EPISTEMOLOGICAL ASPECTS OF BELIEFS

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Basis of the following article is the analysis of mathematical beliefs held by university mathematics teachers with regard to aspects of their epistemological world views. For the theoretical framework we provide an overview about the research related to this topic in mathematics education and psychology. We employed quantitative methods to analyze these epistemological world views of university teachers about their subject, namely mathematics, and their domain. By means of factor analysis we obtained seven components representing different aspects of epistemology. With regard to our findings we are reserved with generalizing aspects of epistemology over all domains and claim for a closer look to mathematical domain specifics.

A. Motivation. Initial points of our paper are theoretical developments in the study of beliefs within psychology and research on mathematics education. For more than 30 years now, mathematics educators have dealt with beliefs about mathematics and analyzed these for different groups (students at various ages, teachers, adults) under diverse conditions. The early papers by Thompson (1992) and Pajares (1992) related to this subject are again and again cited; the more recent work by Leder, Pehkonen and Törner (2002) tries to update the discussion and bring together the results of different domains of research. Meanwhile new handbooks on mathematics education appeared contributing general articles to our subject (see Forgasz & Leder (2007), Philipp, 2007). When comparing the two aforementioned papers by Thompson and Pajares, which both appeared almost at the same time, one becomes aware about the different fields the researchers are involved in, as there is on the one hand mathematics education and on the other hand psychology. In his work, Pajares emphasizes the epistemological character of beliefs whereas in the work of Thompson the word "epistemology" is not even mentioned - with respect to epistemology we refer to the article of Sierpinska and Lerman (1996). But it is apparent that quite similar constructs have been discussed against a different background and therefore different classifications have been made. Both disciplines acknowledge the role of beliefs as subjective theories, mostly philosophically based, and in the literature often considered as "world views". We refer to Schoenfeld (1985) who stated that one reason for the failure of introducing *Problem Solving* in curricula of the United States lay in not appropriate "world views" of teachers.

B. Theoretical framework. By understanding mathematical beliefs primarily as epistemological views we were motivated to revisit older beliefs discussions and to analyze them from an epistemological viewpoint. Beliefs are concerned with truth,

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meanings and certainty – and these are with no doubt epistemological issues. In particular, we want to reopen a discussion on interesting data from the nineties, it is just a revisiting of old data, however investigating within a new light. The sample under discussion consists of university mathematics teachers. We therefore refer to the work of mathematics educators as for example Mura (1993, 1995) and Pehkonen (1997, 1999). Mura asked university mathematicians and university mathematics educators at Canadian universities how they would define mathematics. By means of content analysis, she assigned the answers of 106 mathematicians into 12 categories (Mura, 1993) and the answers of 51 university mathematics educators (teacher trainers) into 14 ones (Mura, 1995). These categories can be viewed as dimensions structuring thoughts of university teachers about mathematics, in which they are holding attitudes towards mathematics. In his study, Pehkonen (1997) investigated the beliefs of mathematics professors *"to find out what kind of mathematical beliefs are submitted to teacher students during their university studies at mathematics departments"* (p. 92). In a later study (1999) he pursued the question what kinds of beliefs mathematics professors have on school mathematics.

Although there are only a few papers addressing epistemology in mathematics education, epistemological issues are implicitly a central point in many discussions with roots going back to philosophical positions towards mathematics (Hersh, 1991; Hersh, 1997; Sierpinska & Lerman, 1996; see also Byers, 2007). For us, this focus is legitimated by the fact that epistemology affects learning of mathematics while epistemological views held by university professors influence prospective teachers' actions in class (e.g. Carter & Norwood, 1997; Schraw & Olafson, 2002), and this again might explain the various beliefs of their students. More generally we like to point out that it was René Thom, a famous mathematician and Fields medallist, who postulated this link in 1973: *"In fact, whether one wishes it or not, all mathematical pedagogy even if scarcely coherent, rests on a philosophy of mathematics"* (p. 204). His paper was in particular aimed at the "New Math" movement which by that time had already turned out to be a failure, and Thom argued that this should be traced back to some fundamental misunderstanding of – in our terminology – epistemological mathematical positions.

Of course, one can ask for similarities to numerous results in psychology. For an overview about related research in this area we restrict ourselves to refer to the paper of Muis (2004), which provides a well-elaborated overview. However, we should not ignore the famous paper by Schommer (1990), which is often cited and finally we refer to Schommer-Aikins' (2002, 2004, p.19) concise description to recall five dimensions for epistemological characteristics. These five dimensions exist on continua and are characterized by the following poles:

- **Structure of knowledge**: ranging from isolated bits and pieces to integrated concepts
- **Stability of knowledge**: ranging from unchanging knowledge to tentative knowledge
- **Source of knowledge**: ranging from omniscient authority to reason and empirical evidence
- **Ability to learn**: ranging from fixed at birth to improvable
- **Speed of learning**: ranging from quick or not-at-all to gradual.

The first three dimensions relate to knowledge itself while the two others to acquisition of knowledge (Clarebout, Elen, Luyten & Bamps; 2001).

C. Methodology. The empirical study presented here partly deals with epistemological world views on mathematics held by university mathematics teachers in countries of German as a first language. We used a questionnaire which was given to participants of an annual meeting of the German Mathematical Society; 119 were filled out and handed back. The study was already conducted in the nineties by the second author and Dr. Grigutsch; however these results have only been published as a preprint.

The questionnaire has been well established before (Grigutsch, 1996; Grigutsch, Törner & Raatz, 1998) and let to reliable scales for measuring mathematical beliefs. The statements in the questionnaire were grouped round the following topics:

- A: My experiences with usual study of mathematics at university (items 1-21)
- B: Mathematics as a science from my point of view (items 24-50)
- C: On the origin of mathematics (items 51-63)
- D: Mathematics and reality (items 64-77)
- E: Statements of mathematicians about mathematics (M1-M14)

Items A to D deal with a more general attitude towards mathematics and its teaching and learning. In addition, we employed a new approach (items E) by using quotations of famous mathematicians. All items are concerned with epistemology as they deal with the nature of mathematics as well as characteristics of learning and teaching mathematics.

D. Results. We employed factor analysis for grouping the statements; in particular principal component analysis with varimax rotation was used. The scree test suggested the extraction of 5 to 10 factors. The decision about how many factors are meaningful and important we based on the following criteria: the statements within a factor have to be homogenous in respect of content, Cronbach's Alpha for each factor has to be sufficiently high and there must be a sufficient number of items loading high on the factor. According to the aforementioned criteria we chose to extract 7 factors; all items had good communality. We compared this solution with other ones where the number of components varied from 5 to 10 and found highest Cronbach's Alphas for the seven-component solution (table 1). Due to space restrictions we only list the three highest loading items for each component.

Table 1. The seven-component solution.

Principal components and statements		Loadings
F1 Characteristics of mathematics (alpha = 0.86)		
30.	Very essential aspects of mathematics are its logical strictness and precision, i.e. "objective" thinking.	0.676
48.	The crucial fundamental elements of mathematics are its axiomatic and the strict, deductive method.	0.672
38.	Mathematical thinking is determined by abstraction and logic.	0.671
F2 Main features of mathematical learning (alpha = 0.784)		
24.	Mathematics is a collection of procedures and rules, which precisely determine how a task is solved.	0.647
44.	Mathematics consists of learning, recalling and applying.	0.616
39.	Mathematics is the memorizing and application of definitions, formulas, mathematical facts and procedures.	0.604
F3 Mathematics and real world (alpha = 0.765)		
72.	Mathematics helps to solve daily tasks and problems.	0.664
68.	Knowledge of mathematics is very important for students later in life.	0.637
71.	Mathematics is of use to any profession.	0.598
F4 Prerequisites to 'do' mathematics (alpha = 0.716)		
41.	Above all, mathematics requires intuition as well as thinking and arguing, both related to contents.	0.667
46.	Central aspects of mathematics are contents, ideas and cognitive processes.	0.643
31.	Mathematics requires new and sudden ideas.	0.519
F5 Instructions or 'how to learn' mathematics (alpha = 0.625)		
4.	In the daily studies of mathematics it is often more important to learn facts and results than to find ideas and continuing questions by oneself.	0.569
11.	In the study of mathematics it is sufficient for the students to learn only those things which are asked for in exams and written tests.	0.528
20.	In order to be successful in the study of mathematics, one has to learn many rules, terms and procedures.	0.514
F6 Philosophical aspects of mathematics (alpha = 0.578)		
M3	It cannot be denied that a large part of elemental mathematics is of considerable, practical use. However, these parts of mathematics appear rather boring when observed as a whole. These are those parts which possess the least aesthetic value. "Real" mathematics from "real" mathematicians such as Fermat, Gauß, Abel and Riemann is almost totally "useless".	0.593
M4	If we can approach to the godliest on no other way but by symbols, we will use the mathematical symbols because they possess undestroyable certainty.	0.546
M6	The mathematicians, who are only mathematicians, are correct in their thinking, but only in the sense that all things can be explained to them using definitions and principles; otherwise their ability is limited and intolerable, because their thinking is only correct when it concerns only extremely clear principles.	0.484
F7 Experts view on mathematics (alpha = 0.604)		
52.	Mathematical tasks and problems can be solved correctly on different ways.	0.734
58.	There is usually more than one way to solve tasks and problems.	0.582
77.	Some mathematical knowledge is important for some chosen professions.	0.463

Two of the principal components relate primarily to university professors' views on what mathematics is about *(F1: characteristics of mathematics)* and what it means to learn mathematics in general *(F2: Main features of mathematical learning)*, one to a utility resp. application aspect *(F3: Mathematics and real world)*, one to what students need to deal with mathematics *(F5: Instructions or 'how to learn' to learn mathematics)*, and three to expert views on mathematics *(F4: Prerequisites to 'do' mathematics, F6: Philosophical aspects of mathematics, F7: Experts view on mathematics)*.

Further, we were interested in university professors' world views within the seven dimensions. In what follows, we set up a scale value for each participant and factor. Since each dimension is described by a different amount of items the absolute value varies accordingly to this. By a process of linear transformation we obtained a common scale for all factors varying from 0 to 50 whereby the range is from 0 - 10 for fully disagreement to 40 - 50 for fully agreement. We are now able to position our sample of university professors within the dimensions of epistemological world views.

F1 Characteristics of mathematics

This factor is founded by 17 items, which clearly emphasize a view on mathematics that is characterized by strictness and objectivity varying in the level of terminology. Mathematical knowledge is seen as based on axiomatic and deduction, and mathematical thinking guided by abstraction and logic. Consequently, there is a focus on exactly defined terms and a precise expert language. This aspect is also reflected in university teachers' beliefs about how their students should learn. They expect them to use the mathematical terms correctly and explain everything precisely. For this factor the mean is 31.65 (std. error of mean 0.68, std. deviation 7.22) and therefore lies in the range of agreement.

F2 Main features of mathematical learning

In this factor mathematics is described as a collection of algorithms, procedures and routines. Accordingly, mathematical learning is seen as recalling and applying definitions, rules and procedures out of this toolbox. However, there is also the aspect included that schemata and algorithms can be valued as "elaborate tricks" and "ornate methods" when solving mathematics problems is compared to doing a crossword puzzle. This dimension is closely related to the *structure dimension* in Schommer-Aikin's (2004) theory of epistemological beliefs but also contains mathematics specificities. University teachers strongly reject a view on learning mathematics where schematic aspects are dominating (mean 12.36, std. error of mean 0.54, std. deviation 5.65).

F3 Mathematics and real world

This factor stresses the practical use of mathematics and the importance for the world we are living in. Mathematics is seen as essential for students later on in life and of "general use for society". This factor consists of 15 items mainly dealing with aspects of application and the mean is located in the neutral range (mean 27.50, std. error of mean 0.69, std. deviation 6.97).

F4 Prerequisites to 'do' mathematics

In this dimension ideas and intuition are regarded as the main prerequisites to deal with mathematics. Besides understanding facts and realizing relationships, new and sudden ideas are of importance for further development in the field. Since we found in this factor evidence for conceptualizing mathematics as dynamic and of changing nature we can state a relation to Schommer-Aikins' dimension of *certain knowledge* that ranges from knowledge as absolute and unchanging to tentative and evolving.

University teachers clearly agree with this view of mathematics (mean 38.28, std. error of mean 0.54, std. deviation 5.88).

F5 Instructions or 'how to learn' mathematics

This factor is concerned with aspects of mathematics that are restricted more to learning facts than discovering and finding ideas. Contrary to the factor mentioned before learning mathematics is not seen as a creative process of discovering but as a collection of rules to apply. This factor is related to Schommer-Aikins' dimension *source of knowledge* ranging from the belief that knowledge is handed down by authority to the belief that knowledge is constructed by the students themselves. The university professors reject this view on learning mathematics (mean 19.38, std. error of mean 0.56, std. deviation 6.05).

F6 Philosophical aspects of mathematics

As remarked before, for the first time we used quotations out of the literature marking some philosophical positions, however, not covering the whole spectrum of philosophical mathematical strands and we were wondering how our test persons would respond. The quotations grouped together by the factor analysis describe mathematics as an highly intellectual activity with nearly no link to reality. We cannot object that in pure mathematics this position is not completely denied. E.g., it was the famous mathematician Halmos who proudly claimed that his research has no applicational use, although this hypothesis is meanwhile denied by some deep applications of number theory in coding theory and cryptography. Although these quotations stand for themselves and describe some mental artificial reality, which should be accepted, it is not surprising that a majority of the university professors is refusing these positions (mean 19.61, std. error of mean 0.84, std. deviation 8.74).

F7 Sophisticated views on mathematics

The items gathered under factor F7 describe positions, which are self-evident for mathematicians, however not as dominant for teachers and students. In many situations, problems can be encountered by different solving strategies; there is often more than one solution for deep problems. Nearly all participants agree with this position, which is marked by a mean of 45.62 (std. error of mean 0.44, std. deviation 4.72).

A further approach is to describe the relations between the dimensions, particularly the structural features generated by the obtained factors. We therefore calculated Pearson correlation for the factors F1 to F7 (table 2).

Table 2: Correlations between the dimensions (* = significant at the 0.05 level).

	F1	F2	F3	F4	F5	F6	F7
F1	1	.161	.225*	-.048	.093	-.087	-.002
F2		1	-.046	.056	.207*	.153	-.241*
F3			1	.237*	-.026	.039	-.053
F4				1	-.102	.039	.114
F5					1	-.032	-.134
F6						1	.111
F7							1

Although the correlations are quite moderate they provide insight in structural relations. The dimension F3 *Mathematics and real world* correlates significantly positive with F1 *Characteristics of mathematics* and F4 *Prerequisites to 'do' mathematics*. Further, the dimension F2 *Main features of mathematical learning* correlates significantly positive with F 5 *Instructions or 'how to learn' mathematics* and negative with F7 *Sophisticated views on mathematics*. Numerous interpretations seem to be plausible for the first mentioned positive relationship. Obviously, the more formalistic aspect of mathematics as represented in F1 and the more procedural one of developing ideas in F4 is application-related and practically useful. These formal elements of mathematics together with new and sudden ideas are important for solving tasks and problems even for modelling reality or providing applications. The second mentioned correlations are not astonishing either since the view on instructions in F5 is closely related to the view on learning as presented in F2. Describing mathematics as a collection of algorithms, procedures and routines goes well together with characterizing successful learning as applying facts and rules. In this regard, the negative relationship to *sophisticated views on mathematics* is quite obvious. Since the quotations in F6 are of philosophical character, which is not an explicitly mentioned part in the other factors, it is not surprising that this factor is totally isolated.

E. Conclusions. In our findings there is some evidence for epistemological characteristics as suggested by Schommer-Aikens (2004). Our first four components remind of the often-mentioned dimensions *structure of knowledge, stability of knowledge* and *source of knowledge*. However, it is not surprising that with respect to our sample, namely university mathematics teachers, the aspects represented by the items are more advanced, differentiated and sophisticated. Furthermore, the components reveal specific aspects, which are not covered by Schommer's interpretation of the epistemological dimensions. And last-not-least, we belief that there are still more, however - minor relevant factors which should not be ignored. The reader is referred to the book of Schmalz (1993), which is a rich source for quotations on mathematics revealing dimensions, which may be identified

as independent factors in some kind of questionnaire. Further, some historical trends in mathematics could not be understood unless one accepts such views on mathematics (see also Davis & Hersh, 1980). Of course, they might be estimated irrelevant for mathematics teaching at school but they cannot be ignored for balanced mathematical education at university.

In psychology, the question whether epistemology is domain-specific or not is a central one. Our results indicate that there are some mathematics related specifics in the views on mathematics and its teaching and learning held by university teachers. From their point of view they can clearly state what is required to do mathematics (F4) and this differs from more standard views on students covered by factor (F5). In addition, the components (F4), (F6) and (F7) represent an elaborated view on mathematics, namely that of an expert. E.g., the sixth factor consists only of 'M-items', which are derived from quotations of famous mathematicians mostly covering philosophical aspects. It seems to us that generating items by using quotations is promising. We should point out once more that the book of Schmalz (1993) is a rich source for those items and we would like to recommend this approach.

With respect to domain-specificity, we finally remind that Goldin (2003) is principally critical whether psychological dimensions based on constructivism can straightforwardly be applied to mathematics, since the underlying assumptions do not sufficiently match the conditions in mathematical research and hence also in mathematics education.

Last not least, in the literature there cannot be much found on the question how epistemological views are differing with respect to particular groups. Our findings show that university teachers are a special sample; the numerous factors – even so found by Mura (1993, 1995) - point to the fact that domain independent generalizations do not catch domain specifics. We believe that there are more aspects of relevance and our results strengthen Goldin's assumption that the hitherto categorisations of epistemology are rather raw for a special group and therefore need further development.

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