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Methodological Considerations on Belief Research and Some Observations

1. Introduction

It is the purpose of this paper to present a short discussion on a few methodological concepts on belief research. We would especially like to point out the meaning of metaphors as linguistic variables.

Beliefs, belief systems and mathematical world views respectively will be understood as attitude structures (see [17]). In this case of identifying beliefs one could actually fall back on the corresponding methods used in psychology. The analogue transferral of statements from the research into attitudes establishes itself only as being conditionally suitable (see the discussion in [9]), because teaching mathematics must be understood as an integral, multi-layered process. In this respect one will want to develop specific methodologies in researching mathematical beliefs.

A widely accepted definition states that, attitudes consist of coherent, cognitive, emotional and action-relevant components. So far, the question arises to which extent the methodological concepts, namely identifying beliefs, carry the responsibility for the parallelism among structures. If one transfers the elements from BLANEY's theories (see [6]) onto the research into beliefs, it must be assumed, that the cognitive net of mathematical contents overlaps the emotional net. We can also assume that single, mathematical objects, i.e. the theorems, the methods, the formulas, the terms etc. have an emotional loading. The spectrum of it all reaches past the every-day observances from the negative estimations through the indifferent ones right up to the positive estimations

2. Evaluational approaches

The reform of teaching mathematics during the 1960s assumed unexplicitly a monolithic view of mathematics which was, however, internally specified in the sense of BOURBAKI's axiomatics. The disallusioning questions along with the reform among others have made clear that one can not be fair with this image of mathematics in relevance to the school classroom. It is surprising, how qualifying changes in the estimations of the philosophy of mathematics coincide with the observation that individual views of mathematics must be assigned to a psychologically greater meaning in the learning and teaching processes. With this idea the research into beliefs and world views, respectfully, won tremendous notoriety.

In the meantime the tool used by DIONNE to evaluate perceptions ([5]) is recognized as classical, namely to coordinatize belief structures by vectorial distributions of weights with respect to some basis components. It was the idea of DIONNE that beliefs constitute a mixture of (basic) components.

For his research, DIONNE [5] used the following three perspectives of mathematics:

(A) Mathematics is seen as a set of skills (*traditional* perspective): Doing mathematics is doing calculations, using rules, using procedures and using formulas.

(B) Mathematics is seen as logic and rigor (*formalist* perspective): Doing mathematics is writing rigorous proofs, using a precise and rigorous language and using unifying concepts.

(C) Mathematics is seen as a constructive process (*constructivist* perspective): Doing mathematics is developing thinking processes, building rules and formulas from experiences on reality and finding relations between different notions.

In his book, ERNEST [7] describes three views of mathematics: instrumentist, platonist and problem solving. These correspond more or less to the three perspectives of Dionne (1984) mentioned above. Furthermore, the same three-component model was rediscovered ten years later by TÖRNER / GRIGUTSCH [27] calling (A) the *toolbox aspect*, (B) the *system aspect* and (C) the *process aspect*. In order to determine a person's mathematical beliefs, this person must be asked to distribute a total of 30 points to these components.

The methodical problems appearing as a result are three-fold in nature. Firstly, the problem is the prior identification of the assumed dimensions. Regarding the above mentioned basis categories it can be accepted that these factorally analytical examinations have pointed themselves out to be quite stable although they seem to be in need of completion. Recent research shows that there are at least four basic components in an individual's view of mathematics: the fourth component seems to be application (GRIGUTSCH / RAATZ / TÖRNER 1997).

Secondly, even more difficult is ascertaining the vectoral data from the test persons in question. Almost no considerations of plausibility are available to the test subjects. Therefore, his appraisals should not be over interpreted.

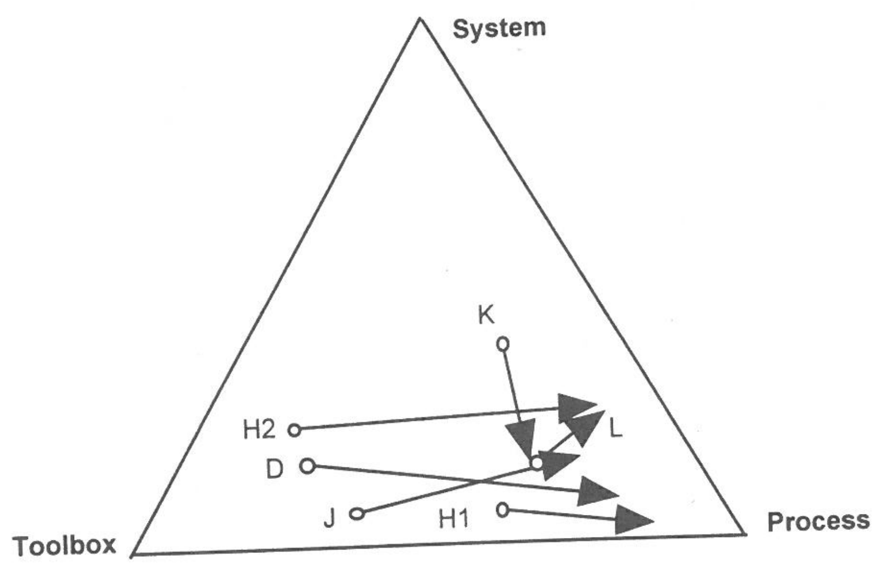

Figure 1: The self-estimation of six teachers in the aspects of toolbox, process and system in the view of real and ideal teaching of mathematics, respectfully (the arrow shows the tendency of change from the real to the ideal teaching situation).

Finally the basic question is presented to which extent the ascertained point distribution can justify the statement: the theoretical construction of 'beliefs.' In other words, the problem presents itself to which extent to the cognitive, emotional and action-related components can be understood with a numerical list.

In each case the Dionnian statement must be classified as coarse. It should however be noted as positive that this entirely questionable method is accessible only on the elementary level. It proves itself in practice to be robust and convincingly provides the proof that there are various, subjective estimations of the raised attitude objects.

PEHKONEN / TÖRNER [18] have recently refined the basic DIONNE fundamentals into a graphic illustration of components which is more than just a complementary for of presentation. Test persons were asked to graphically mark their self-estimations on an equilateral triangle in regard to the practical and the ideal teaching situation of mathematics. We are referring to the work appearing subsequently. The result is that such a graphical evaluation of data is not redundant across a clear listing of the data, but rather produces a complementary character throughout. The following diagram from [18] contains the self-estimation value of the test persons through which we, then, clarified the tendencies of change (real to ideal teaching situation) by means of vectors.

This method allows in principle only two free variables. In practice one can assume that three variables in their relationship to one another can be described, and a normalization can be assumed.

These complementary statements are to be recognized as advantages:

- the method combines the advantages of DIONNE's approach
- it serves as an additional data source
- vectorial data are represented graphically
- it assists the interviewed person to generate data
- different data can be visualized in one figure
- tendencies can be made apparent

3. Procedure for factor analysis

The prevailing number of quantitative works under consideration about the beliefs of teachers and students (compare with the literature [28]) serves as methods by which certain factors are analyzed. Beside the fundamental problem of quantitative procedures the statements from the questionnaires must be kept out of our sight in this context and classified. The expense on the part of the test persons is not insignificant, especially if going on the assumption that the simultaneous reaction to analogue, linguistic charms is necessary for considering an attitude. To this extent each component must be represented repeatedly in the questionnaires. As a result, the number of the testable items is inevitably limited. Because the discussion about the consistency-theorem has not found a satisfying conclusion until now, the consistency between the cognitive and emotional net must be postulated *a priori*.

In contrast to statements up until now GRIGUTSCH, RAATZ and TÖRNER have (compare also with [29] and the dissertation by GRIGUTSCH [11]) constructed graphs on the basis of the partial correlations. The vertices are the factors whereas the edges of the graph are defined through the significant correlations. It is obvious that these structures have to be interpreted. The produced structure will be labelled by us as a *factor analytical model of a belief system*. This net structure clarifies especially the relative stability of mathematical views of the world. From [10] will quote the following diagram which touches on a partial correlation (n = 253) of a factor analysis with the four factors: formalism (F), application (A), process (P) and system (S).

It must, once again, be established that this methodological statement doesn't provide an explicit differentiation between the cognitive aspects and the emotional loadings. Formalism is, for example, for the individual in question neither *a priori* positively nor negatively loaded.

	F	A	P	S
F	1,000	,042	-,127 *	,364 ***
A	,042	1,000	,127 *	,087
P	-,127 *	,127 *	1,000	-,146 *
S	,364 ***	,087	-,146 *	1,000

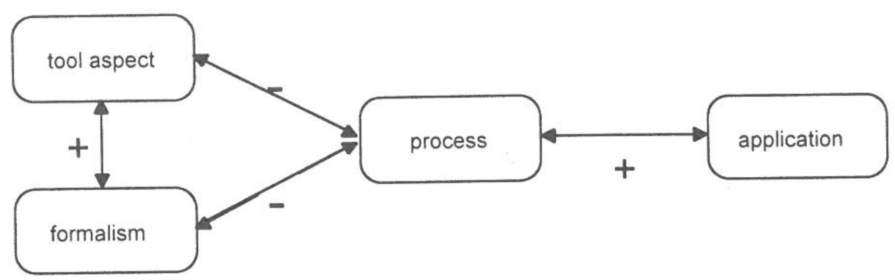

Figure 2: Intercorrelative relationships between the scales.

The method portrayed in Section 3 is trivially statistical in nature and can state nothing specific in regard to the individual case. It only makes clear that belief structures are complex and must be classified as stable. It does not make any reference, however, to how these structures could possibly be changed. Thus, it is quite natural to ask for more individual, quantitative indicator.

4. Beliefs as linguistic variables

4.1 The role of metaphors

According to MAIER [15, p. 119] language (also in the mathematics education) is the most important medium which is readily available to the teacher. Language accompanies and even initiates each constructivistic learning process. Also, the choice of language medium in the classroom is of extreme importance as in almost no other field regarding technical language, idioms and the actual language competence of the students.

In addition, the translation process, the process of modelling using mathematics, is too a significant theme of the lesson. The process of translation (everyday language to mathematical terminology) gains further degrees of freedom as well increases in complexity if internal language levels attracts some attention through the use of figures of speech.

It is primarily through the work of LAKOFF [12] (see also [1] and [15]) to bring attention to the meaning of figures of speech in teaching and learning processes, in particular in mathematics. These figures of speech are not simply a beautified decoration in the process of communication but rather specific elements which are brought about, formed and presumably initiated.

It was BAUERSFELD and ZAWADOWSKI [1] who, already in 1981, emphasized the meaning of metaphors for the classroom:

> We want to mention a further figure of speech. ... Metaphors, quite in contrast to this, are nearly always formed deliberately and with intention and sometimes with much intellectual effort. In order to the use of metonymies we produce metaphors in cases when we want to evoke a certain understanding, we want accentuate an aspect, or to lay emphasis on certaon properties, and yet we are in lack of common words for it with an established meaning.
>
> In this sense a metaphor is like the square root of 2 expressed within the rationals. Such situations occur very often not only in living speech but also in mathematics teaching, where they seem to build the rule rather than rare exceptions.

In this regard we refer to the chapter entitled 'Mathematik und Rhetorik' in [3] in which the meaning of rhetorical elements by means of mathematics is enlightened.

(1) Some cognition scientists see a close connection between the learning and the use of metaphors in the following way:

> The metaphor is one of the central tools in overcoming the epistemological gap that exists between the old and the new knowledge. (Petri)... Simplifying thinks a little bit, one may say that, in the first case, the metaphor serves understanding, and, in the second, that it serves explanation. (compare wih [19], pp. 92)

(2) Memory psychologists favour the coding of stored information using pictures. Here the metaphors play the role of basis macros:

> The role of metaphor for organizing and communicating thoughts about one's personal reality is central to a constructivist's approach to language which views individual constructions of personalized realities as limited by individual knowledge and language. (cf. [8]. p. 104)

(3) For constructivists metaphors are powerful aids:

> We must give students "tools to think with" - and these are not merely formulas and algorithms. They include concepts and powerful metaphors... What kinds of experience does school need to provide to children? ... we might list four ... (2) deliberately created "assimilation paradigms" - that is to say, carefully designed metaphors that correctly mirror the structural features of various pieces of mathematics, and which therefore give the student a basis for powerful mental representations... (cf. [4], pp. 188)

4.2 Metaphors and beliefs

It is probably more than just coincidence that the platonian view of mathematics, hence the classical prototype of a mathematical world view, was clarified by PLATO using metaphors as a medium:

> ... hunters and fishermen hand over their catch to the cooks... but the surveyor, arithmetician and the astronomer are also hunters, because they do not create their figures and rows of numbers, rather these already exist and one only comes across them as they already are and hand them over to the dialecticians (transitional dialogue EUTHYDEM 290 B.C.).

It was TOBIN especially[17a] who brought attention to the role of metaphors by identifying mathematical beliefs and who was supported by his empirical results.

But why can we expect that metaphors illuminate mathematical beliefs? In order to understand teaching and learning processes in mathematics, an analysis of the different underlying relations is specifically required. There is the teacher; there is the student and his class, and their social interrelations. Then, we have the fixed curriculum on the other side the mathematical content. In short, there are many highly woven complex relationships among different subjects and objects which have to be modelled and partly personally evaluated.

Metaphors are of relational character.

In a sense, teaching and learning within a class is like playing ball. There is the ball (= material), there are players and so on. It is obvious that illuminating metaphors produce a corresponding, relational structure. We can also speak of a 'salesman' and 'clients' and the 'ball' is the product which has to be sold or bought respectively.

LEINO, A.-L / DRAKENBERG, M. [15] approach this theme from the other side and name as the core of suited metaphors the categories of schools, up-bringing and curriculum. Previous research (TOBIN [20]; TOBIN / LAMASTER [25]) suggested metaphors for teaching typically describe three distinct roles of teachers: teaching, assessing, and classroom management. It has the appearance that the relevant, fundamental categories, which have the character role of the teacher, have not yet been definitively nor thoroughly discussed.

Starting points suited to particular professions seem to be metaphors in that they concern the interaction between student, teacher and material because these produce similiar relations in regard to the clients (customers) of the time and the products in question. Certain occupations produce moreover the advantage that their characteristics can be knowingly assumed as being sufficient.

Without continuing the discussion , we will mention at this time some professions which are named in the literature in the context of mathematics lesson: gardener, gas station attendant, construction worker, guide (BERRY / SAHLBERG [2]); intimidator (TOBIN / GALLAGHER [22]); preacher (TOBIN / ESPINET [19]); policeman, mother hen?, entertainer (TOBIN / JAKUBOWSKI [23]); captain (TOBIN / KAHLE / FRASER [24]); saintly facilitator, manager, assessor, comedian (TOBIN / ULERICK [26]); comedian, miser, social director, researcher, mentor, coach, city-planner, telephone-operator, mother, magician. (FLEENER ET AL. [8]).

Already at first glance this list makes clear that these professions display typical characteristics of certain attitudes and are always in a context associated with feelings for the world as if they played particular roles. Also the dual relationship between teacher and student is modelled as typical of the profession. To a certain extent the relative diversity of the belief constructions carries the responsibility.

In this lies the second advantage for the use of metaphors:

Figures of speech, e.g. the metaphors, carry subjective emotional loadings.

Our basic concern is, as mentioned above, the description of beliefs by the utilization of adequate metaphors whereby the aspect of the roles and the material must carry the main responsibility.

4.3 Metaphors as methodical tools in the research of beliefs

In addition to the fundamental roles described above in context with beliefs figures of speech can be seen as a methodological tool. The following categories appear to be meaningful, which are already mentioned in various papers.

(a) Metaphors help to make abstract belief constructs concrete.

Refering to well-known relational figures, it is possible to generate belief constructs:

(b) Metaphors help to conceptualize beliefs.

Naturally the test person does not want to inevitably be identified with an analoging metaphor situation. This separation from a declination up until an agreement must be valued as a terrible process of self-reflection which can be described as a conceptualization of beliefs.

(c) Metaphors help to group beliefs as belief systems.

Because the figures of speech in question display, on the most part, many relative "lefts" and are associated also with emotional components (e.g. orders from a captain) not only are the single beliefs addressed but entire belief systems are grouped.

(d) Through metaphors beliefs become conscious and are open for reflections.

Just as it was addressed in (b), the self-reflection leads to an awareness of possible unconsciously functioning beliefs.

(e) Metaphors help to verbalize and represent beliefs.

A critical consideration of related figures of speech support a verbalization of beliefs. At the same time a representation independent of language is made possible as the paper by BERRY / SAHLBERG [2] shows. Here, metaphors are being represented through comics.

(f) Through metaphors comparisons of beliefs become possible.

On the same note, the paper of BERRY / SAHLBERG [2] shows that beliefs represented by metaphors are accessible to an international comparison independent of language. A recent paper of LERMAN [15] shows that

(g) Through metaphors different psychological theories in mathematics education can be made apparent.

Admittedly the statements sketched here nothing more than plausible and conclusive hypotheses of this paper which allow themselves to be justified by their subsequent productivity. In this context LAKOFF's work (see [13]) raised the question: *Invariance Hypothesis: Do Metaphors Preserve Cognitive Topology?* we may ask the following questions:

- Do metaphors preserve affective contexts of beliefs?
- Do metaphors preserve behavorial aspects of beliefs?
- Do metaphors preserve cognitive structures of beliefs?

Here, certainly many questions still remain unanswered.

4.4 Metaphors as Methodical Tools in the Influence of Beliefs

At this point it should be briefly mentioned that Tobin stresses the fact that the reflecting of the teacher's behavior on the basis of metaphors can, as a master switch, ultimately lead to a change in the teacher's own individual behavior.

4.5 Using metaphors as linguistic variable to measure beliefs

It has become clear that metaphors can be understood as linguistic variables. A similar statement is known as the Fuzzy Control Theory. In this case linguistic variables model physical conditions, whereby the coordinates are not of acute but rather linguistical value, which are coupled over the function of membership (value distribution between 0 and 1) with the acute values. The linguistical variable "age" can be understood and compared with the conditions "young", "middle-aged", "old" and "very old", whereby it requires a subjective interpretation in order to characterize, for example, a person 50 years of age as being 80% "middle-aged" and 15% "old". The value of the function of membership for "young" can be shown as 0% for a person 80 years of age.

We will apply this statement by concerning ourselves with the linguistic variable "distance" in a teacher/student relationship. Apparently this proximity is within the parameters of the teaching/learning process and overpowers varying estimations. This lingustic variable "proximity" is coordinatize by the metaphors "entertainer" and "captain" which play the role of functions of membership. The question about beliefs on teaching mathematics would at the same time be seen as the question, to which extent does the teacher play the role more or less of the "entertainer" or the "captain". Here the person in question can graphically integrate him/herself. A corresponding picture reveals this point as the following:

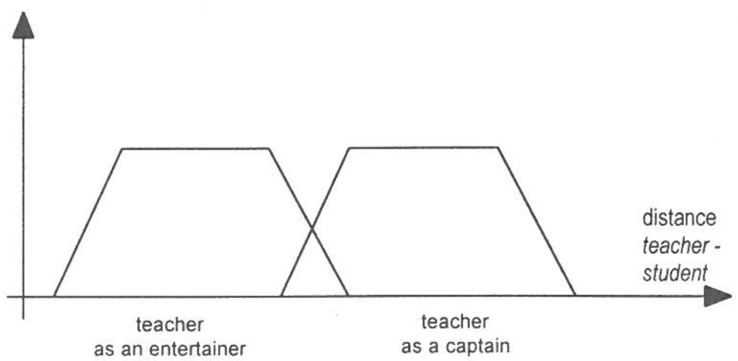

Figure 3: Membership functions of metaphors with respect to the linguistic variable 'distance'.

Apparently more variables play a role in the teaching / learning process; therefore, not only the management and entertainment aspects, respectfully, but also the aspects of training, of being an expert, etc. play roles as well. On the other hand, these linguistic variables are roughly typified by the use metaphors which then take on the role of function of membership.

5. Summary

On this note, the statements of this paper should be portrayed as an overview of an understanding of belief structures. In addition to the classical, quantitative analysis, the qualitative and apparently also the linguistic statements seem to be promising. Not much can be addressed in further detail at this point concerning the application of the teacher interviews

which were conducted, but these connections will be addressed and presented in a further detailed paper.

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