# A synthesis of footloose-entrepreneur new economic geography models: When is agglomeration smooth and easily reversible?

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**Abstract**

Models of the new economic geography share a number of common conclusions, but also exhibit notable differences, in particular with respect to the shape of the location pattern. Some models imply a catastrophic agglomeration process with hysteresis, so that concentration in one region is not easily reversible. Other models suggest that agglomeration may be smooth, easily reversible and not necessarily feature extreme 'bang-bang' outcomes. These differences reflect the fact that new economic geography models have relied heavily on specific functional forms. In this paper we approach the properties of the particular class of 'footloose entrepreneur'-models with a unifying framework based on the indirect utility function of mobile agents. We are able to provide general, yet handy, formulae to determine the break point and the bifurcation pattern. An application of this framework allows us to show how specific results in the literature can be reconciled as special cases, so that the origin of their differences can be highlighted.

Keywords: new economic geography, agglomeration, location pattern, bifurcation

JEL-classification: R12, R50, F12, F15, F22

## 1. Introduction

The new economic geography has become a burgeoning research field. After the development of the seminal core-periphery model by Krugman (1991) numerous analyses appeared which extended his basic framework. Building on these models, there has recently been an explosion of work which addresses policy issues such as trade policy, taxation or regional redistribution.[1] New economic geography models share a number of properties. One crucial common result is the prediction that, due to market size effects ('linkages', 'pecuniary externalities'), an asymmetric spatial structure may emerge endogenously from a situation with ex-ante identical regions as trade barriers are continuously removed from prohibitive levels. However, a more detailed inspection reveals that new economic geography models exhibit notable differences, too. In this paper we address one key difference, namely their predictions concerning the shape of the location pattern (i.e. the bifurcation pattern).

We use a particular class of new economic geography models, namely the analytically tractable 'footloose entrepreneur' models, where agglomeration is the result of mobile agents who render the local market size endogenous.[2] The core-periphery model by Krugman (1991) is represented in this class of models through the analytically tractable version developed by Forslid and Ottaviano (2003). The contributions by Ottaviano, Tabuchi and Thisse (2002) and Pflüger (2004) belong to this same class of footloose-entrepreneur models. These models

[1] Comprehensive treatments of the new economic geography are provided in Fujita et al. (1999), Fujita and Thisse (2002), Baldwin et al. (2003) and Ottaviano and Thisse (2004).

[2] This framework originates in independent research by Forslid and Ottaviano which later got published in this journal as Forslid and Ottaviano (2003). An alternative approach, where endogenous agglomeration is the result of vertical linkages between firms, has been developed by Venables (1996) and Krugman and Venables (1995). Ottaviano and Robert-Nicoud (2006) provide a synthesis of this approach, and Robert-Nicoud (2005) works out similarities in the mathematical structure of the labour mobility class of models with those based on vertical linkages. Baldwin (1999) has developed a further approach where agglomeration is the result of endogenous capital accumulation (see also Baldwin et al. (2003)).

differ, in particular, in their assumptions regarding the functional form of individual preferences, and that they make different predictions concerning the shape of the bifurcation pattern.

The core-periphery model (Krugman 1991, Forslid and Ottaviano 2003) assumes an upper-tier utility function of the Cobb-Douglas-type, and CES sub-preferences over manufacturing varieties. The bifurcation pattern of this *CP-model* is such that symmetry is the only stable location equilibrium for high levels of trade costs. Once trade costs have fallen to a critical level, the so-called 'break point', all mobile economic activity concentrates immediately in a single region. This model exhibits hysteresis. A reversal of the trade cost decline need not restore symmetry, because the core-periphery pattern is stable up to a 'sustain point' which obtains at a strictly higher level of trade costs than the break point. In contrast, in the model by Ottaviano, Tabuchi and Thisse (2002), where the upper-tier utility is a quadratic quasi-linear function, the break point and the sustain point coincide. Hence, this model, which we term *QLQuad-model*, predicts catastrophic agglomeration without hysteresis. Yet other papers provide results that break even more drastically with the idea of catastrophic agglomeration. The model by Pflüger (2004) which assumes a logarithmic quasi-linear upper-tier utility, predicts a smooth transition from symmetry to agglomeration in the course of trade integration. Hence, in this *QLLog-model*, there are stable interior equilibria with some, but not all mobile workers concentrated in one region. Figure 1 summarises the location pattern of these three models.

**[Figure 1 about here]**

The precise shape of the location pattern is not only a minor technical property of new economic geography models. Rather, it has potentially profound policy implications. For example, a model with the property of locational hysteresis implies that a small change in trade costs may induce large changes in the spatial structure of the economy, which are hardly reversible by policy. These issues are of lesser concern when the location pattern does not exhibit hysteresis, but implies a smooth transition from symmetry to agglomeration. Furthermore, some economic geography models feature a location pattern that can only lead to 'bang-bang' outcomes: Either the two regions are completely symmetrical, or all mobile agents are concentrated in one of the regions. Other models leave room for the more realistic result of stable partial agglomeration.

So far, no systematic analysis has been undertaken that explains how different preferences translate into different equilibrium location patterns. Our approach in this paper differs from the usual one. Rather than starting with specific functional forms for individual preferences, we present a generalised framework based on the indirect utility function of mobile entrepreneurs. This approach has several payoffs. We are able to provide general, yet handy, formulae to determine the break point and the bifurcation pattern. An application of this framework allows us to show how the three specific footloose-entrepreneur models mentioned above can be reconciled as special cases, thereby allowing us to highlight the origin of their differences. With this unifying approach we hope to shed some light on the robustness of results that have been provided in the new economic geography, and to strengthen their scope and applicability for policy purposes.

We want to make it clear from the outset that we do not provide a fully general analysis of the new economic geography in this paper. First, as already explained, we focus on one particular class of models. Second, even within this class of footloose-entrepreneur models we are forced to make restrictive assumptions, in particular regarding technology (see section 3).[3]

[3] The principal reason why a fully general analysis is presumably out of reach is that the new economic geography builds on the assumption that firms produce under increasing returns and do have market power. The quest for a general model of imperfect competition has (so far) been elusive.

Finally, we focus on the positive properties of these models only.[4] Still, we do believe that our framework provides a useful way to organise results which have been derived in the literature. The rest of this paper is organised as follows. In the next section we introduce our unifying framework. In sections 3 and 4 we apply this framework to the specific cases and we highlight the origin of their differences with respect to their location pattern. Section 5 provides a brief summary and outlook.

## 2. A unifying framework

### 2.1. Basic assumptions

This section presents our framework based on a general indirect utility function. As most of the new economic geography, we consider a $2 \times 2 \times 2$ trade model. The economy is composed of two regions (labelled 'home' and 'foreign') with identical tastes and technologies. There are two types of households with perfectly inelastic factor supply. The first type, call it 'skilled labour' or ' entrepreneurs' ($K$), is mobile across regions. The second type, unskilled labour ($L$), is immobile and equally distributed across the two locations. Both household types derive utility from the consumption of two types of goods. There is an agricultural good ($A$), which is homogeneous, traded without cost and produced competitively with a unit input requirement of $L$. This good serves as the numéraire and is assumed to be produced in both regions throughout the analysis. It follows from these assumptions that the wage (income) of the unskilled workers is fixed at unity in both regions. The second good is a manufacturing aggregate $X$ that consists of a large variety of differentiated products. Individual utility functions exhibit a 'love of variety effect', i.e. for a given income level, consumers achieve a higher utility level the higher is the number of available varieties. Each manufacturing good is

[4] The working paper version of this paper, in addition, addresses the welfare properties of these models under the restrictive assumption that the social welfare function is utilitarian (Pflüger and Südekum 2006).

produced by a single firm under internal economies of scale with strictly positive fixed costs and positive marginal costs. The fixed cost component is due to the compensation of skilled labour $K$ of which one unit is needed to produce at all. Unskilled labour $L$ is the only variable input in this industry. Each firm has a negligible impact on the aggregate market outcome and there is no strategic interaction between firms. Due to the fixed input requirement, the supply of skilled labour determines the number (mass) of firms. Firms make zero profits in a long-run equilibrium. The wage rate of skilled labour adjusts to ensure this. Hence, the wage of a firm's skilled worker is the operating surplus of a firm, i.e. the difference between its revenues and its variable costs. Trade of manufacturing varieties across regions is inhibited by transport costs which are measured by some parameter $\tau > 1$.

It is worthwhile to briefly comment on our assumptions concerning technologies. We hold these constant throughout the paper. In contrast to the core-periphery model of Krugman (1991), where skilled labour is used for both the fixed and variable cost to produce a variety of $X$, our setting makes the assumption that the factor intensity of fixed costs differs from the factor intensity of the variable cost in an extreme way.[5] Concerning the agricultural good, the assumptions are made that the input requirement is constant (at unity), that the good is traded without cost and that it is always produced in equilibrium. Even though these assumptions are in line with the core-periphery model and several other analyses, they are restrictive but considerably simplify the analysis.[6]

## 2.2. Indirect utility function and the equation of motion

The long-run spatial equilibrium is determined by the migration decision of the footloose entrepreneurs ($K$). They respond to differentials in (indirect) utility levels across locations. In

[5] This assumption can be traced back to Lawrence and Spiller (1983) and to Flam and Helpman (1987). It was introduced by Forslid and Ottaviano (2003) in the new economic geography as the footloose-entrepreneur model.

[6] These assumptions are relaxed e.g. in Puga (1999) and Fujita et al. (1999).

contrast to previous analyses we do not specify individual preferences. We simply assume the existence of a utility function with standard continuity and non-satiation properties (see Mas-Colell et al. 1995: 56) which gives rise to an indirect utility function of the form

$$V = V\left(Y\left(\lambda,\tau\right), P\left(\lambda,\tau\right), p_A\right) \tag{1}$$

The utility of mobile workers living in the domestic region depends on attainable income $Y$ (i.e. the wage of skilled workers) and on consumer prices of the goods $X$ and $A$. The price of the agricultural numéraire good is equal to one, $p_A = 1$, and will henceforth be suppressed. With some loss of generality we assume that the consumer prices of the manufacturing aggregate can be aggregated into a single number, a scalar price index $P$.[7] By analogy, the indirect utility in the foreign region (distinguished by an asterisk *) is represented by $V^* = V^*\left(Y^*, P^*\right)$.[8]

Nominal income of skilled workers $Y\left(\cdot\right)$ and the scalar price index $P\left(\cdot\right)$ depend on the endogenous share $0 < \lambda < 1$ of mobile workers located in the domestic region, on the exogenous level of transport costs $\tau$, and on the other primitives of the model. Both functions $Y\left(\cdot\right)$ and $P\left(\cdot\right)$ are assumed to be differentiable with respect to $\lambda$. Furthermore, $V\left(\cdot\right)$ is assumed to be continuous and differentiable with respect to both arguments. Letting subscripts denote partial derivatives, and suppressing the arguments of $V\left(\cdot\right)$, we have $V_Y > 0$,

[7] In general, *P* is a *vector* of consumer prices. Under standard separability assumptions on the utility function, this vector can be aggregated into a scalar (e.g. Varian 1992, chapter 9.3). These assumptions are fulfilled in most models of the new economic geography, and so in particular in the three applications that we contemplate below. An example where *P* is not a scalar is the recent CARA specification of Behrens and Murata (2007).

[8] Since the agricultural good is assumed to be produced in both regions and can be traded without costs, the law of one price implies that its price is unity in the foreign region as well.

$V_P < 0$, $V^*_{Y*} > 0$, $V^*_{P*} < 0$ (Mas-Colell et al. 1995).[9] Using the common assumption of myopic adjustment, the equation of motion for mobile households is given by[10]

$$\frac{d\lambda}{dt} \equiv \dot{\lambda} = \left(V(\cdot) - V^*(\cdot)\right) \cdot \lambda \cdot (1-\lambda) \tag{2}$$

There are two types of equilibria in this model, interior ones with $V(\cdot) = V^*(\cdot)$ where either symmetry ($\lambda = \frac{1}{2}$) or stable partial agglomeration ($0 < \lambda < 1, \lambda \neq \frac{1}{2}$) obtains, or core-periphery equilibria with $V(\cdot) \geq V^*(\cdot)$ and $\lambda = 1$, or $V(\cdot) \leq V^*(\cdot)$ and $\lambda = 0$.

### 2.3. Determination of the bifurcation point

Due to the symmetry of the two regions, an equal division of mobile workers across locations ($\lambda = \frac{1}{2}$) is always an equilibrium which is assumed to prevail at initially high trade costs. This equilibrium is not necessarily stable, however, because there is an interaction of destabilising agglomeration forces and of stabilising dispersion forces. The stability of the symmetric equilibrium can be addressed by evaluating the sign of $\partial\left(V(\cdot) - V^*(\cdot)\right)/\partial\lambda$ at the symmetrical configuration $\lambda = \frac{1}{2}$.[11] Of course, agglomeration forces may be so weak that symmetry never becomes unstable, or they may be so strong that symmetry becomes unstable even at infinitely high trade costs. Both cases are usually excluded by appropriate restrictions on the model

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[9] We assume a strictly negative response of the indirect utility with respect to the manufacturing price scalar.

[10] See Baldwin (2001), Neary (2001) and Ottaviano and Thisse (2004) for a critical assessment.

[11] The dynamic adjustment mechanism underlying our analysis is based on (indirect) utility driven migration and assumes that firms are always in zero profit equilibrium (i.e. the wages of the skilled adjust infinitely faster than migration). The pioneering analysis of the stability of the symmetric equilibrium by Puga (1999) which synthesises and generalises the contributions by Krugman (1991) and Krugman and Venables (1995), reverses these assumptions (i.e. the adjustment mechanism involves the entry and exit of firms in his specification). However, these two adjustment mechanism are equivalent in terms of the long-run equilibria and their stability properties (see Puga 1999 and Neary 2001).

primitives (e.g. the 'no black hole'-condition). Ruling out $\partial\left(V(\cdot)-V^*(\cdot)\right)/\partial\lambda > 0$ and $\partial\left(V(\cdot)-V^*(\cdot)\right)/\partial\lambda < 0 \;\; \forall\tau$ at $\lambda=\frac{1}{2}$, there is a critical level of trade costs, the 'break point' $\tau_b$, at which the symmetric equilibrium becomes unstable and, hence, the regions start diverging. This bifurcation point is formally implied by the condition

$$\left.\frac{\partial\left(V(\cdot)-V^*(\cdot)\right)}{\partial\lambda}\right|_{\lambda=1/2} = 0 \;\Leftrightarrow\; \tau_b \tag{3}$$

There need not be a unique solution for $\tau$ in the relevant range. The lowest level of $\tau$ that satisfies eq. (3) is called the 'break point'. In the models that are summarised in figure 1, the bifurcation point is unique, i.e. there is only one value of $\tau$ such that (3) is satisfied.[12]

Using eq. (1), the first derivative of the indirect utility differential $V(\cdot)-V^*(\cdot)$ is given by

$$\frac{\partial\left(V(\cdot)-V^*(\cdot)\right)}{\partial\lambda} = V_P P_\lambda - V^*_{P*}P^*_\lambda + V_Y Y_\lambda - V^*_{Y*}Y^*_\lambda \;, \tag{4}$$

where all terms on the right hand side depend on $\lambda$ and $\tau$. Since regions have identical preferences, $V^*_{P*}=V_P$ and $V^*_{Y*}=V_Y$. Moreover, at $\lambda=\frac{1}{2}$ we have $P^*_\lambda=-P_\lambda$ and $Y^*_\lambda=-Y_\lambda$. Applying (3) at $\lambda=\frac{1}{2}$ we obtain:

$$\left.\frac{\partial\left(V(\cdot)-V^*(\cdot)\right)}{\partial\lambda}\right|_{\lambda=1/2} = 2\left(V_P P_\lambda + V_Y Y_\lambda\right)=0 \quad\Leftrightarrow\quad -\frac{V_P}{V_Y}=\frac{Y_\lambda}{P_\lambda} \tag{5}$$

This condition captures the central forces of the new economic geography: The supply linkage as an agglomeration force works through the effects of the number of firms on the price level,

[12] One possibility to obtain more than one solution is to introduce a congestion force into the model, such as – for example – a fixed housing stock (as in Pflüger and Südekum 2007), a spatial extension involving a linear city with a central business district (as in Tabuchi 1998 or Ottaviano et al. 2002) or decreasing returns to scale in the agricultural sector (as in Puga 1999). The consequence is that below some critical level of trade costs, symmetry reemerges as the unique stable equilibrium, i.e. eq. (3) is also satisfied at this 're-dispersion point'. We abstract from such congestion forces in the present analysis.

and hence on utility ($V_P P_\lambda$). The demand linkage and the competition effect work through the influence of the number of firms on the wage for skilled workers that firms can just afford to pay to break even, and hence on utility ($V_Y Y_\lambda$). The demand linkage is an agglomeration force and the competition effect is a dispersion force. Applying Roy's identity, condition (5) can also be written as $Y_\lambda / P_\lambda = C^X \geq 0$, where $C^X$ denotes demand for the aggregate of manufactures. This formulation shows that $P_\lambda$ and $Y_\lambda$ must have the same sign at the break point. In standard models, $P_\lambda$ is negative due to the supply linkage, as the price index declines when more firms locate in the domestic region. Hence, at $\tau_b$ the skilled wage must also be decreasing in $\lambda$, i.e. $Y_\lambda < 0$. Under our assumption that the indirect utility is continuously differentiable, this same inequality must hold in a neighbourhood of $\lambda = 1/2$. Hence, moving away from the symmetric configuration always leads to a lower nominal income. This is an important insight, because it shows that, in the broad family of models that we consider, initial migrants must be driven by commodity price considerations only.[13] Summarising the results of this section, we have

**Proposition 1**

*Let the location choice of mobile skilled workers be governed by the indirect utility differential* $V(Y(\cdot), P(\cdot)) - V^*(Y^*(\cdot), P(\cdot)^*)$ *as specified above. The following results hold:*

(i) *Provided a break point exists, it is determined by the condition* $-V_P / V_Y = C^X = Y_\lambda / P_\lambda$.

(ii) *At the break point and in its neighbourhood, the wage of skilled workers falls when more skilled workers concentrate in one region (*$Y_\lambda < 0$ *as long as* $P_\lambda < 0$*), i.e. the demand linkage is dominated by the competition effect.*

[13] An explicit analytical and graphical example is provided in Pflüger and Südekum (2007, appendix C).

## 2.4. The location pattern

The bifurcation pattern that emerges at the break point is determined by higher order derivatives of the indirect utility differential evaluated at $\lambda = \frac{1}{2}$ and at the critical level of trade costs, $\tau_b$ (e.g. Fujita et al. 1999; Grandmont 1988; Guckenheimer and Holmes 1983). A necessary condition for a bifurcation to occur is that $\partial^2 \left(V(\cdot) - V^*(\cdot)\right) / \partial\lambda^2 = 0$ at $\lambda = \frac{1}{2}$ and $\tau = \tau_b$, i.e. the second order derivative must be zero at the symmetrical configuration and the critical trade cost level. Whether the bifurcation is a sub-critical pitchfork ('tomahawk') as in fig. 1a, a super-critical pitchfork ('pitchfork') as in fig. 1b, or the borderline case depicted in fig. 1c, is determined by the sign of the third order derivative, $\partial^3 \left(V(\cdot) - V^*(\cdot)\right) / \partial\lambda^3$, which evaluated at $\lambda = \frac{1}{2}$ and $\tau = \tau_b$ depends only on the model primitives. If this term is greater than (smaller than) zero, a tomahawk (pitchfork) bifurcation is implied, and the borderline case follows if the term is equal to zero. Differentiating (4) with respect to $\lambda$ yields

$$\frac{\partial^2 \left(V(\cdot) - V^*(\cdot)\right)}{\partial\lambda^2} = \left(V_{PP} P_\lambda + V_{PY} Y_\lambda\right) P_\lambda + V_P P_{\lambda\lambda} - \left(V^*_{P*P*} P^*_\lambda + V^*_{P*Y*} Y^*_\lambda\right) P^*_\lambda - V^*_{P*} P^*_{\lambda\lambda} \tag{6}$$

$$+ \left(V_{YP} P_\lambda + V_{YY} Y_\lambda\right) Y_\lambda + V_Y Y_{\lambda\lambda} - \left(V^*_{Y*P*} P^*_\lambda + V^*_{Y*Y*} Y^*_\lambda\right) Y^*_\lambda - V^*_{Y*} Y^*_{\lambda\lambda}$$

Using the fact that preferences are identical across regions (i.e. $V^*_{P*} = V_P$, $V^*_{Y*} = V_Y$ and so forth) and taking into account that at $\lambda = \frac{1}{2}$ we have $P^*_\lambda = -P_\lambda$, $Y^*_\lambda = -Y_\lambda$, $P^*_{\lambda\lambda} = P_{\lambda\lambda}$ and $Y^*_{\lambda\lambda} = Y_{\lambda\lambda}$ due to symmetry, one immediately obtains the result that eq. (6) is equal to zero at $\lambda = \frac{1}{2}$. Hence the necessary condition for a bifurcation is always fulfilled.

Moving to the third order derivative, using $\lambda = \frac{1}{2}$, homogeneous preferences, and the fact that the cross derivatives of the utility function are identical (e.g. $V_{PY} = V_{YP}$, $V_{PYY} = V_{YPP}$ and so forth), we obtain after straightforward if somewhat tedious manipulations

$$\Delta(\cdot) \equiv \frac{\partial^3 \left(V(\cdot) - V^*(\cdot)\right)}{\partial \lambda^3}\Bigg|_{\lambda=1/2,\, \tau=\tau_b} = 2 \cdot \frac{\partial^3 V(\cdot)}{\partial \lambda^3}\Bigg|_{\lambda=1/2,\, \tau=\tau_b} = 2\left[\Lambda_P(\cdot) + \Lambda_Y(\cdot) + \Lambda_{YP}(\cdot)\right] \quad (7)$$

where $\Lambda_P(\cdot) = \left[V_P P_{\lambda\lambda\lambda} + 3V_{PP} P_\lambda P_{\lambda\lambda} + V_{PPP}(P_\lambda)^3\right]$

$$\Lambda_Y(\cdot) = \left[V_Y Y_{\lambda\lambda\lambda} + 3V_{YY} Y_\lambda Y_{\lambda\lambda} + V_{YYY}(Y_\lambda)^3\right]$$

$$\Lambda_{YP}(\cdot) = 3\left[V_{YP}\left(P_{\lambda\lambda} Y_\lambda + Y_{\lambda\lambda} P_\lambda\right) + V_{YPP} Y_\lambda (P_\lambda)^2 + V_{YYP} P_\lambda (Y_\lambda)^2\right]$$

The shape of the location pattern depends on the sign of $\Delta(\cdot)$ which we have decomposed into three parts. The terms $\Lambda_Y$ and $\Lambda_P$ collect direct derivatives of $V(\cdot)$ with respect to the skilled wage and the scalar price index and derivates of $Y(\cdot)$ and $P(\cdot)$ with respect to $\lambda$. The term $\Lambda_{YP}$ captures all cross-derivatives of $V(\cdot)$.

The first, second and third derivatives of $Y(\cdot)$ and $P(\cdot)$ with respect to $\lambda$ indicate in what way (i.e. linearly, degressively or progressively) the skilled wage and the scalar price index are affected by an inflow of footloose entrepreneurs (i.e. firms). The direct derivatives and cross derivatives of the indirect utility characterise how utility responds to income and the price index. Apart from these observations, the term $\Delta(\cdot)$ is rather unrevealing, in general. However, once we impose restrictions on preferences as we do so often in qualitative and quantitative applications of demand theory (e.g. Cornes 1992), the term $\Delta(\cdot)$ takes on much simpler forms. For example, if we assume preferences to be quasi-linear, $V_Y = 1$ and all cross derivatives and higher order income derivatives are zero. With homothetic (e.g. Cobb-Douglas) or quasi-homothetic preferences, $V_Y$ depends on the scalar price index and all income derivatives of higher order are zero. The detailed response of the skilled wage and the scalar price index, on the other hand, depends on the utility specification and the particular setup of the model. The simplification obtained when preferences and the detailed model setup is specified will become clear below when we turn to applications of our broad framework. Before turning to these, we summarise the results of this section in

**Proposition 2**

*The bifurcation pattern that unfolds at the break point is determined by the term* $\Delta(\cdot)$ *as specified in eq. (7). If this term is positive (negative), the model exhibits a tomahawk (pitchfork) bifurcation. For* $\Delta(\cdot)=0$*, the borderline case is implied.*

## 3. Applications

In this section we apply our framework to the three models which we have introduced in sections 1 and 2: the core-periphery (CP) model, the quasi-linear logarithmic (QLLog) model, and the quasi-linear quadratic (QLQuad) model. Analytical details for these three models are reported in appendices A1, A2 and A3, respectively.

### 3.1. The CP-model

The CP-model by Forslid and Ottaviano (2003) features a Cobb-Douglas upper-tier utility function $U(X,A)=X^{\mu}A^{1-\mu}$, $0<\mu<1$, with CES sub-preferences over manufacturing varieties $X$. Consumer prices of the symmetrical varieties differ across regions by a multiplicative constant due to the assumption of iceberg trade costs. Hence, $P(\cdot)$ is a scalar which is given by the perfect CES price index, where the constant elasticity of substitution between any two manufacturing varieties is denoted by $\sigma>1$. The Cobb-Douglas preference structure implies an indirect utility function of the form $V(\cdot)=Y(\lambda,\tau)\cdot(P(\lambda,\tau))^{-\mu}$. Hence, $V_{YY}=V_{YYY}=V_{YYP}=0$. Using the expressions for $Y(\lambda,\tau)$ and $P(\lambda,\tau)$ from Forslid and Ottaviano (2003), taking derivatives $Y_{\lambda}$ and $P_{\lambda}$, using $\lambda=\frac{1}{2}$, and applying (5) yields the break point $(\tau_{b,CP})^{1-\sigma}=(\sigma-\mu)(\sigma-1-\mu)/(\sigma+\mu)(\sigma-1+\mu)$ as reported by Forslid and Ottaviano (see also equation A4 in the appendix).

Turning to the shape of the location pattern, we insert the functional forms of this model in equation (7). We then apply $\lambda = \frac{1}{2}$ and $\tau = \tau_{b,CP}$, and rearrange terms to obtain an expressions for $\Delta_{CP}\left(\frac{1}{2}, \tau_{b,CP}\right)$, which is unambiguously positive (see eq. A5 in the appendix). This verifies that the CP model exhibits a tomahawk bifurcation as depicted in fig. 1a.

### 3.2. The QLLog-model

The QLLog-model by Pflüger (2004) has the same production structure as Forslid and Ottaviano (2003). The crucial difference is the form of the upper tier utility function, which is given by $U(X, A) = \alpha \ln X + A$. All income effects are eliminated from manufacturing and channelled into the agricultural sector. The sub-preferences for the manufacturing varieties are of the CES form, and trade costs are of the iceberg type as in the CP-model. Hence, the demand curves for individual varieties have constant price elasticity $\sigma > 1$.

In the QLLog-model the indirect utility function is given by $V(\cdot) = Y(\cdot) - \alpha \ln P$, so that $V_Y = 1$, and $V_P = -\alpha / P(\cdot)$. The CES price index $P(\lambda, \tau)$ corresponds to the one of the CP model. Applying (5) together with $\lambda = \frac{1}{2}$ and using the expression for $Y(\lambda, \tau)$ and $P(\lambda, \tau)$ from Pflüger (2004) yields the break point as given in (A8) in the appendix: $\left(\tau_{b,QLLog}\right)^{1-\sigma} = \left(\sigma(2\rho - 1) - 2\rho\right) / \left(\sigma(3 + 2\rho) - 2(1 + \rho)\right)$. The location pattern is now even simpler to determine, since we have $V_{YY} = V_{YYY} = V_{YP} = V_{YPP} = V_{YYP} = 0$ with quasi-linear preferences. Inserting $\tau = \tau_{b,QLLog}$ in equation (7) yields $\Delta_{QLLog}\left(\frac{1}{2}, \tau_{b,QLLog}\right) < 0$. This verifies that the QLLog-model exhibits a pitchfork bifurcation (see fig. 1b).

### 3.3. The QLQuad-model

The QLQuad-model by Ottaviano, Tabuchi and Thisse (2002) differs more profoundly from the two previous frameworks in that it is neither based on CES preferences, nor on iceberg trade costs. However, it is related to the QLLog-model, because it also assumes quasi-linear preferences. The upper-tier utility function is given by $U(X, A) = X + A$ with subutility

$$X = \alpha \int_{i=0}^{N+N^*} x_i \, di - \frac{\beta - \delta}{2} \int_{i=0}^{N+N^*} x_i^2 \, di - \frac{\delta}{2} \left( \int_{i=0}^{N+N^*} x_i \, di \right)^2 .$$

Demand curves for the single varieties are linear rather than isoelastic. Imposing the restrictions $\beta > \delta > 0$ and $\alpha > 0$, these preferences exhibit the 'love for variety'-effect. Turning to the supply side, Ottaviano, Tabuchi and Thisse (2002) assume price competition on segmented markets, and that trade costs are incurred in units of the numéraire good by the parameter $\tau$. In contrast to the two previous models, the markups of producer prices on marginal costs are no longer constant.

The QLQuad-model leads to a particularly simple additive separable form of indirect utility $V(\lambda, \tau) = Y(\cdot) - P(\cdot) + \overline{q}_0$, where $\overline{q}_0 > 0$ is a constant. Hence, $V_Y = 1$, $V_P = 0$ and all higher order derivatives of $V(\cdot)$ are zero. The impact of prices on indirect utility can be characterised inversely by the consumer surplus function $P(\lambda, \tau)$, a scalar. This scalar, and the income function $Y(\lambda, \tau)$ are both quadratic in $\lambda$. Hence, $Y_{\lambda\lambda\lambda} = P_{\lambda\lambda\lambda} = 0$. The break point can be determined by solving the equation $-(V_P / V_Y) = (Y_\lambda / P_\lambda) = 1$ for $\tau$.

This yields the break point as given in Ottaviano and Thisse (2002: 420):

$$\tau_{b,QLQuad} = \frac{4aF(3bF + 2cK)}{\left(2bF(3bF + 3cK + cL) + c^2 K(L + K)\right)}$$

For details about this derivation, and for the definition of the symbols, refer to appendix A3. Regarding the shape of the location pattern, due to $V_{PP} = V_{PPP} = 0$, eq. (7) reduces to $\Delta_{QLQuad} = 2(P_{\lambda\lambda\lambda} + Y_{\lambda\lambda\lambda}) = 0$. It follows that the bifurcation pattern is a borderline case between tomahawk and pitchfork bifurcation (as illustrated in fig. 1c).

## 4. Model comparison

The three models that we have presented as special cases of our framework share the properties $V_{YY} = V_{YYY} = V_{YYP} = 0$ since the indirect utility function is linear in $Y$ in all these cases. However, the marginal utility of income is independent of prices only in the quasi-linear models. As a result of income effects in the $X$ -sector, $V_Y$ is decreasing in $P$ ($V_{YP} < 0$) and always below unity ($0 < V_Y < 1$) in the CP-model. Table 1 summarises the signs of the utility, price and income responses evaluated at their break point for the three models.

**[Table 1 about here]**

An inspection of this table shows that there is actually just one single case where a derivative has a different sign across models, namely $Y_{\lambda\lambda}$ is positive in the CP- and the QLLog-model, but negative in the QLQuad-model. This wide correspondence of signs is useful for pinning down *why* these three models differ with respect to their location pattern.

The shape of the bifurcation of the QLQuad-model is easily rationalised by the fact that the indirect utility $V\left(Y(\cdot), P(\cdot)\right)$ is linear in both arguments, and that $Y_{\lambda\lambda\lambda} = P_{\lambda\lambda\lambda} = 0$ due to the assumption of a quadratic sub-utility for the manufacturing aggregate $X$ . To understand why the bifurcation pattern is a tomahawk in the CP-, but a pitchfork in the QLLog-model we use the decomposition of $\Delta(\cdot)$ into parts which entails only direct derivatives ($\Lambda_P$ and $\Lambda_Y = V_Y Y_{\lambda\lambda\lambda}$) and the term $\Lambda_{YP}$ that captures the cross-partial derivatives $V_{YP}$ and $V_{YPP}$. This term $\Lambda_{YP}$ is equal to zero in the QLLog-model (as in any quasi-linear model), but it is positive in the CP-model. Furthermore, the two models share the properties $\Lambda_P > 0$, $\Lambda_Y < 0$ and $\left(\Lambda_P + \Lambda_Y\right) < 0$ (see appendix B). Hence, the positive overall sign of $\Delta(\cdot)$ in the CP-case must be due to the fact that the positive term $\Lambda_{YP}$ dominates the negative term $\left(\Lambda_P + \Lambda_Y\right)$. Economically speaking, $\Lambda_{YP}$ represents income effects in the demand for manufactures $X$

which are absent in quasi-linear models. Hence, our analysis suggests that income effects are the cause of the tomahawk shape of the location pattern.[14]

## 5. Conclusion

In this paper we have developed a unifying framework of different footloose-entrepreneur new economic geography models. We have discussed how the results of three specific models with regard to the shape of the location pattern can be reconciled. Our framework can be used to pin down the properties which explain the differences across models. For example, the core-periphery framework is the only model that features income effects in the demand for manufacturing varieties. We have shown that this distinctive feature explains why the CP-model exhibits a tomahawk bifurcation.

On a more general level, our analysis suggests that certain insights of the new economic geography are robust. Trade integration will in fact lead to endogenous agglomeration since some type of bifurcation surely arises at the break point, and this agglomeration is initially driven by commodity price considerations. The further details of this agglomeration process (catastrophic or smooth, with or without hysteresis) are, however, not robust across different models and depend on quite specific technical properties. Putting too much emphasis on the particular implications of the tomahawk bifurcation type that results in the seminal Krugman-model ('history matters') is, therefore, unwarranted because this bifurcation type will only result under certain conditions of the underlying individual preferences.

Although our unifying framework appears to be of little use if preferences are completely arbitrary, it may be used to develop further specific models based on preferences with other,

[14] Note, however, that absence of income effects ( $\Lambda_{YP} = 0$ ) does not necessarily imply a pitchfork bifurcation. Although $\left(\Lambda_P + \Lambda_Y\right) < 0$ holds in the QLLog-model, this need not be true in general; e.g. a tomahawk shape follows if $\Lambda_P$ is sufficiently strongly positive, whereas $\Lambda_Y$ is sufficiently close to zero, which in turn might be due to a low value of $V_Y$ .

possibly more modest, restrictions than those used so far in the new economic geography. Future work might also address ways to relax some of the restrictive assumptions concerning technology that we have imposed. Lastly, it would also be desirable, with suitable modifications, to develop a similar unifying framework that comprehends further types of new economic geography models, notably those in which agglomeration is due to vertical linkages between firms.

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## Appendix A1 CP-model (version by Forslid and Ottaviano 2003)

In the CP-model we have

$$V(\cdot) = Y(\lambda,\phi)\cdot\left(P(\lambda,\phi)\right)^{-\mu} \tag{A1}$$

with $$P(\lambda,\phi) = \left[\lambda + (1-\lambda)\phi\right]^{\frac{1}{1-\sigma}}, \tag{A2}$$

$$Y(\lambda,\phi) = \frac{\rho\,\mu\left(2\sigma\phi\lambda + \left(\sigma-\mu+(\mu+\sigma)\phi^2\right)(1-\lambda)\right)}{(\sigma-\mu)\left(\sigma\phi\lambda^2 + \left(\sigma-\mu+(\mu+\sigma)\phi^2\right)\lambda(1-\lambda) + \sigma\phi(1-\lambda)^2\right)} \tag{A3}$$

where $\sigma > 1$, $0 < \mu < 1$, $0 < \lambda < 1$.

Eqs. (A1)-(A3) make use of the common notation in CES-based models where $0 < \phi \equiv \tau^{1-\sigma} < 1$ is an inverse measure of the iceberg trade costs. Note that (A3) is a slightly rewritten version of eq. (16) in Forslid and Ottaviano (2003).

Break point determination ($\lambda = \frac{1}{2}$):

$$-\left(\frac{V_P}{V_Y}\right) = \frac{\mu\cdot Y(\cdot)}{P(\cdot)} = \frac{Y_\lambda}{P_\lambda} \qquad \Rightarrow \phi_{b,CP} = \left(\tau_{b,CP}\right)^{1-\sigma} = \frac{(\sigma-\mu)(\sigma-1-\mu)}{(\sigma+\mu)(\sigma-1+\mu)} \tag{A4}$$

$\phi_{b,CP} > 0$ due to assumption $(\sigma-1-\mu) > 0$ ("no black hole"-cond.); see eq. (26) of Forslid and Ottaviano (2003).

Location pattern

$$\Delta_{CP}\left(\tfrac{1}{2},\phi_{b,CP}\right) = \frac{64\mu^5\rho(\sigma-1+\mu)(2\sigma-1)^3\cdot\Gamma_{CP}}{(\sigma-\mu)(\sigma-1)^3\left(\mu^2+\sigma(\sigma-1)\right)^3}\cdot(\sigma-1-\mu) > 0 \tag{A5}$$

where $\Gamma_{CP} \equiv \left(\frac{\mu^2+\sigma(\sigma-1)}{(\mu+\sigma)(\mu+\sigma-1)}\right)^{\frac{\mu}{(\sigma-1)}} > 0$, $\rho \equiv L/K > 0$

## Appendix A2 QLLog-model (Pflüger 2004)

In the QLLog-model the indirect utility, income and price function read as

$$V(\cdot) = Y(\cdot) - \alpha \cdot \ln P(\cdot) \tag{A6}$$

with
$$Y(\lambda, \phi) = \frac{\alpha}{\sigma}\left[\frac{\rho+\lambda}{\lambda+(1-\lambda)\phi} + \frac{\phi(\rho+1-\lambda)}{\phi\lambda+(1-\lambda)}\right] \tag{A7}$$

$$P(\lambda, \phi) = \kappa\left[\lambda + (1-\lambda)\phi\right]^{\frac{1}{1-\sigma}}$$

where we also use the trade freeness measure $0 < \phi \equiv \tau^{1-\sigma} < 1$.

Break point determination ( $\lambda = \frac{1}{2}$ ):

$$-\left(\frac{V_P}{V_Y}\right) = \frac{\alpha}{P(\cdot)} = \frac{Y_\lambda}{P_\lambda} \qquad \Rightarrow \phi_{b,QLLog} = \left(\tau_{b,QLLog}\right)^{1-\sigma} = \frac{\sigma(2\rho-1)-2\rho}{\sigma(3+2\rho)-2(1+\rho)} \tag{A8}$$

Location pattern

$$\Delta_{QLLog}(\cdot) = -\frac{64\alpha(2\sigma-1)^3}{(1+2\rho)^3(\sigma-1)^4} < 0 \tag{A9}$$

(cf. Pflüger 2004: 569 and footnote 10 in the original paper).

## Appendix A3 QLQuad-model (Ottaviano, Tabuchi and Thisse 2002)

The income function $Y(\cdot)$ and the scalar price function $Y(\cdot)$ in the QLQuad-model are given by (see Ottaviano and Thisse 2002, eqs. (7) and (10) and pages 439-440)

$$Y(\cdot)=\frac{bF+cK}{4F^2\left(2bF+cK\right)^2}\left[\left(2aF+\tau cK(1-\lambda)\right)^2\left(\frac{L}{2}+\lambda K\right)+\left(2aF-2\tau bF-\tau cK(1-\lambda)\right)^2\left(\frac{L}{2}+(1-\lambda)K\right)\right]$$

$$P(\cdot)=\frac{K}{8F^2\left(cK+2bF\right)^2}\left[\begin{array}{c} c^3K^3\tau^2(1-\lambda)\lambda+c^2K^2\tau F(1-\lambda)(b\tau(4+\lambda)-8a)- \\ 8cKF^2\left(a^2+2ab\tau(1-\lambda)-b^2\tau^2(1-\lambda)\right)-4bF^3\left(3a^2+2ab\tau(1-\lambda)-b^2\tau^2(1-\lambda)\right)\end{array}\right]$$

In order to avoid confusion we have changed some symbols compared to the original notation of Ottaviano, Tabuchi and Thisse (2002). In particular, the symbols F, K and L are originally labelled $\phi$, L and A. Furthermore, the symbols a, b and c are defined as follows:

$$a\equiv\alpha\big/\left[\beta+\left(N-1\right)\delta\right],\qquad b\equiv1\big/\left[\beta+\left(N-1\right)\delta\right],\qquad c\equiv\delta\big/\left[\left(\beta-\delta\right)\left(\beta+\left(N-1\right)\delta\right)\right]$$

where $N=K/F$ denotes the total number of varieties in the economy.

Break point determination ($\lambda=\frac{1}{2}$): (cf. Ottaviano and Thisse 2002: 420)

$$-\left(\frac{V_P}{V_Y}\right)=1=\frac{Y_\lambda}{P_\lambda}\qquad\Rightarrow\tau_{b,QLQ}=\frac{4aF\left(3bF+2cK\right)}{2bF\left(3bF+3cK+cL\right)+c^2K(L+K)}\tag{A10}$$

## Appendix B Model comparison

**CP-model**

$$\Lambda_P = \frac{16\mu^5\rho(2\sigma-1)^3\left(2(\sigma-1)-\mu\right)\left(\sigma-1-\mu\right)\cdot\Gamma_{CP}}{(\sigma-\mu)(\sigma-1)^3\left(\mu^2+\sigma(\sigma-1)\right)^3} > 0, \quad \Gamma_{CP} \equiv \left(\frac{\mu^2+\sigma(\sigma-1)}{(\mu+\sigma)(\mu+\sigma-1)}\right)^{\frac{\mu}{(\sigma-1)}}$$

$$\Lambda_Y = -\frac{96\mu^5(2\sigma-1)^2\cdot\Gamma_{CP}}{(\sigma-\mu)(\sigma-1)^2\left(\mu^2+\sigma(\sigma-1)\right)^2} < 0$$

$$(\Lambda_P+\Lambda_Y) = -\frac{16\mu^5\rho(2\sigma-1)^2\left(2\sigma(\sigma+2\mu)-5\mu-2\right)\left(\sigma-1+\mu\right)\cdot\Gamma_{CP}}{(\sigma-\mu)(\sigma-1)^3\left(\mu^2+\sigma(\sigma-1)\right)^3} < 0$$

$$\Lambda_{YP} = \frac{48\mu^5\rho(2\sigma-1)^2\left(2\sigma(\sigma-1)-\mu\right)\left(\sigma-1+\mu\right)\cdot\Gamma_{CP}}{(\sigma-\mu)(\sigma-1)^3\left(\mu^2+\sigma(\sigma-1)\right)^3} > 0$$

$\Rightarrow 2\left[(\Lambda_P+\Lambda_Y)+\Lambda_{YP}\right] = \Delta_{CP}(\cdot)$, see eq. (A5)

**QLLog-model**

$$\Lambda_P = \frac{16\alpha\left(2\sigma-1\right)^3}{\left(2\rho+1\right)^3\left(\sigma-1\right)^4} > 0 \qquad \Lambda_Y = -\frac{48\alpha\left(2\sigma-1\right)^3}{\left(2\rho+1\right)^3\left(\sigma-1\right)^4} < 0 \qquad \Lambda_{YP} = 0$$

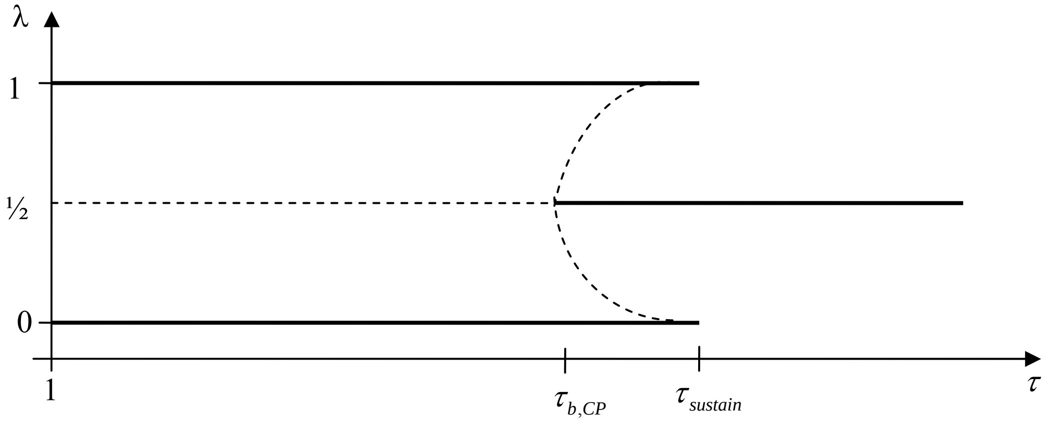
$\Rightarrow 2(\Lambda_P+\Lambda_P) = \Delta_{QLLog}(\cdot) < 0$, see eq. (A9)

**Table 1: Properties of the specific models** (at $\lambda = \frac{1}{2}$ and $\tau = \tau_b$)

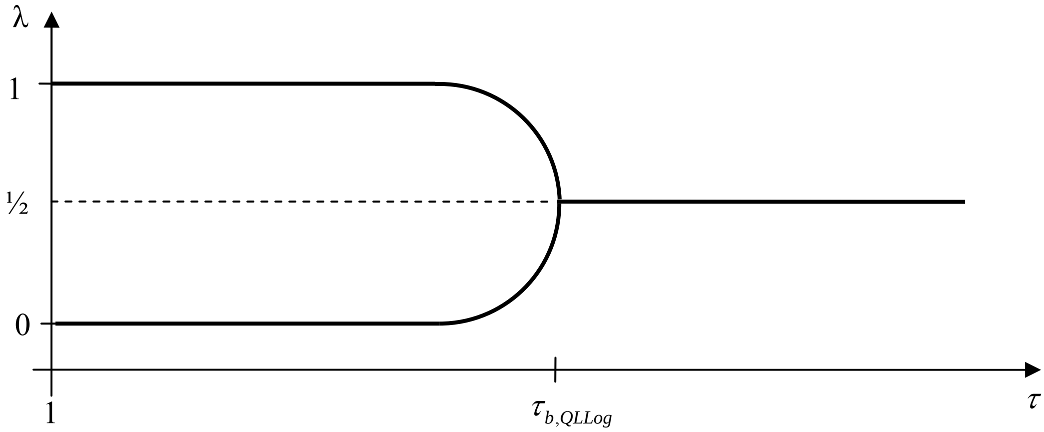
| | **CP** | **QLLog** | **QLQuad** |
|---|---|---|---|
| $\mathbf{V_Y}$ | $0 < V_Y < 1$ | **1** | **1** |
| $\mathbf{V_{YP} = V_{PY}}$ | ⊖ | **0** | **0** |
| $\mathbf{V_{YPP} = V_{PYY}}$ | ⊕ | **0** | **0** |
| $\mathbf{V_P}$ | ⊖ | ⊖ | **-1** |
| $\mathbf{V_{PP}}$ | ⊕ | ⊕ | **0** |
| $\mathbf{V_{PPP}}$ | ⊖ | ⊖ | **0** |
| $\mathbf{P_\lambda}$ | ⊖ | ⊖ | ⊖ |
| $\mathbf{P_{\lambda\lambda}}$ | ⊕ | ⊕ | ⊕ |
| $\mathbf{P_{\lambda\lambda\lambda}}$ | ⊖ | ⊖ | **0** |
| $\mathbf{Y_\lambda}$ | ⊖ | ⊖ | ⊖ |
| $\mathbf{Y_{\lambda\lambda}}$ | ⊕ | ⊕ | ⊖ |
| $\mathbf{Y_{\lambda\lambda\lambda}}$ | ⊖ | ⊖ | **0** |

## Figure 1 Characteristics of different agglomeration models

a) Cobb-Douglas/CES model (Krugman 1991, Forslid and Ottaviano 2003) – tomahawk bifurcation at $\tau_{b,CP}$




b) Quasi-linear logarithmic model (Pflüger 2004) – pitchfork bifurcation at $\tau_{b,QLLog}$




c) Quasi-linear quadratic model (Ottaviano, Tabuchi and Thisse 2002) – borderline bifurcation type at $\tau_{b,QLQuad}$