FLOW CONTROL IN A FAILURE-PRONE MULTI-MACHINE MANUFACTURING SYSTEM *

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Abstract:

The problem of computing optimal production rates for a multi-machine multi-product manufacturing system subject to machine failure is considered. The inventory balance equation is represented by a discrete-time flow model with Markovian jumps to take into account machine breakdown. When defining quadratic cost functions, the associated optimal control problem leads to a set of strongly coupled Riccati-like equations. A necessary and sufficient condition for the existence of a positive semidefinite solution for such equations is given. By verifying a simple matrix inequality, it is shown that such solution exists and can be obtained as a limit of a monotonic sequence. This leads to a straightforward numerical algorithm for the computation of the solution. In this way the optimal production rates for each product type can be determined. An example involving 9 machines and 6 product types is given to illustrate the proposed approach.

1. Introduction

A special attention has been given during the last few years to the problem of production control in manufacturing systems with failure prone machines [2, 3, 6, 8-10]. In the seminal paper of Kimemia and Gershwin [8], the problem is formulated in a dynamic programming framework and a hierarchical control scheme is proposed. The main idea in such a hierarchy is to treat events and variables according to the speed of their dynamics such that at any control level some events may be considered as static at one end of the time scale, i.e. the levels of the hierarchy correspond to classes of events that have distinct frequencies of occurrence. For example, if a three level control hirarchy is adopted, the highest level (or level 1) will be dealing with long term, low frequency, strategic and managerial decisions; at level 2, medium term planning and organization decisions must be taken, while the lowest level (level 1) will be treating short term, high frequency scheduling and operational decisions. For variables with fast variations (with respect to a given time scale), an averaging model must be determined. Flow models are often used in the intermediate level to represent the inventory balance equation of the production system. The aim is to compute an average production rate for each product type in order to meet the demand with minimum surplus or backlogging costs. During the optimization procedure capacity changes due to machine failures must be taken into account. In [3, 6, 8], changes in the system's capacity due to machine failure are described as a function of the state of the system and since machine breakdowns cannot be predicted, capacity is considered as a stochastic set. This means that for the dynamic system representing the flow of parts, a sudden change in the system's state due to machine failure is transformed to a change (or a jump) for the capacity constraint. This formulation leads to rather difficult stochastic optimization problems and it is well known [3, 10] that if the system can produce enough to meet the demand, the optimal production strategy will transfer the inventory level toward a specific target called *the hedging point* as quickly as possible, and will remain at this point as long as no change in the machine states occur. However, computing the hedging point is not an easy task [2, 3, 10], and to have a tractable solution only examples with a limited number of machines and products could be treated.

In this paper we propose to introduce the changes in machine states, i.e. capacity changes, directly in the flow model. This means that, depending on the system's state, several flow models should be used during the optimization procedure. It is shown that such a formulation leads to an optimal control problem involving linear discrete-time systems with Markovian jumps and an efficient solution of such a problem is given.

The paper is structured as follows: in section 2, the problem to be solved is formulated. Section 3 is dedicated to theoretical results giving a necssary and sufficient condition for the existence of a solution. Also, an efficient and convergent algorithm to solve the set of strongly coupled Riccati-type equations arising in the optimization procedure is given in. An example with 9 machines and 6 product types is treated in section 4 to illustrate the proposed approach.

2. Problem Formulation

We consider a manufacturing system with m machines $M_1, M_2, \cdots, M_m$ where each machine can be in either functional or breakdown state. This system is producing p types of parts $p_1, p_2, \cdots, p_p$ and each part type require processing for a specified

time on some of the machines. The machines are flexible in the sense that they can process several part types with virtually no setups but not necessarily with the same production rate. Therefore, each part type has many predetermined possible paths through the system and each path corresponds to a sequence of machines where such a part can be processed. For example, parts of type p_2 may be processed at machine M_1, then at M_2, then at M_3; they may also follow the path M_1, M_4, M_5 or M_2, M_4, M_6. The choice of the most appropriate path for parts of type p_i $(1 \leq i \leq p)$ at time t_j depends on machine availability in the system and on the demand for part p_i. Due to machine failures and repairs, the system's capacity may fluctuate in large proportions. It should be noted that a single machine failure may cause a serious drop in capacity if it is common to several paths. Therefore, breakdown and repair events must be taken into account quickly in order to keep up with the demand.
To take into consideration the changes in the machine states, a discrete-time inventory balance equation with Markovian jumps is chosen,i.e.

$$x_{k+1} = x_k + \hat{B}(\hat{r}_k)\hat{u}_k - d_k; \quad 0 \leq k \leq N, \tag{1}$$

with initial state $x(0) = x_0,\ \hat{r}(0) = \hat{r}_0$, where $x_k \in \mathbf{R}^p$, is the available stock vector, $\hat{u}_k \in \mathbf{R}^r$ is the control vector representing the production rates and $d_k \in \mathbf{R}^p$ is the demand.
Here k is the time index, $\hat{r}_k$ is the form process taking values in the set $\mathbf{F} = \{1, 2, ..., F\}$ and being a finite state discrete time Markov chain with transition probabilities

$$\mathbf{Pr}\{\hat{r}_{k+1} = j | \hat{r}_k = i\} = \hat{p}_{ij} \ , \ 1 \leq i, j \leq F, \text{ with } \hat{p}_{ii} > 0.$$

It is required to determine $\hat{u}_k$ in order to keep x_k as close as possible to zero. In an optimal control framework, this is equivalent to compute $\hat{u}_k$ that minimizes the cost criterion

$$\hat{J}(x_0, \hat{r}_0) = \lim_{N\to\infty} \mathbf{E}\,[\sum_{k=0}^{N-1}(x_k^T Q(\hat{r}_k)x_k + \hat{u}_k^T \hat{R}(\hat{r}_k)\hat{u}_k) + x_N^T \hat{K}_T(\hat{r}_N)x_N], \tag{2}$$

where $Q(\hat{r}_k) \geq 0$, $\hat{R}(\hat{r}_k) > 0$, $\hat{K}_T(r_N) \geq 0$ for all k.

Discrete-time flow models are used here since they reflect in a natural way the computer controlled aspects in a manufacturing system. However, it should be noted that all results given in this paper could be easily extended to the case of continuous time models.

It is assumed that the events of machine breakdown or machine repair are independent and that the controller has complete information about the state of all machines. In other words, the mode $\hat{r}_k$ of the system is known. The transition rates of the Markov chain are functions of the mean time between failure (MTBF) and the mean time to repair (MTTR) of the machines. Matrix $\hat{B}(\hat{r}_k)$ is the control matrix when the system is in mode $\hat{r}_k$ corresponding to a given configuration of operating machines. The elements of $\hat{B}(\hat{r}_k)$ will be zero when a machine is out of order and/or if it is not used to process the considered part type. With m machines and two possible states per machine, $F = 2^m$ modes should be considered. Such a task becomes unrealistic even for a relatively small number of machines. By using the formulation with predetermined paths for each part type it is possible to reduce considerably the number of modes to be studied. The basic idea is to consider the state of paths instead of considering the state of machines. In fact, each machine state is directly related to a path state and a single machine failure may cause a breakdown state for several paths. A path is considered in a breakdown state when at least one of the machines in the path is down. This corresponds to a cautious strategy where parts are only introduced in paths with operating machines. Thus, instead of solving the optimal control problem associated to machine states (equations 1-2), we obtain a modified problem associated to path states defined as

$$x_{k+1} = x_k + B(r_k)u_k; \;\; 0 \leq k \leq N, \tag{3}$$

with initial state $x(0) = x_0$, $r(0) = r_0$, where $x_k \in \mathbf{R}^p$, is the available stock vector, $u_k \in \mathbf{R}^s$ is the control vector to be determined representing the production rates for paths. r_k is the form process taking values in the set $\mathbf{f} = \{1, 2, ..., f\}$ and being a finite state discrete time Markov chain with transition probabilities

$$\mathbf{Pr}\{r_{k+1} = j | r_k = i\} = p_{ij} \;,\; 1 \leq i, j \leq f, \text{ with } \; p_{ii} > 0.$$

f is the total number of authorized paths. It should be noted that by considering the states of paths $f << 2^m$. However, there is no simple relation between f and $F = 2^m$. The cost criterion to be minimized is:

$$J(x_0, r_0) = \lim_{N\to\infty} \mathbf{E}\,[\sum_{k=0}^{N-1} (x_k - d_k)^T Q(r_k)(x_k - d_k) + (u_k - u_{dk})^T R(r_k)(u_k - u_{dk})$$
$$+ (x_N - d_N)^T K_T(r_N)(x_N - d_N)], \tag{4}$$

where $Q(r_k) \geq 0$, $R(r_k) > 0$, $K_T(r_N) \geq 0$ for all k. These weithing matrices are mode dependent, and thus they could be chosen to give a priority to some production paths in function of the operating mode. Note that the demand has been included in the cost function and u_d is an average production rate that could be imposed by a higher control level.

Equations (3-4) define a discrete-time Markovian-jump linear-quadratic optimal control problem [4, 5, 7]. It is well known that the solution of such a problem leads to the mode dependent feedback control law:

$$u_k(x, i) = u_{dk} - \left[R_i + B_i^T G_j B_i\right]^{-1} B_i^T G_j (x_k - d_k) \tag{5}$$

where the notation $B(r_k) = B_i$ is used when $r_k = i$, and $G_j = \sum_{i=1}^{f} p_{ji} K_i$ and $1 \leq j \leq f$.

$K_i = K(r_k)$ if $r_k = i$, are obtained by solving the system of coupled Riccati-like equations

$$Ki = G_j + Q_i - G_j B_i [R_i + B_i^T G_j B_i]^{-1} B_i^T G_j \tag{6}$$

Solving equation (6) is a major difficulty in computing the production rates sought. In fact, according to [5] it is not possible to write (6) as a higher dimensional single Riccati equation. Therefore, it is clear that known results and algorithms to solve the linear-quadratic optimal control problem cannot be applied directly to jump-linear systems.

The purpose of the next section is to give necessary and sufficient conditions for the existence of a positive semidefinite solution for the set of coupled Riccati equations (6). Moreover, a simple algorithm to compute the solution sought is given.

3. Existence and Monotonicity Results for Coupled Riccati Equations

Discrete time markovian jump-linear quadratic optimal control problems have been studied in several papers. In [4] the finite-time horizon case is solved. In [5, 7] necessary and sufficient conditions for the existence of a steady-state solution of (6) are derived; however, these conditions are not easy to test. We recall in this section some of the main results given in [1]. These results are adapted here to the case of flow control in manufacturing systems.

If $r_k = j$ we shall use the following notations

$$\bar{B}_j := \sqrt{p_{jj}} B(r_k) R(r_k)^{-1/2} \, , K_j := K(r_k) \ , \quad Q_j := Q(r_k) \ , \quad \text{and} \quad \pi_{ij} = \frac{p_{ij}}{p_{ii}},$$

$1 \leq i, j \leq f$. Using these abbreviations we can write (6) as

$$K_j = F_j + Q_j - F_j \bar{B}_j [I + \bar{B}_j^T F_j \bar{B}_j]^{-1} \bar{B}_j^T F_j =: \varphi_j(F_j, Q_j) \ , 1 \leq j \leq f \ , \tag{7}$$

where $F_j = \sum_{i=1}^{f} \pi_{ji} K_i = K_j + \sum_{i \neq j} \pi_{ji} K_i$ and where φ_j is defined by (7).

Let $K_{j0} \geq 0 \ , 1 \leq j \leq M$, then we define the sequence $(K_j(\nu))_{\nu \in \mathbf{N_0}}, 1 \leq j \leq f$ by

$$K_j(0) = K_{j0}$$

and

$$K_j(\nu + 1) = \varphi_j(K_j(\nu) + \sum_{i \neq j} \pi_{ji} K_i(\nu), Q_j) \ \ , \tag{8}$$

$1 \leq j \leq f, \nu \in \mathbf{N_0}$.

The following hypothesis ensures that the sequence $(K_j(\nu))$ is decreasing (see [1] for details).

(**H**): There exist matrices $K_{j0} \geq 0, 1 \leq j \leq f$ such that $K_j(1) \leq K_{j0} \ , 1 \leq j \leq f$.

Theorem 1.

Under hypothesis (**H**) for $1 \leq j \leq f$ the limit

$$K_j^\infty := \lim_{\nu \to \infty} K_j(\nu)$$

exists ; furthermore we have the following monotonicity properties

$$0 \leq K_j^\infty \leq K_j(\nu + 1) \leq K_j(\nu) \ , \nu \in \mathbf{N_0}. \tag{9}$$

The Proof of Theorem 1 is given in [1].

Corollary 1.

The coupled system (7) has a set of positive semidefinite solutions K_j^∞ , $1 \leq j \leq f$, if and only if (**H**) is satisfied.

Lemma 1.
The sequences $(K_j^0(\nu))_{\nu\in\mathbf{N_0}}$ defined by $K_j^0(\nu+1) = \varphi_j(K_j^0(\nu)+\sum_{i\neq j}\pi_{ji}K_i^0(\nu), Q_j)$ and $K_j^0(0) = 0$ for $\nu \in \mathbf{N_0}$ are nondecreasing. These sequences are bounded if and only if $(\mathbf{H})$ is valid; in this case $K_j^{0,\infty} := \lim_{\nu\to\infty} K_j^0(\nu)$ exists and defines a solution of the coupled system (7).
For the proofs of Corollary 1 and Lemma 1 see [1].
Hypothesis $(\mathbf{H})$ is important from theoretical point of view since it ensures the existence of positive semidefinite solutions of (7). However, in practical applications one can always start with the zero matrix since $(\mathbf{H})$ is automatically fulfilled if the sequences $(K_j^0(\nu))$ turn out to be bounded. On the other hand, if $(\mathbf{H})$ is satisfied and according to Theorem 1, the sequences $(K_j^0(\nu))$ are bounded and converge monotoni to $K_j^{0,\infty}$. Considering these results, the set of coupled Riccati-like equations is integrated as follows:

$$\begin{aligned} K_j(\nu+1) &= (I+\bar{B}_jH_j(\nu))^TF_j(\nu)(I+\bar{B}_jH_j(\nu)) + H_j(\nu)^TH_j(\nu) + Q_j \\ &=: \Phi_j(K_j(\nu)+\sum_{i\neq j}\pi_{ji}K_i(\nu), Q_j)) \end{aligned}$$

with $H_j(\nu) = -(I+\bar{B}_j^TF_j(\nu)\bar{B}_j)-1B_j^TF_j(\nu)A_j$ and $F_j(\nu) = K_j(\nu)+\sum_{i\neq j}\pi_{ji}K_i(\nu),\ 1 \leq j \leq f$. The iterative procedure is stopped if $\max\|E_j(\nu)\| \leq \epsilon,\ 1\leq j\leq f$, where $\|\cdot\|$ is the spectral norm and

$$E_j(\nu) = K_j(\nu+1) - \Phi_j(K_j(\nu)+\sum_{i\neq j}\pi_{ji}K_i(\nu), Q_j)).$$

4. Example

We consider a manufacturing system producing 6 product types using 9 machines. Each product type could be processed following two different paths. In our case the predetermined paths are:

p_1	M_1, M_2, M_3	or M_4, M_8, M_9
p_2	M_4, M_5, M_6	or M_7, M_3, M_5
p_3	M_7, M_8, M_9	or M_1, M_6, M_2
p_4	M_1, M_2, M_3	or M_7, M_4, M_6
p_5	M_4, M_5, M_6	or M_1, M_2, M_3
p_6	M_7, M_8, M_9	or M_2, M_8, M_9

Note that the paths are strongly interconnected. For such a configuration the number of modes for paths $f = 55$, i.e., 55 coupled Riccati-like equations of order 6 have to be solved. With MTBF = 10 and MTTR = 1 for the different machines, the discrete path state transition probability matrix is computed and the algorithm described in section 3 is applied. It required less than 10 minutes on a workstation using MATLAB to converge to the solution of the Riccati equations and to compute the optimal controls. Figures 1 and 2 give the available stock and the production rates for p_1 and p_3. It is clear that the system tries to compensate a drop in the production of one part type due to machine breackdowns by increasing the production rates of the functional path. The computed average production rates must be transmitted to the lower level to be used in the scheduling algorithm where other operational constraints are considered to achieve real-time routing and dispatching of parts. The final aim is to design a consistent hierarchical decision strategy for manufacturing systems control.

Figure 1

Figure 2

References

[1] Abou-Kandil, H., G. Freiling and G. Jank (1995). On the solution of disdrete-time Markovian jump linear quadratic control problems. *To appear, Automatica*, **31**, 765-768.

[2] Akella, R., B.H. Krogh and M. R. Singh (1990). Efficient computation of co-ordinating controls in hierarchical structures for failure-prone multi-cell flexible assembly systems. *IEEE Trans. Robotics and Automation*, **6**, 659-672.

[3] Akella, R. and P. Kumar (1986). Optimal control of production rate in a failure-prone manufacturing system. *IEEE Trans. Automatic Control*, **31**, 116-126.

[4] Blair, W.P. Jr. and D.D. Sworder (1975). Feedback control of a class of linear discrete systems with jump parameters and quadratic cost criteria. *Int. J. Control*, **21**, 833-841.

[5] Chizeck, H.J., A.S. Willsky and D. Castanon (1986). Discrete-time markovian-jump linear quadratic optimal control. *Int. J. Control*, **43**, No. 1, 213-231.

[6] Gershwin, S.B. (1989). Hierarchical flow control: a framework for scheduling and planning discrete events in manufacturing systems. *Proceedings of the IEEE*, **77**, 195-209.

[7] Ji, Y. and H.J. Chizeck (1988). Controllability, observability and discrete-time markovian jump linear quadratic control. *Int. J. Control*, **48**, No. 2, 481-498.

[8] Kimemia, J.G. and S.B. Gershwin (1983). An algorithm for the computer control of production in flexible manufacturing systems. *IIE Trans.*, **15**, 353-362.

[9] Maimon, O.Z. and S.B. Gershwin (1988). Dynamic scheduling and routing for flexible manufacturing systems that have unreliable machines. *Operations Research*, **36**, 279-292.

[10] Sharifnia, A. (1988) Production control of a manufacturing system with multiple machine states. *IEEE Trans. Automatic Control*, **33**, 620-625.

[11] Sharifnia, A. (1994) Stability and performance of distributed production control methods based on continuous-flow models. *IEEE Trans. Automatic Control*, **39**, 725-737.