

On pitchforks and tomahawks[♣]

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Abstract

The core-periphery model of the new economic geography has two 'dramatic' implications: catastrophic agglomeration and locational hysteresis around the symmetry breaking level of trade freeness. In this note we study a generalized version of this model with CES instead of Cobb-Douglas upper tier preferences. The possibility of a continuous and easily reversible transition from symmetry to agglomeration now arises. One of the most prominent results of the new economic geography literature – the catastrophic consequences of small parameter changes – therefore hinges crucially on specific functional forms for consumer preferences.

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1. Introduction

Two of the most widely emphasized features of the seminal model of the new economic geography (NEG), the core-periphery model by Krugman (1991), are catastrophic agglomeration and locational hysteresis. In the basic framework with two regions, an equal division of economic activity is the only stable equilibrium for low levels of trade freeness. Once trade freeness has risen to a critical level, the so-called 'break point', all mobile activity agglomerates catastrophically and fully in one region. A reversal of the trade integration need not restore symmetry, however, because the core-periphery pattern is stable up to a 'sustain point' which lies at a strictly lower level of trade freeness than the break point. Technically speaking, the core-periphery model exhibits a *tomahawk bifurcation* at the break point.¹

In this note we show that a generalization of the individual preference structure within the original modelling framework suffices to change these stark features. We assume a CES- rather than a Cobb-Douglas upper tier utility function. Our main finding is that the endogenous agglomeration process which arises in the course of trade integration no longer needs to be catastrophic, and need not feature locational hysteresis. Subject to specific parameter constellations there can rather be a continuous and easily reversible transition from symmetry to agglomeration (a [super-critical] *pitchfork bifurcation*, cif. footnote 1).

Why is the shape of the bifurcation pattern important? Firstly, it derives its particular relevance from policy implications – an insight that is worked out comprehensively in Baldwin et al. (2003, ch.1, 2.2, 9.2). Under a tomahawk bifurcation, small changes in economic policies may have catastrophic effects. For example, a small infrastructure improvement that slightly reduces inter-regional transport cost can lead to a sudden and discontinuous change in the spatial structure of the economy, maybe comparable to how a sudden outburst of volcanoes or earthquakes reshape the earth's physical geography. The

¹ The tomahawk bifurcation is also known as the “*sub-critical pitchfork bifurcation*”. To avoid confusion with the alternative “*super-critical pitchfork bifurcation*” we use a simpler terminology. We distinguish the *tomahawk* (sub-critical pitchfork) and the *pitchfork* (super-critical pitchfork) bifurcation. See Fujita et al. (1999), or Grandmont (1988) for an introduction to bifurcations.

effects of this small policy shock may then become long-lasting and hardly reversible. When the economy is shifted from one to another (possibly less desirable) equilibrium, due to locational hysteresis, a reversal of this shift requires policies that are much larger than the initial ones. A pitchfork bifurcation, in contrast, implies a continuous emergence of agglomeration without lock-in effects, and therefore has far less dramatic implications when it comes to the spatial consequences of economic policies. A second reason why an analysis of the bifurcation pattern is important is that the implications of Krugman's standard core-periphery framework have attracted huge attention in the empirical literature (see, e.g., Davis and Weinstein 2002, 2008; Bosker et al. 2007; Redding et al. 2007). This literature does not seem to substantiate the particular features of the tomahawk shape, but seems to regard the pitchfork as inherently more realistic (see in particular Head and Mayer, 2004, sect.7.2).

For both reasons, policy and empirical relevance, it is important to understand under which theoretical conditions such a pitchfork pattern can emerge. Here it should be noted that previous contributions have already shown that the tomahawk bifurcation is not a robust feature of NEG models in general. Puga (1999) was probably the first to notice this (see, in particular, footnote 18 of that paper). Further examples of NEG models with different location patterns include Helpman (1998), Tabuchi (1998), Ludema and Wooton (1999), Tabuchi and Thisse (2002), Ottaviano et al. (2002), Murata (2003), Pflüger (2004), Nocco (2009), or Berliant and Kung (2009). The difference between these papers and ours is that we do not add any new elements (like mobility or urban costs, heterogeneous locational tastes, more than two regions, etc.), but stay as close as possible to the original framework by Krugman (1991). Our paper shows that a generalization of upper tier preferences (from Cobb-Douglas to CES) can already overturn the tomahawk pattern. The two widely emphasized features of the canonical NEG model – catastrophic agglomeration and locational hysteresis – therefore hinge crucially on a specific functional form for individual preferences, more specifically on the upper tier substitution elasticity. This is the new result of this paper.

2. The model

To make our point in the simplest possible way, we build on the analytically tractable version of the core-periphery model developed by Forslid and Ottaviano (2003).² The economy is composed of two regions $r = 1, 2$, two industries A (agriculture) and M (manufacturing), and two types of individuals. Skilled workers are mobile across regions. Their total endowment is normalized to one, and the share λ_r is located in region r . Unskilled workers are geographically immobile, with ρ denoting the factor endowment of either region.

In the perfectly competitive A -sector one unit of the homogeneous output is produced with one unit of unskilled labor. In the monopolistically competitive M -sector each variety i is produced by a single firm. It takes one unit of skilled labor as a fixed input to produce. Then, one unit of output is produced with c units of unskilled labor. Cross-border transport of M -varieties is subject to iceberg costs $\tau > 1$. The A -good is traded without costs and is chosen as the numéraire. Its price and, therefore, the wage of unskilled workers is unity in both industries and locations, $p_r^A = w_r^U = 1$.

The only difference between our model and Forslid and Ottaviano (2003) is the specification of individual preferences. Rather than using a Cobb-Douglas upper tier utility function we assume that upper tier preferences of individuals in region r are given by a CES function:

$$U_r(A_r, M_r) = \left[A_r^{\frac{\varepsilon-1}{\varepsilon}} + M_r^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad M_r = \left(\int_0^{n_r} m_{rr}(i)^{\frac{\sigma-1}{\sigma}} di + \int_{n_r}^{n_r+n_s} m_{sr}(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

The elasticity of substitution between the aggregates M and A is given by ε , and σ expresses the substitution elasticity between any two M -varieties. In this paper we shall

² It is well known that the original Krugman-model can only be solved with numerical simulations whereas the version by Forslid and Ottaviano (2003) allows for more analytical solutions. Robert-Nicoud (2005) has proven that these two models are formally equivalent in terms of their long-run spatial equilibrium configurations. In the web supplement to this paper (see references) we also analyze the original Krugman-model with CES upper tier preferences. This yields results that are qualitatively identical to those reported in the paper.

focus on the economically plausible 'normal case' where $\sigma > \varepsilon \geq 1$, i.e., it is easier to substitute one manufacturing variety with another variety than with the agricultural good.³

The total mass of varieties is $n = n_r + n_s$, where $s = 1, 2$ and $s \neq r$. Region r 's consumption of local and imported varieties are denoted by $m_{rr}(i)$ and $m_{sr}(i)$, respectively. Denoting aggregate income in region r by Y_r and producer prices by $p_r(i)$ and $p_s(i)$, the budget constraint is given by:

$$Y_r = A_r + \int_0^{n_r} p_r(i) m_{rr}(i) di + \int_{n_r}^{n_r+n_s} \tau p_s(i) m_{sr}(i) di \quad (2)$$

The following demand functions can then be derived from standard utility maximization:

$$A_r = (G_r)^{\varepsilon-1} Y_r, \quad M_r = (G_r^M)^{-\varepsilon} (G_r)^{\varepsilon-1} Y_r, \quad (3)$$

$$m_{rr}(i) = p_r(i)^{-\sigma} (G_r^M)^{\sigma-\varepsilon} (G_r)^{\varepsilon-1} Y_r, \quad m_{sr}(i) = (\tau p_s(i))^{-\sigma} (G_r^M)^{\sigma-\varepsilon} (G_r)^{\varepsilon-1} Y_r, \quad (4)$$

where the overall price index G_r and the standard CES-price index for manufacturing varieties G_r^M are given by

$$G_r = \left[1 + (G_r^M)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad G_r^M = \left[n_r p_r^{1-\sigma} + n_s (\tau p_s)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (5)$$

Notice that the demand for a specific variety in (4) rises in response to an increase in the general manufacturing price level G_r^M , which follows from our assumption $\sigma > \varepsilon \geq 1$.

Region r 's aggregate income Y_r consists of the wage incomes for unskilled ($w_r^U = 1$) and for skilled workers (w_r), $Y_r = \rho + w_r \lambda_r$. Taking into account transport losses, total output of any variety is given by $q_r(i) = m_{rr}(i) + \tau \cdot m_{rs}(i)$. With mill price $p_r(i) = \bar{p}$, firm's profits are

³ In principle it is also possible to allow for the case $\varepsilon > \sigma > 1$ in our analysis. This would imply, however, that the substitution possibility within the group of manufacturing varieties is worse than the possibility to substitute any variety with the agricultural good. We believe that this case has little economic relevance. Helpman (1990) and Antras and Helpman (2004) essentially make the same assumption that the substitution elasticity is higher *within* the group of varieties.

then given by $\pi_r(i) = (p_r(i) - c) q_r(i) - w_r$. Profit maximization yields $\bar{p} = \sigma \cdot c / (\sigma - 1)$, and when choosing units such that $c = (\sigma - 1) / \sigma$, we can henceforth use $\bar{p} = 1$.

In a zero profit equilibrium all operating profits accrue to skilled labor, hence the wage for skilled workers is given by $w_r = q_r(i) / \sigma$ and the number of firms in r is determined by the supply of skilled workers ($n_r = \lambda_r$). We can solve for the wages of skilled workers (w_r) for a given distribution of skilled labor (λ_r):

$$w_r = \frac{\rho \cdot \left[\sigma(\phi \cdot b_r + b_s) - (1 - \lambda_r)(1 - \phi^2) \right]}{\left[(\lambda_r - \sigma \cdot b_r)(1 - \lambda_r - \sigma \cdot b_s) - \lambda_r(1 - \lambda_r)\phi^2 \right]}, \quad (6)$$

where $b_r \equiv (G_r)^{1-\varepsilon} (G_r^M)^{\varepsilon-\sigma}$. Here, $\phi \equiv \tau^{1-\sigma} \in [0, 1]$ denotes the usual measure of trade freeness.

Using (6), all other endogenous variables can readily be inferred. In particular, associated with (1) is an indirect utility function for skilled workers of the form $V_r = w_r / G_r$, whose explicit expression for given λ_r follows directly from (6) and (5).

3. Symmetry breaking and bifurcation pattern

Symmetry breaking. In the long-run, skilled workers move in response to utility differences across regions. As regions are ex-ante identical, symmetry ($\lambda_r = 1/2$) is always an equilibrium. However, since the model features the usual agglomeration forces (supply and demand linkage) the symmetric equilibrium is not necessarily stable. Counteracting these agglomeration forces is a dispersive market crowding effect, and the balance of these forces depends on the level of trade freeness ϕ .

The critical level ϕ_b at which the symmetric equilibrium turns unstable can be determined by the condition $d(V_r - V_s) / d\lambda_r = 0$ at $\lambda_r = 1/2$ (see Puga 1999 on this). It is convenient to apply

the technique developed by Pflüger and Südekum (2008) to solve for the break point. They show that the break point ϕ_b lies where the following condition is satisfied at $\lambda_r = 1/2$:

$$-\left[\frac{\partial V_r / \partial G_r}{\partial V_r / \partial w_r}\right] = \frac{dw_r / d\lambda_r}{dG_r / d\lambda_r} \Leftrightarrow \phi = \phi_b \quad (7)$$

Noting that $-(\partial V_r / \partial G_r) / (\partial V_r / \partial w_r^M) = w_r / G_r$ in the present case (since $V_r = w_r / G_r$), we can rewrite (7) as $\left[(1/w_r) \cdot (dw_r / d\lambda_r) - (1/G_r) \cdot (dG_r / d\lambda_r)\right] = 0$. Using (5) and (6) we arrive at the following implicit function

$$F(\phi, \sigma, \varepsilon) = \frac{1}{1+z} - \left[\frac{(\sigma-1)[(1+\phi) - \sigma(1-\phi)] - \sigma(\sigma-\varepsilon)(1-\phi)z}{(1-\phi) - \sigma(1+\phi)(1+z)} \right] = 0 \quad (8)$$

where $z \equiv 2^{\frac{\varepsilon-1}{\sigma-1}} (1+\phi)^{\frac{1-\varepsilon}{\sigma-1}}$.⁴ Even though it is not possible to solve (8) explicitly for the break point $\phi_b(\sigma, \varepsilon)$, we can use the implicit function theorem to prove the following result:⁵

Proposition: *For the core-periphery model with CES upper tier preferences the following result holds: Assuming $\sigma > \varepsilon \geq 1$ the break point $\phi_b(\sigma, \varepsilon)$ is unambiguously increasing in the substitution elasticity across the single varieties ($d\phi_b / d\sigma > 0$) and decreasing in the upper tier substitution elasticity ($d\phi_b / d\varepsilon < 0$).*

Economically, the first result is well known from standard NEG models. As varieties become closer substitutes there is agglomeration over a smaller parameter domain of ϕ . The latter result refers to the novel parameter of our analysis. With $\varepsilon > 1$, the fall in prices that is

⁴ Notice that the endowment of unskilled labor ρ does not affect the break point. This is due to the fact that ρ enters the wage equation (6) in multiplicative form and therefore does not affect relative regional wages.

⁵ The proof and more details about the derivation of (8) can be found in the web supplement (see references).

induced by trade integration increases the expenditure share devoted to manufactured goods. This should then lead to agglomeration over a larger range of ϕ the higher ε is.⁶

The bifurcation pattern. The bifurcation type that unfolds at the break point can be addressed by evaluating the sign of the third derivative of the utility differential for mobile skilled workers. If the third derivative, evaluated at $\lambda_r = 1/2$ and at the symmetry breaking level of trade freeness is positive, we have the catastrophic tomahawk bifurcation. If it is negative the smooth pitchfork bifurcation follows (see also Grandmont 1988).⁷

$$\tilde{V}_{(3)} \equiv \left. \frac{d^3(V_r - V_s)}{d\lambda_r^3} \right|_{\lambda_r=1/2, \phi=\phi_b}, \quad \begin{aligned} \tilde{V}_{(3)} > 0 &\rightarrow \text{tomahawk-bifurcation} \\ \tilde{V}_{(3)} < 0 &\rightarrow \text{pitchfork-bifurcation} \end{aligned} \quad (9)$$

As no closed-form solution for ϕ_b can be provided we cannot solve explicitly for $\tilde{V}_{(3)}$. However, since the break point $\phi_b(\sigma, \varepsilon)$ is determined by two parameters only, we can proceed conveniently with a numerical approach. Assuming some initial values for σ and ε , we can solve for $\phi_b(\sigma, \varepsilon)$ and obtain the corresponding value of $\tilde{V}_{(3)}(\sigma, \varepsilon, \phi_b(\sigma, \varepsilon))$. Repeating this procedure for arbitrary many combinations of σ and ε , we can—in principle—cover the complete admissible parameter range where a break point $\phi_b \in [0, 1]$ exists, so that our numerical results are exhaustive and not merely a collection of selective examples. The essence of this analysis is summarized in figure 1.

⁶ This intuition is similar as in Yu (2005), who studies the *home market effect* with CES instead of Cobb-Douglas upper tier preferences. It is possible to decompose (8) into parts attributable to the linkages and the market crowding effect, respectively (see the web supplement to this paper). This exercise shows that a thorough intuition for the comparative statics of the break point in terms of the three location forces is actually quite cumbersome, because each force is in a complex way affected by ε and σ . However, the proposition shows that there is still an unambiguous net effect on the break point when either ε or σ is increased.

⁷ In graphical terms, a tomahawk bifurcation arises if the utility differential changes from concave to convex at the break point, whereas the pitchfork emerges if the utility differential changes from convex to concave. The associated curvatures already imply that the tomahawk with $\tilde{V}_{(3)} > 0$ must then exhibit sustainable full agglomeration equilibria at lower trade freeness than the break point, i.e., a sustain point ϕ_s that is smaller than the break point ϕ_b for given parameters. Analogously, $\tilde{V}_{(3)} < 0$ is equivalent to $\phi_s > \phi_b$ which is characteristic for the pitchfork bifurcation. In the web supplement we study the sustain point of our model in greater detail, and show that our analysis in terms of $\tilde{V}_{(3)}$ and a comparison of break and sustain point yield identical results.

We represent the sign of $\tilde{V}_{(3)}$ for different values of $\{\sigma, \varepsilon\}$, with the value of σ increasing along the vertical and the value of ε along the horizontal axis. Our assumption $\sigma > \varepsilon \geq 1$ implies that we focus on the area below the main diagonal. The light grey area characterizes the set of parameters where the model exhibits the catastrophic tomahawk bifurcation. The dark area characterizes the parameter range where a pitchfork obtains. Note that the standard core-periphery model is included at the left borderline of the box ($\varepsilon = 1$, varying σ), where in fact the tomahawk bifurcation is observed.

FIGURE 1 ABOUT HERE

Drawing on the characterization of the model's behavior in figure 1, we can state the following result which concludes this paper:

Result: *In the core-periphery model with CES upper tier preferences, assuming $\sigma > \varepsilon \geq 1$, there exists a range of parameter values $\{\sigma, \varepsilon\}$ for which the model exhibits a pitchfork bifurcation rather than a tomahawk bifurcation.*

A value of the upper tier substitution elasticity ε larger than one is not a sufficient, but a necessary condition for the pitchfork. The range of ε -values for which the model delivers a pitchfork increases at larger values of σ , and this location pattern requires that ε is neither too close, nor too far away from the value of σ . As discussed in the introduction, a continuous pitchfork bifurcation appears more plausible from the point of view of descriptive realism (Head and Mayer 2004, sect. 7.2.). We have shown that this pattern can indeed follow within the original modeling framework of the canonical NEG model once the individual preference structure is generalized from a Cobb-Douglas to a CES upper tier utility function. The more general message of our paper is that NEG-theory provides no robust rationale why small parameter changes should have catastrophic consequences.

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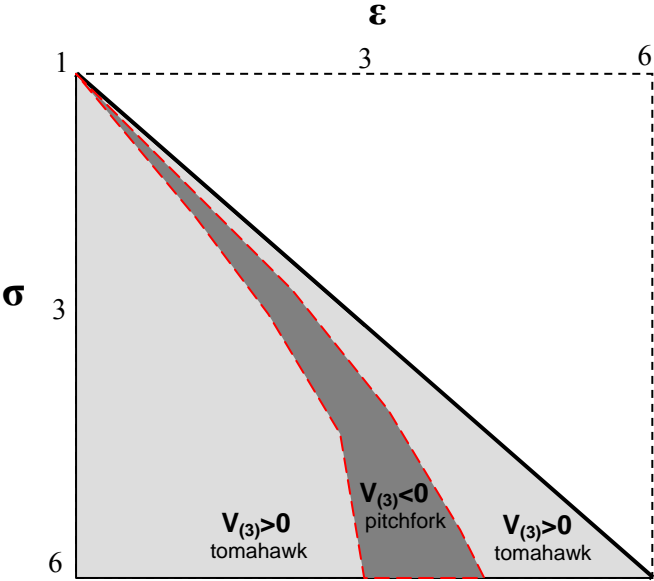
The web supplement is available as a ZIP-File under: www.uni-due.de/js/CESsuppl.zip

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Figure 1: Third derivative of indirect utility differential



Note: In the supplementary material we provide further details on the numerical computations underlying this figure