematics teacher? In G. Törner (Ed.), Current State of Research on Mathematical Beliefs; Proceedings of the MAVI Workshop, University of Duisburg, October 4-5, 1995 (pp. 57 – 61). *Preprint Nr. 310. Gerhard-Mercator-Universität Duisburg Gesamthochschule: Schriftenreihe des Fachbereichs Mathematik.*

Appendix: Addresses of the contributers

Dr. Andras Ambrus, Department of Mathematics, Edetvös Lorand University, Racoczi-utca 5.III.336, H-1088 Budapest, HUNGARY

Mr. Peter Berger, Institute of Mathematics, University of Duisburg, D-47048 Duisburg, GERMANY

Prof. Fulvia Furinghetti, University of Genua, Department of Mathematics, Via Dodecanneso 35, I-16146 Genua, ITALY

Prof. Günter Graumann, Institute of Mathematics, University of Bielefeld, PB 100131, D-33501 Bielefeld, GERMANY

Mr. Markku Hannula, Department of Teacher Education, University of Helsinki, PB 38 (Ratakatu 6A), FIN-00014 Helsinki, FINLAND

Ms Kirsti Hoskonen, Varkaus Secondary School, Kumputie 7, FIN-79700 Heinävesi, FINLAND

Dr. Sinikka Lindgren, Department of Teacher Education in Hämeenlinna, University of Tampere, Erottajankatu 12, FIN-13130 Hämeenlinna, FINLAND

Prof. Hermann Maier, Department of Mathematics Education, University of Regensburg, D-93040 Regensburg (Grosskunden-PLZ), GERMANY

Ms. Marja-Liisa Malmivuori, Department of Teacher Education, University of Helsinki, PB 38 (Ratakatu 6A), FIN-00014 Helsinki, FINLAND

Mr. Christoph Oster, Institute of Mathematics, University of Duisburg, D-47048 Duisburg, GERMANY

Dr. Erkki Pehkonen, Department of Teacher Education, University of Helsinki, PB 38 (Ratakatu 6A), FIN-00014 Helsinki, FINLAND

Prof. George Philippou, Department of Education, University of Cyprus, P.O.Box 537, CYP-Nicosia, CYPRUS

Prof. Ildar S. Safuanov, Komarova, 1, kv. 24, GUS-423806, Nabereshnie Tschelni -6, RUSSIA

Dr. Hans-Joachim Sander, FB 13 Natural Sc. & Mathematics in Vechta, University of Oldenburg, PB 1553, D-49364 Vechta, GERMANY

Prof. Günter Törner, Institute of Mathematics, University of Duisburg, D-47048 Duisburg, GERMANY