

## NARRATION AS A TOOL FOR ANALYZING BELIEFS ON CALCULUS – A CASE STUDY

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### The context of the case study

The following study is initiated on the whole by four circumstances. First of all by the discussion on calculus lessons in Germany and their importance for school mathematics education in the Sekundarstufe II. In the German educational system calculus plays a central role in the secondary grades (age groups 16-19) (see details of the German system in [Robitaille 97]). This circumstance dates back to the initiative of the famous mathematician Felix Klein at the beginning of this century. His concepts are still markedly present in the German mathematics curriculum of today. This results in the fact that calculus contents are always compulsory parts in the Abitur examination (the final school-leaving examination). The demands in the mathematics Leistungskurse in Germany (equivalent to A-Level courses in Great Britain) are greater compared to those examinations equivalent in the United States (NCTM 1991). Calculus also already plays a dominant role in the first semester of university prospective teacher courses for the Sekundarstufe II. These students visit the same lectures and seminars as those studying for a purely academic degree.

However, it can not be ignored that calculus lessons in schools are increasingly slipping into an orientation crisis. The unfulfilled, exaggerated expectations towards New Maths can be held partially responsible - which views calculus primarily as a propaedeutic field of science. After all, the present discussion is also being pushed forward by the availability of CAS-Tools in pocket calculators featuring graphics display facilities (DERIVE). These calculators reduce the usual traditional curve discussions and calculus extrema exercises (suitable for schema-oriented lessons) down to literally trivial example applications. This view amongst teachers, however, has at present not yet affected the traditionally oriented calculus canon at university. In contrast to the United States, a calculus debate has not yet got underway in Germany.

Nonetheless, new demands are being discussed in occasionally offered lectures on mathematics education for prospective teachers. However, such seminars do not predominantly serve the acquisition of methodical aspects of calculus but intend to exemplary introduce general and specific didactic aspects related to mathematics lessons. The subject material employed in this paper originated from such a seminar.

Due to the central role for school and for teachers' training, sensitive issues related to teachers training can be ascertained by research related to the scope of this field. The fundamental question arises concerning the effects and the results of present teachers' training.

The third motive is related to the search for adapted tools for the initiation of self-reflection on those who do mathematics. Reflection on mathematics lessons can itself be carried out on different levels and be subject to widely differing objectives. In this

paper the question is discussed to what extent self-reflection in the form of narration elements can offer highly differing information on system-internal states which might reveal concealed parameter. The innate ideas of the learner are also articulated here. The subject material employed for this question were topics of papers by 7th semester students who were requested to write essays on this in a seminar on mathematics education focusing in particular calculus.

In the literature reflection is emphasized as having a central role as a structuring facility. Both for oneself and for the external observer reflections reveal beliefs and also conceptual systems expressing teachers' thoughts and actions. Reflection has, therefore, gained significant acceptance as a basis of prospective teacher education. Chapman (1998) (cf. the literature quoted there) emphasizes that in general preservice teachers tend to rely on their personal experiences as learners in constructing meaning for classroom events. Self-reflection is also employed as a means to accompany and support processes of change. Self-reflection, therefore, is also a basis for a prospective teacher's professional progress. Different possibilities arise for the initiation of self-reflection. The use of narration in our study is based on the view that narration is a way in which one makes sense of the world (Bruner, 1986, see Chapman, 1998) by incorporating in particular one's own experiences.

Finally, it is our interest to better understand the role of beliefs for mathematical learning processes. There are good reasons for globally analyzing beliefs on mathematics. We can hereby speak of a top-down-approach. Here we are interested in the beliefs of different groups of persons: university teachers, teachers and pupils. In accordance to this, research into the German school system has been conducted by us. On the other hand one can assume that belief and knowledge structures are closely interwoven with each other and constitute themselves across diverse contents. It is therefore decisive to survey content-specific beliefs and to analyze their structural features.

### The research questions

According to Schoenfeld (1998), the interdependency of teacher's goals, beliefs and knowledge is decisive for the teaching process. From this we derive the following questions for our case study:

- Which beliefs on mathematics, in particular on calculus, are characteristic for prospective teachers? When were they developed?
- In which dimensions are calculus lessons perceived by prospective teachers: (A) the self-experienced lessons (in school, at universities)? (B) lessons considered desirable according to one's own experiences at university and on the basis of knowledge gained in seminars on mathematics education? (C) lessons given in the future as a qualified teacher?
- To what extent can permanent temporal lines of consistent views be determined? To what extent are self-experienced teaching situations criteria for self-planned lessons?

### Method and procedure

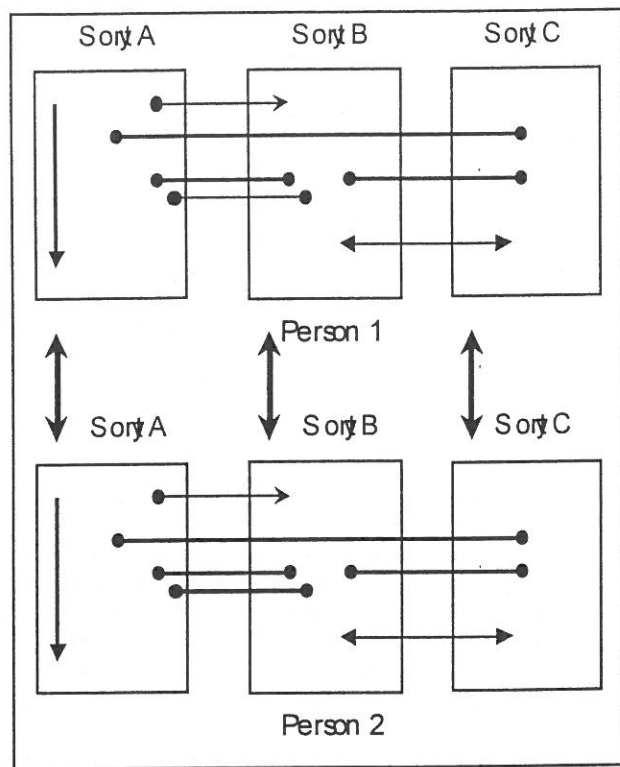
Within the framework of a seminar on calculus the students ( $n = 10$ ) were asked to write essays (1 – 2 pages) on the topics:

- (A) Calculus and me: how I (have) experienced calculus at school and university.
- (B) How I would have liked to have learned calculus.
- (C) How I would like to teach calculus.

These essay topics were subsequently handed out at intervals of three weeks. The students knew nothing of the actual case study approach of the author. All together six students handed in contributions on all three topics. Their essays are used here for this case study.

In the first viewing of the essays the given topics can be categorized as being similar. The research approach realized here is classified as triangulation (Cohen; Manion 1994). This procedure can be justified through the following basic assumptions: Learning and teaching are dual processes that can be individually considered as linked together. Possibly experienced deficits are categorized - when viewed positively - as points of emphasis of one's own responsibly conducted lessons. Positive experiences lead to reinforcement of one's own actions towards others. In this respect a temporal invariant consistency in the evaluation of one's own teaching and learning processes is presumed, whereby one must bear in mind that repeated mentioning of its aspects could lead to its confirmation.

Due to the limitation of number of pages an exhausting presentation of the three topics by the students cannot be expected here. No content-related expectations were placed on the students so that freely written essays were ensured. The location and topic change, induced by the respective question formulation, enables new reflection impulses and recapitulates new aspects of a topic from the viewpoint of the others. As the three topics are intended to illuminate different time concepts (A – past, B – present, C – future) it is to be expected that through the essays it will be possible to determine time-invariant lines.



Drawn together, two different patterns are on the whole the result of the evaluations. We speak of a horizontal evaluation when the positions in the three stories of the individual persons compare or confront. And we will speak of a vertical evaluation when we point out differing persons on similar or controversial positions in the stories A resp. B resp. C. It may be possible to formulate generalizations here when individual coherence or contrast patterns occur for a number of persons or content themes. The diagram above visualizes possible clusters of evaluation.

### Results and evaluation

The lines given by the three topics ((A), (B), (C)) offer text contributions which can be assigned to the following fields: (1) objectives of mathematics lessons and calculus, (2) views on mathematics, in particular calculus, (3) calculus and formal elements of mathematics, (4) calculus and learning / teaching mathematics, (5) calculus and demonstration, (6) calculus and the emotional dimensions of mathematics, (7) personal specifica. These fields cover aspects on cognition, beliefs and goals (see Schoenfeld [98]). Not all essays contain explicit contributions to each

of these fields. For reasons of concise presentation only a few general results can be presented here, reported rather as a tendency:

Calculus in the upper secondary grades is perceived by pupils as an independent and new mathematics field, whereas at the lower Sekundarstufen-I grades (age groups 10-16) mathematics is viewed rather as a whole. A satisfying link to the contents of Sekundarstufen-I mathematics appears as a rule to not to be taken for granted. University mathematics reinforces the impression of calculus being an independent field of mathematics, in particular as the student becomes familiar with further fields of mathematics. It would be complementary, however, in the interest of the students, to go beyond the individually specialized fields and instead enable (again) an integral all-encompassing view of mathematics to be experienced. Here deficits were clearly articulated.

Differences between the individual fields of mathematics are defined by the students primarily with relation to individual task types. In the minds of the students differences as compared to common aspects and features of mathematical structuring patterns are dominant.

The students formulate a clear discrepancy between school and university education. While the school view of calculus is schema-oriented, calculus at university possesses primarily a formal orientation. It seems that stimulation towards a stronger problem-oriented viewpoint is hardly given. In this respect their views towards calculus and mathematics in general present on the whole a one-sided picture.

It is surprising that not the application-nearness of calculus is stressed by the students, but the enormous logic-biased mathematization patterns. A satisfying answer to the fundamental applicability of mathematics is still not given by the university. Intensified research into the historical evolution of termini is however positively noted here. Such efforts increase the relation content of the research material.

Their views on ideal calculus lessons seem to be nurtured from their own positive experiences in lessons or to be complemented through self-experienced deficits in school and at university. Within this framework calculus lessons are experienced at university as being not very constructive. Admittedly, calculus seminars serve a universal function in academic mathematics, whereby aspects of educating prospective teachers how to teach mathematics only plays a subordinate role. On the other hand, the few chances for illuminating the relevance of academic mathematics for school teaching are apparently hardly made use of.

In all the six essays emotional value judgments are given towards calculus, whereby in particular negative statements stand out towards calculus experienced at university. The scope lies between 'interesting but difficult', 'found it very hard' to 'more brutal than any other mathematics seminar'.

One must hereby note that in particular those students who successfully completed calculus under considerable effort have the tendency to exaggerate the formal aspect (for instance the logical structures of calculus terms) as its important characteristic.

Such a formalistic view of calculus for a prospective teacher is, however, possibly more than slightly responsible for learning difficulties of their future pupils.

### Conclusions

It hardly needs mentioning that these results cannot be considered representative either for calculus lessons in the upper secondary grades in Germany, or in respect to statements of prospective teachers in general.

Nonetheless, the evaluations offer the author a number of valuable insights, in particular it provides a deeper and better understanding of the psychological aspects of teaching and learning topics around the calculus. Whereas the literature offers an abundance of papers on teachers' and pupils' beliefs, descriptions of specifically related beliefs are not abundant at all. In comparison, papers on calculus (in the upper secondary grades) are hard to find (see e.g. Fox 1998). More concern must be given to the circumstance that calculus in the evaluation of future mathematics teachers is high emotionally loaded - however not primarily in a positive manner.

In the discussion on the content and function of didactics seminars for teacher training students one often finds the remark that it is necessary to work on deficits in general mathematical fields. Even though this applies primarily more to the fields of stochastics or linear algebra than to calculus, the students nonetheless point out in their reports that the general world view on mathematics needs a more fundamental correction or modification. Whether this circumstance can be achieved by academic educators of mathematics can be viewed as doubtful. Of greater importance appears to be the working through and consolidation of affective contexts. Almost every report of the students revealed deficits here.

Formulated in a slightly exaggerated manner one can stipulate that the enormous effort of academic mathematical education and training concerning analysis has to be judged reservedly. The formative function of a university course from a cognitive point of view has to be judged with reserve in the light of the present papers; scars are more to be found in the emotional field.

Finally, action is required when one feels compelled to note that university seminars on calculus do not seem to offer a convincing contribution towards the professionalization of future teachers. It appears that the well-known quotation of F. Klein (1908) on the so-called 'two-fold discontinuity' still has its relevance after almost one hundred years:

*The young student sees himself at the beginning of his university course confronted with problems which in no point remind him of things he was concerned with at school; of course this is why he forgets all these things rapidly and thoroughly. However, when he enters a teaching position after completion of study, he is expected to teach traditional elementary mathematics in the traditional school manner; as he can hardly relate this to his university mathematics, he will in most cases embrace traditional teaching within short time, and the university course will remain to him only a more or less pleasant memory that has no influence on his lessons.'*

### Literature

- Chapman, O. 1998. Narrative as a tool for facilitating preservice mathematics teacher development. In Berenson, S.B. et al. Proceedings of the 20th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Volume 2. (pp. 662 - 667). Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education.
- Cohen, L.; Manion, L. 1994. *Research Methods in Education*. London and New York: Routledge.
- Fox, Thomas B. 1998. Teacher change in a reform calculus curriculum: Concepts related to the derivative. In Berenson, S.B. et al. Proceedings of the 20th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Volume 1 (pp. 221 - 225). Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education.
- Grigutsch, S.; Raatz, U.; Törner, G. 1998. Einstellungen gegenüber Mathematik bei Mathematiklehrern. *Journal für Mathematikdidaktik* 19 (1), 3 - 45.
- Klein, F. 1908. *Elementarmathematik von einem höheren Standpunkt*. Berlin: Springer-Verlag. Neuauflage.
- National Council of Teachers of Mathematics. 1991. Professional standards for teaching mathematics. Reston (VA): National Council of Teachers of Mathematics.
- Reichel, H.-Chr. 1992. Grundhaltungen, Motive und der 'Seelische Bodensatz' von Lehramtskandidaten - ein persönlicher Bericht und ein Versuch einer Analyse. In H. Schumann (Ed.), Beiträge zum Mathematikunterricht 1992. Vorträge auf der 26. Bundestagung für Didaktik der Mathematik in Weingarten (pp. 359 - 362). Hildesheim: Franzbecker.
- Robitaille, D.F. (Eds.). 1997. National Contexts for Mathematics and Science Education. An encyclopedia of the education systems participating in TIMSS. Vancouver (Canada): Pacific Educational Press.
- Schoenfeld, A. 1998. Toward a theory of teaching-in-context (draft version). <http://www-gse.berkeley.edu/faculty/aschoenfeld/TeachInContext/tic.html>
- Toerner, G. & Grigutsch, S. 1994. Mathematische Weltbilder bei Studienanfängern - eine Erhebung. (German) (Mathematical world views on first-year-students-a study). *Journal für Mathematik-Didaktik* 15 (3/4), 211 - 252.