Mathematical Beliefs and Their Impact on the Students' Mathematical Performance

Questions Raised by the TIMSS Results

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TIMSS has played a major role in political discussions regarding education in the past few months due to the fact that German students only within the middle range pertaining to classroom performance. Unfortunately, the present discussion over language reforms in Germany seems to have pushed us and this most interesting topic aside. It is obvious, however, that one should pursue this question to its furthest possible extent in which the direction of our research may deliver a explainable contribution.

The following representations are to be understood as a preliminary listing; they describe more the actual state of didactics research as it is presented in the literature, and they contain annotations to the few TIMSS results which have been accessible to all up to now.

The confinements of brevity prevent us from providing detailed descriptions of definitions of beliefs at this time. The reader is referred to Thompson (1992), Pehkonen & Törner (1996) and Törner & Pehkonen (1996).

1. Some Fundamental Remarks

Of course one can basically have second thoughts about discussing the priority of teaching mathematics from the point of view of efficiency. I would like to quote the mathematics philosopher Reuben Hersh (1986):

> *Anyone who has even been in the least interested in mathematics, or has even observed other people who were interested in it, is aware that mathematical work is work with ideas. Symbols are used as aids to thinking just as musical scores are used as aids to music. The music comes first, the score comes later. Moreover, the score can never be a full embodiment of the musical thoughts of the composer. (p. 18 - 19)*

Hersh´s comparison of mathematics with music more likely raises the question, to which extent one can do justice to the teaching of mathematics by the successful measuring of beliefs when concerning aspects of "virtuosity" of mathematics in the classroom. Furthermore Thompson (1992) expands on the idea by saying:

> *Yet, as noted in the Standards (NCTM, 1989), traditional teaching emphases have been on the mastery of symbols and procedures, largely ignoring the processes of mathematics and the fact that mathematical knowledge often emerges from dealing with problem situations. Indeed, the converse of Hersh's statement can be used to*

characterize typical school mathematics - first comes the score, but the music never follows.

In this respect many results are clouded by fundamental doubts concerning the success of teaching mathematics, especially those conclusions ascertained by the TIMSS. Yet the measurability of success by schooled learning is a political axiom. Only measurable success is actual success!?...

2. Mathematical Performance and Beliefs

And why should the innerlying beliefs of the teachers and the students be an important part to the character of the explanation? It is shown that the question why problem solving associated with individual cases has produced only minimal success refers directly to mathematical beliefs. And yet I have become more cautious (reserved) through my investigations as I will subsequently reveal.

2.1 Conceptual Frame for Interrelated Curricula

If one looks for didactical papers on mathematics which provide evidence for a connection between beliefs on mathematics and mathematical performance, one shall come across various articles in the literature of the early 80´s in the area of problem solving (e.g. Buchanan, 1987; Cobb, 1984; Garafalo, 1989; Grover & Kennedy, 1994; Schoenfeld, 1983; Wheatley, 1984). The timely frequence of such articles is not astonishing, however, due to the fact that the style and character of teaching mathematics through problem solving was a main theme of didactics during those years. On the other hand informed persons were not astonished by the observations made at the time originating in a more open teaching situation.

The shortcomings described in these papers highlight fundamental limitations in when approaching mathematics. Attitudes were criticised which, for example, reduced the teaching of mathematics to the simple memorising of facts and algorithms. Such an opinion would naturally produce a negative effect on the will to memorise formulas if one believes that mathematics could be subdivided in isolated sections and that the material need only be retained until the next classroom test. He who favours formulas exclusively and depreciates their derivations as such, only supports a mechanical approach to mathematics. As representative of other quotations we refer to the following:

> *Recent research in mathematics education has shown that success or failure in solving mathematics problems often depends on much more than the knowledge of requisite mathematical content... Other factors, such as decisions one makes and the strategies one uses in connection with the control and regulatio of one's actions (...), the emotions one feels while working on a mathematical task (e.g. anxiety, frustration, enjoyment), and the beliefs one holds relevant to performance on mathematical tasks, influence the direction and outcomes of one's performance. ([Ga89])*

If one takes these observations seriously, it is not so much the specific views of mathematics which are to be criticised, but it is rather the implementation of the mathematics curricula in the classroom; it is the reduction of the Intended Curriculum to the Implemented Curriculum in the daily lessons. These reductions occur yet again at further points, for example reduction from the Implemented Curriculum offered in the classroom to curriculum attained by the individual. In analogy of the considerations from the framework of the TIMSS (see [Ro93]), the following diagram describes the pattern of the transformation:

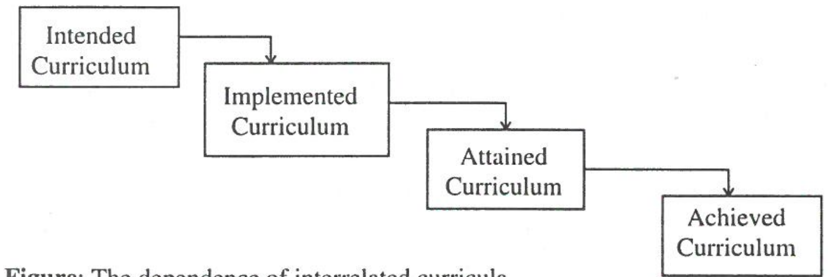

Figure: The dependence of interrelated curricula

These connections and indirect influences can rarely be altered through curricular modifications. Moreover, they can only be described to a certain degree and conditionally brought into question. Each teacher would attest to the indispensable compromises and inherent necessities which actually lead to a rather grey representation of real mathematics teaching in his or her classes originating from a colourful illustration of ideal mathematics teaching.

Let us assume for the sake of simplicity of our exposition a linear input-output model. The influencing effect of beliefs should be of interest. The box 'Intended Curriculum' serves as the input variable. Robitaille understands Intended Curriculum to be the following:

> *The question of who makes curriculum decisions is a fundamental and timeless issue... The array of participants who officially designated or who function through default to make curriculum decisions is complex enough, but the question centers around not only who makes them, but also what type of curriculum decision is under discussion.*

The intervention of influence results from the many different players and interest groups, with which among others teacher training programs at the university are associated. These involvements crystallise into a concrete framework regarding content by school administrators. The curricula having originated as such may be monolithic in character, but they can reflect also inconsistent majority votes (see Ernest [Er93]). In this respect the intended curricula are never neutral concerning a world view; in fact, they are full of beliefs. In general, however, the curricula are organised in a plural fashion on the grounds of a necessary and widespread suitability. They are exemplary mostly of the idea of 'compromise.' Here there is room for a view of mathematics as an art form as well as addressing the toolbox aspect of mathematics. In addition, it allows for the idea which describes mathematics as a system. Finally the relevance of the applications will also not be concealed. In this regard one can assume that generally the official curricula can be classified as tolerant in respect to world views.

At the point between 'Intended Curriculum' and 'Implemented Curriculum' the teacher takes on a major role of importance:

> *Teachers fulfill a variety of functions regarding the creation and implementation of curriculum materials, their curriculum 'texts'... The interpretation of curriculum materials allows teachers to express their individual approaches to teaching, as well as their responses to the needs of their specific classroom situation.*

There are social and cultural stipulations surrounding this concept which decide the coupling gradients. It can be assumed, that the transition must generally be understood as a process of adaptation, which levels off peaks and fills in holes. Everything is being smoothed out and simplified as well as nonessentials being left out and avoided, whereby overdrawn requirements and expectations in the curricula may not be made responsible. In specific cases attention may be observed regarding additional, individual profiles and emphases with the conversion of the 'Intended Curriculum.' However, refinements will be used, in the author's opinion, quite seldom. Comparable processes can be assumed when forming and changing beliefs.

The phenomenon occurring at the interfaces between 'Implemented Curriculum - Attained Curriculum' and 'Attained Curriculum - Achieved Curriculum,' respectfully, shall not be dealt with at this time. On the whole it must be assumed that this coupling process specifically relating to mathematics is rarely dealt with in the literature. The same is true of the formation of beliefs induced at the time.

Back to the observations quoted above about limited mathematical beliefs: basically two clarifications must be admitted. Deficits can be assigned to the parties immediately involved such as teachers, school administrators, students, etc. Because the couplings described above (see Figure) are not, as a rule, fully fitting, it can be presumed with equal justification that the deficits are

immanently induced from the transformation process (in a constructivist view: construction process, respectfully). For this reason the conclusions drawn from input variables (the role of specific beliefs), are flawed with a very large amount of uncertainty; hastily drawn conclusions cannot be scientifically responsible.

Independent of this, however, the question arises to which extent varying mathematical world views are resistant to transformation at the given segments (see Figure).

2.2 The Role of Self-Concepts

The self-concepts of the students play an important role in the literature in connection with beliefs and their prognosa character (Cooper & Robinson, 1991; Evans, 1987; Hannula & Malmivuori, 1996; Kloosterman, 1991; Malmivuori & Pehkonen, 1996). The effect of gender-specific factors would be suitable to be discussed here. The total immediate connections to the detailed mathematical beliefs appear unclear and require a more exact analysis within the structure of case studies.

3. Mathematical Beliefs and TIMSS' performance

3.1 The Philippou-Hypothesis

At the present time there is not enough data collected from the results of the TIMSS to draw key conclusions from the full spectrum of the performance of the individual countries. It will still take a fairly long time until the material in this point of view can be made accessible. In the first attempt, the work of Philippou represents the world views of 7th grade teachers. The author distinguishes the grouping of TIMSS countries into three categories. He characterises a top group, to which Singapore, Hong Kong, Korea, and Japan belong. The middle group comprises the countries of England, Germany, Belgium and Sweden. The bottom group is represented by Greece, Cyprus, Colombia and Iran. These groups are chosen with respect to the performance of the students in these countries. The responses of teachers in each group of countries (drawn from the report made available by Cyprus' national representative) were combined together (weighed average) to form one entity i.e. the agreement / disagreement proportion for each group of countries. Using some kind of Median Polishing Analysis he is able to determine significant differences concerning the conceptions about the nature of mathematics. The following table is deduced from data from the paper (Philippou, 1997) and only contains tendencies (++ / + / - / --) regarding items from the international questionnaire which we further quote below. In respect to the particular details we refer to the original work:

Countries / Item	N1	N2	N3	N4	N5
Top group		+	+	--	
Middle group	--	+	-	++	++
Bottom group	-	-	-	++	
Total	--	--	--	--	++

	Mathematics conceived as a set of rules
N1	Mathematics should be learned as sets of algorithms that cover all possibilities.
N2	Basic computational skills are sufficient for teaching primary school mathematics.
N3	Mathematics is primarily an abstract subject.
	Mathematics conceived as a connected and coherent body of knowledge
N4	Mathematics is primarily a formal way of representing the real world.
N5	Mathematics is primarily a practical and structured guide for addressing real situations.

The table above makes clear various view of mathematics. For brevity we shall not pay mention to obvious interpretations and will refer to the actual work

3.2 Are the TIMSS questions suitable to determine world views?

A major point of interest are the teacher questionnaires, whose results are only partly known in the public. The following questions obviously touch on the aspects of beliefs:

	In order to be good in mathematics in school, how important is it for students to...according to your opinion? (not important, somewhat important, very important)
15a	To be good in math is important to remember formulas and algorithms
15b	To be good in math is important to think logically and consistently
15c	To be good in math is important to understand mathematical concepts, fundamentals and strategies.
15d	To be good in math is important to be able think creatively.
15e	To be good in math is important to understand how mathematics is used in the real world.
15f	To be good in math is important to be able to provide reasons to support their conclusions.
	To which extent do you agree with the following statements? (strongly disagree, disagree, agree, strongly agree)
16a	In the first place, mathematics is an abstract field.
16b	Mathematics is primarily a formal way of representing the real world
16c	Mathematics is primarily a practical and structured guide to approach a real situation.
16d	If students are having difficulty, an effective approach is to give them more practice by themselves during the classes.

16e	Some students have a natural talent for mathematics and others do not
16f	More than one representation (picture, concrete materials, symbol, etc.) should be used in teaching a mathematics topic.
16g	Mathematics should be learned as systems of algorithms and rules which uncover all possibilities.
16h	Basic computational skills on the part of the teacher are adequate to teach the primary level of mathematics.
16i	Attachment to and understanding for the students are essential for teaching mathematics

Some of the results can be found in corresponding (see Beaton[2] et al.). The complete factor analysis has not been published and - as far as the author knows - does not reveal some convincing pattern.

Thus, the German TIMSS-team developed additional items which we list below. For brevity, however, we must omit a presentation of the results and their interpretation.

1	Mathematics affects each one of us everyday.
2	Mathematics helps in explaining economic occurrences.
3	Mathematics helps in solving technical problems.
4	Mathematics contains the aesthetics associated with formulas.
5	What I learn in mathematics I can use in other courses of study.
6	A mathematical theory is similar to a work of art because both are the result of creativity.
7	The goal of mathematical theories is to make life more comfortable.
8	Proof or derivation of a formula is not important. The important thing is that I can apply it.
9	One day the mathematicians will have discovered all of which mathematics can provide.
10	In mathematics there is always only *one* way to solve problems.
11	Mathematics is the remembering and application of definitions and formulas, all from mathematical facts and procedures.
12	Mathematics is just a game with figures, pictures and formulas.
13	Mathematics is of no use to me in other courses of study.
14	Most of the mathematical problems have already been solved.
15	Mathematics is used in many tasks in everyday life.
16	Almost all mathematical problems can be solved by the direct application of known rules, formulas and procedures.
17	Mathematics is essentially a game.
18	The goal of mathematical theories is to solve practical problems.
19	Doing mathematics means applying general laws and procedures to specific tasks.
20	Mathematics is a language which provides its own stimulation.

The upcoming papers will present and interpret the results.

[2] http://wwwcsteep.bc.edu/timss

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