Vanishing theorems

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Let Y be a regular projective variety of dimension n over the field of complex numbers $\mathbb{C}$ and ω_Y the dualizing sheaf. The vanishing theorem of K. Kodaira says that for any ample invertible sheaf $\mathscr{L}$ the higher cohomology groups

$$H^p(Y, \mathscr{L} \otimes \omega_Y) = 0 \quad \text{for} \quad p > 0.$$

In this paper we want to weaken the condition "$\mathscr{L}$ ample".

Notation 0. 1. If b is a real number, let $[b]$ denote the integral part of b, i.e.: $[b] \in \mathbb{Z}$ and $[b] \leqq b < [b]+1$. Suppose now that E is an effective divisor and $E = \sum \nu_j E_j$ its decomposition into prime divisors. Let $\mathscr{M}$ and $\mathscr{L}$ be invertible sheaves such that for some $N > 0$ we have $\mathscr{L}^N = \mathscr{M} \otimes O_Y(\sum \nu_j E_j)$. Then we define

$$\mathscr{L}^{(i)} = \mathscr{L}^i \otimes O_Y\left(-\sum \left[\frac{\nu_j \cdot i}{N}\right] E_j\right).$$

Theorem I. *Using the notations* (0. 1) *we assume that E is a divisor with regular components intersecting transversally. Let $c_1(\mathscr{M})^n > 0$ and $c_1(\mathscr{M}) \cdot C \geqq 0$ for every curve C in Y. Then*

$$H^p(Y, \mathscr{L}^{(i)} \otimes \omega_Y) = 0 \quad \textit{for} \quad p > 0 \quad \textit{and} \quad i > 0.$$

Remark 0. 2. If one replaces the condition "$c_1(\mathscr{M})^n > 0$" by "$c_1(\mathscr{M})^m \cdot H^{n-m} > 0$ *for some ample divisor H*", one still gets $H^p(Y, \mathscr{L}^{(i)} \otimes \omega_Y) = 0$ for $p > n-m$. In fact one only needs to consider the long exact sequence obtained from

$$0 \to \mathscr{L}^{(i)} \otimes \omega_Y \to \mathscr{L}^{(i)} \otimes \omega_Y \otimes O(H) \to \mathscr{L}^{(i)} \otimes \omega_H \to 0$$

and use induction on n.

As a special case of Theorem I we get the following generalization of the vanishing theorem of C. P. Ramanujam [9].

Corollary 0. 3. *For any invertible sheaf $\mathscr{L}$ with $c_1(\mathscr{L})^n > 0$ and $c_1(\mathscr{L}) \cdot C \geqq 0$ for every curve C in Y, we have*

$$H^p(Y, \mathscr{L} \otimes \omega_Y) = 0 \quad \textit{for} \quad p > 0.$$

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Of course, the type of positivity of $\mathcal{M}$ that we assume in Theorem I, is fullfilled if $\mathcal{M}$ is generated by its global sections and $\kappa(\mathcal{M})=n$. Thus the theorem following is a special case of Theorem I and (0. 2). However we prove Theorem II first (in § 2) and then in § 3 show that both theorems are in fact equivalent.

Theorem II. *Using the notation* (0. 1) *we assume that E is a divisor with regular components intersecting transversally. Assume that $\kappa(\mathcal{M}) \geqq m$ and that $\mathcal{M}^{\mu}$ is generated by its global sections for some $\mu > 0$. Then*

$$H^p(Y, \mathcal{L}^{(i)} \otimes \omega_Y) = 0 \quad \textit{for} \quad i > 0 \quad \textit{and} \quad p > n-m.$$

This theorem is a slight generalisation of the vanishing theorem of H. Grauert and O. Riemenschneider [4], and (taking $E=0$) we also obtain a different proof of this result.

Remark 0. 4. Using the arguments of [9], page 49, one can weaken the condition "*$\mathcal{M}^{\mu}$ is generated by global sections*" to "*the dimension of the base locus of $\mathcal{M}^{\mu}$ is less than or equal to $n-m$*".

The proof of both theorems uses, besides some elementary algebraic arguments, two analytic results for regular projective varieties:

A) The symmetry of the Hodge numbers.

B) The closedness of global logarithmic differential forms (P. Deligne [2]).

The idea of using B) appears in the paper of F. Bogomolov [1]. In the case of a surface ($n=2$) Y. Miyaoka obtained similar vanishing theorems for the "integral part of divisors" [8].

Important remark. During the CIME-Conference at Varenna the author learned that Yujiro Kawamata has obtained independently the same results. In fact he found an elegant and short method to reduce (0. 3) to the vanishing theorem of Kodaira for ample sheaves. However, using (0. 3) one can also obtain the vanishing theorems for the "integral parts" (see [7]).

§ 1. Cyclic coverings

Most of the calculations we describe in this section were done together with Hélène Esnault, who wanted to apply them to Milnor fibres of cones over singular plane curves. For the results which appear in § 1 of her article [3] we shall only give a sketch of the proofs.

(1. 1) Let D be an effective divisor with regular components intersecting transversally, and $D = B + \sum \nu_j E_j$ its decomposition into prime divisors. Writing $\mathcal{M} = O_Y(B)$, suppose that for some invertible sheaf $\mathcal{L}$ and some integer $N > 0$ we have

$$\mathcal{L}^N = \mathcal{M} \otimes O_Y\left(\sum \nu_j E_j\right) = O_Y(D).$$

If s is the section, having D as zero set, s defines a structure of an O_Y-algebra on $\mathcal{A} = \bigoplus_{i=0}^{N-1} \mathcal{L}^{-i}$. Let $\tau' : Y' = \mathbf{Spec}_Y(\mathcal{A}) \to Y$ be the natural map, $n : Y'' \to Y'$ the normalization and $\tau = \tau' \cdot n$. We choose a desingularization $d\colon V \to Y''$ such that for $f = \tau \cdot d$ we have: $D' = f^{-1}(D)$ is also a divisor with regular components and transversal intersections.

(1.2) We call $f\colon V \to Y$ the morphism, obtained by taking the N-th root out of D.

Lemma 1.3 (see for example [10]). *Y'' has only rational singularities (i.e.: $R^i d_* O_V = 0$ for $i > 0$) and τ is flat.*

Lemma 1.4. *Using the notation* (0.1) *one has*

$$f_* O_V = \tau_* O_{Y''} = \bigoplus_{i=0}^{N-1} \mathscr{L}^{(i)-1}.$$

Proof. The Galoisgroup $\mathfrak{G}$ of Y'' over Y is cyclic of order N and operates on $\mathscr{A}$ and $\tau_* O_{Y''}$. We may choose a generator σ and a primitive root of unity e such that σ operates on $\mathscr{L}^{-i}$ as multiplication by e^i. Letting $\mathscr{F}^{(i)}$ denote the subsheaf of $\tau_* O_{Y''}$ such that $\sigma(\delta) = e^i \cdot \delta$ for any local section δ, we have $\tau_* O_{Y''} = \bigoplus_{i=0}^{N-1} \mathscr{F}^{(i)}$.

The sheaves $\mathscr{F}^{(i)}$ must be invertible and so in order to show that $\mathscr{F}^{(i)} = \mathscr{L}^{(i)-1}$, it is enough to consider regular points of D_{red}. The result now follows from the following elementary lemma.

Lemma 1.5. *Let R be a regular local ring, x a regular parameter and u a unit. Let $A = R[t]/t^N - x^\nu \cdot u$ and B be the normalization of A. Then B is generated as R module by*

$$t^i \cdot x^{-\left[\frac{\nu \cdot i}{N}\right]}, \qquad 0 \leqq i < N.$$

Now let $\Omega_Y^p\langle D\rangle$ (respectively $\Omega_V^p\langle D'\rangle$) be the sheaf of meromorphic p-forms having only logarithmic singularities along D_{red} (respectively D'_{red}).

Lemma 1.6. *Using the notation of* (1.1) *we have $f^* \Omega_Y^p\langle D\rangle \subset \Omega_V^p\langle D'\rangle$, and moreover this inclusion is an isomorphism at all points v where f is finite and $v \notin f^{-1}$*$(\mathrm{Sing}(D_{\mathrm{red}}))$.

Remark. A similar statement is true for all morphisms $f\colon V \to Y$, as long as $D' = f^{-1}(D)$ is a divisor with regular components intersecting transversally.

Proof. It is enough to consider the case $p = 1$. Both sheaves being subsheaves of the meromorphic differential forms having poles along D', we may check the statement locally at $v \in V$. We choose regular parameters $x_1, \ldots, x_n$ at $f(v)$ of Y, such that D is defined by $x_1^{\nu_1} \cdots x_r^{\nu_r} = 0$. Hence $\Omega_Y^1\langle D\rangle$ is generated by $\frac{dx_1}{x_1}, \ldots, \frac{dx_r}{x_r}, dx_{r+1}, \ldots, dx_n$ over O_Y.

Since D' has only transversal intersections as singularities we may choose regular parameters $z_1, \ldots, z_n$ of V at v, such that for $i = 1, \ldots, r$ and some units u_i we have $f^* x_i = \prod_j z_j^{\eta_{ij}} \cdot u_i$. One sees immediately that $f^* \frac{dx_i}{x_i} = \sum_j \eta_{ij} \frac{dz_j}{z_j} + u_i^{-1} du_i$ is an element of $\Omega_V^1\langle D'\rangle$. If $f(v)$ belongs to the regular locus of D we have $r = 1$, and if moreover f is finite, then $f^* \frac{dx_1}{x_1}, f^* dx_2, \ldots, f^* dx_n$ generates $\Omega_V^1\langle D'\rangle$ in v.

Lemma 1. 7. *With $f\colon V \to Y$ as in* (1. 1), *we have*

$$f_*\Omega_V^p\langle D'\rangle = \Omega_Y^p\langle D\rangle \otimes f_* O_V$$

and

$$f_*\Omega_V^p \subset \Omega_Y^p \oplus \bigoplus_{i=1}^{N-1} \Omega_Y^p\langle D\rangle \otimes \mathscr{L}^{(i)-1}.$$

Proof. From (1. 6) we know that $f_* f^* \Omega_Y^p\langle D\rangle = \Omega_Y^p\langle D\rangle \otimes f_* O_V$ is a subsheaf of $f_*\Omega_V^p\langle D'\rangle$, isomorphic outside a subvariety of codimension 2. Lemma 1. 3 shows that the former sheaf is locally free and hence it must be equal to the torsion free sheaf $f_*\Omega_V^p\langle D'\rangle$. We define

$$C_D = \Omega_B^{p-1}\langle B \cap \bigcup_j E_j\rangle \oplus \bigoplus_j \Omega_{E_j}^{p-1}\langle E_j \cap (B \cup \bigcup_{k \neq j} E_k)\rangle$$

and similarly for $C_{D'}$.

We have the exact (horizontal) sequences

$$\begin{tikzcd}
0 \arrow[r] & \Omega_Y^p \arrow[r] \arrow[d] & \Omega_Y^p\langle D\rangle \arrow[r] \arrow[d, shift right] & C_D \arrow[d, "\eta"] \\
0 \arrow[r] & f_*\Omega_V^p \arrow[r] & f_*\Omega_V^p\langle D'\rangle \arrow[r] \arrow[u, shift right] & f_* C_{D'}.
\end{tikzcd}$$

Applying (1. 6) to the components of D we find that η is injective, and hence the second statement of (1. 7) follows.

§ 2. The proof of Theorem II

The following theorem is the main ingrediant for the proof of II. For the convenience of the reader we sketch the proof.

Theorem III (F. Bogomolov [1]). *Let Y be a regular projective variety, $\mathscr{F}$ an invertible sheaf with $\kappa(\mathscr{F}) \geqq m$ and D an effective divisor with regular components intersecting transversally. Then*

$$H^0(Y, \Omega_Y^p\langle D\rangle \otimes \mathscr{F}^{-1}) = 0 \quad \textit{for} \quad p < m.$$

Proof. Any section of the above sheaf defines an inclusion of $\mathscr{F}$ as a subsheaf of $\Omega_Y^p\langle D\rangle$. Some power $\mathscr{F}^\nu$ has $m+1$ sections $s_0, \ldots, s_m$ such that $\frac{s_1}{s_0}, \ldots, \frac{s_m}{s_0}$ are algebraic independent rational functions. Let $f\colon V \to Y$ be the morphism obtained by taking the ν-th root out of one of the s_i (as in (1. 2)). Lemma 1. 6 gives $f^*\mathscr{F}$ as subsheaf of $\Omega_V^p\langle D'\rangle$. Repeating this construction we may assume $\nu = 1$. We denote the p-forms induced by s_i under the inclusion of $\mathscr{F}$ in $\Omega_Y^p\langle D\rangle$ also by s_i. From [2] we know that $ds_i = 0$ for $i = 0, \ldots, m$. Hence $0 = ds_i = s_0 \wedge d\left(\frac{s_i}{s_0}\right)$. This means that $d\left(\frac{s_1}{s_0}\right), \ldots, d\left(\frac{s_m}{s_0}\right)$ span a rank m subsheaf of Ω_Y^1 over some open set, which sheaf annihilates $\mathscr{F}$ under the exterior product. This is only possible if $p \geqq m$.

Theorem IV. *Let N, $\mathcal{M}$, $\mathcal{L}$, E and $\mathcal{L}^{(i)}$ be as in* (0.1). *Assume that $\mathcal{M}=O_Y(B)$ for some prime divisor B and that $B+E=D$ is a divisor with regular components and transversal intersections. If $\kappa(\mathcal{L}^{(i)})\geqq m$ for $i=1,\dots,N-1$, then*

$$H^p(Y,\mathcal{L}^{(i)^{-1}})=0 \quad \textit{for} \quad p<m.$$

Remark. IV is slightly stronger than II, as sometimes $\kappa(\mathcal{L}^{(i)})\geqq m$ without $\kappa(\mathcal{M})\geqq m$.

Proof. Choose $f\colon V\to Y$ as in (1.1). Then the symmetry of the Hodge numbers on V together with (1.4) gives

$$h^p(O_Y)+\sum_{i=1}^{N-1} h^p(\mathcal{L}^{(i)^{-1}})=h^p(O_V)=h^0(\Omega_V^p),$$

where $h^p(\ )$ denotes the dimension of $H^p(\ )$. Using (1.7) one obtains the inequality

$$h^0(\Omega_V^p)\leqq h^0(\Omega_Y^p)+\sum_{i=1}^{N-1} h^0(\Omega_Y^p\langle D\rangle\otimes\mathcal{L}^{(i)^{-1}}).$$

Using Theorem III and the symmetry of the Hodge numbers of Y, we see that the second number is equal to $h^p(O_Y)$ if $p<m$.

(2.2) *Proof of Theorem II.* We only have to check that the assumptions of Theorem II imply those of Theorem IV. $\mathcal{M}^\mu$ is generated by global sections. Replacing $\mathcal{L}^N$ by $\mathcal{L}^{N\cdot\mu}$ we may assume $\mu=1$. The Theorem of Bertini ([5] III, 10.9) applied to the linear systems corresponding to:

$$\begin{aligned}
&H^0(Y,\mathcal{M})\\
&\operatorname{Im}\big(H^0(Y,\mathcal{M})\to H^0(E_j,\mathcal{M}\otimes O_{E_j})\big)\\
&\operatorname{Im}\big(H^0(Y,\mathcal{M})\to H^0(E_j\cap E_k,\mathcal{M}\otimes O_{E_j\cap E_k})\big)
\end{aligned}$$

etc. guarantees the existence of some divisor B, such that $D=B+E$ has regular components intersecting transversally. Since $\mathcal{M}^i$ is contained in $\mathcal{L}^{(i)N}$ we have

$$m\leqq\kappa(\mathcal{M})\leqq\kappa(\mathcal{L}^{(i)}).$$

We end this paragraph with the following application of Theorem II which we are going to use in § 3.

Proposition 2.3. *Let Z be a (not necessarily regular) projective normal variety, Y a projective regular variety and $\varrho\colon Y\to Z$ a birational morphism. Let F be a Cartier divisor on Z and $E=\varrho^*F=\sum \nu_jE_j$ an effective divisor on Y having regular components intersecting transversally. If $N>0$ and $i>0$, then*

$$R^q\varrho_*\left(\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)=0 \quad \textit{for} \quad q>0.$$

Moreover, if Z is regular and $F=\sum \nu'_kF_k$ is also a divisor with regular components intersecting transversally, we have

$$\varrho_*\left(\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)=\omega_Z\otimes O_Z\left(-\sum\left[\frac{\nu'_k\cdot i}{N}\right]F_k\right).$$

Remark 2. 4. In the special case that $E=F=0$ this gives the vanishing theorem of H. Grauert and O. Riemenschneider for the higher direct images of the dualising sheaf in the projective case. For the application in § 3 it would be enough to show that, if Z is regular and F a divisor with regular components intersecting transversally, we have

$$R^q\varrho_*\left(O_Y\left(\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)=\begin{cases} O_Z\left(\sum\left[\frac{\nu'_k\cdot i}{N}\right]F_k\right) & \text{if } q=0,\\ 0 & \text{if } q\neq 0.\end{cases}$$

This also follows from the argument given at the end of the proof.

Proof of (2. 3). We may choose invertible sheaves $\mathscr{M}$ and $\mathscr{L}$, such that $\mathscr{L}^N=\mathscr{M}\otimes O_Z(F)$. Replacing $\mathscr{L}$ by $\mathscr{L}\otimes\mathscr{H}^\nu$ for high powers of an ample sheaf $\mathscr{H}$, we may assume that $\mathscr{M}$ and $R^q\varrho_*\left(\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)\otimes\mathscr{L}^i$ are generated by its global sections for $q>0$ and $i=1,\dots,N-1$. Moreover we may assume that for $p>0$ we have $H^p\left(Z,R^q\varrho_*\left(\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)\otimes\mathscr{L}^i\right)=0$. The corresponding Leray spectral sequence is degenerate, and using Theorem II we find that

$$H^0\left(Z,R^q\varrho_*\left(\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)\right)\otimes\mathscr{L}^i\right)=0$$

for $q>0$. Hence we have the first statement of (2. 3). For the second we just have to show, that under our assumptions $\varrho^*\left(\omega_Z\otimes O_Z\left(-\sum\left[\frac{\nu'_k\cdot i}{N}\right]F_k\right)\right)$ is a subsheaf of $\omega_Y\otimes O_Y\left(-\sum\left[\frac{\nu_j\cdot i}{N}\right]E_j\right)$.

Let E_1 be an exceptional component of Y. If $\varrho(E_1)$ is contained in the intersection of r components (say $F_1,\dots,F_r$) of F, then $d=\dim(\varrho(E_1))\leqq n-r$. We have

$$-\sum_{l=1}^{r}\left[\frac{\nu'_l\cdot i}{N}\right]\leqq-\left[\frac{\nu_1\cdot i}{N}\right]+r-1\leqq-\left[\frac{\nu_1\cdot i}{N}\right]+n-1-d.$$

On the other hand the general fibre of $\varrho\colon E_1\to\varrho(E_1)$ is $\mathbb{P}^{n-1-d}$ and

$$\varrho^*\omega_Z\otimes O_Y((n-1-d)E_1)$$

is contained in ω_Y (see for example [5], II, Ex. 8. 5).

§ 3. The proof of Theorem I

Lemma 3. 1. *Let $\mathscr{M}$ and $\mathscr{B}$ be invertible sheaves over Y such that $c_1(\mathscr{M})\cdot C\geqq 0$ for every curve C in V. Then there exist real numbers $a_i>0$ such that*

$$h^i(Y,\mathscr{B}\otimes\mathscr{M}^\nu)\leqq a_i\cdot\nu^{n-i}\quad\textit{for}\quad\nu>0\quad\textit{and}\quad i=0,\dots,n.$$

Proof. The statement being true if $\dim(Y)=0$, we proceed by induction on $n=\dim(Y)$. Let Y' be a regular divisor on Y, such that for $\mathscr{H}=O_Y(Y')$ the sheaf $\mathscr{H}\otimes\mathscr{B}\otimes\omega_Y^{-1}$ is ample. Using the criterion for ampleness of C.S. Seshadri (see for example [6]), we have that $\mathcal{H}\otimes\mathcal{B}\otimes\omega_Y^{-1}\otimes\mathcal{M}^\nu$ is also ample for all $\nu\geqq 0$. Theorem II (or the usual Kodaira Vanishing Theorem) shows that $H^i(Y,\mathcal{H}\otimes\mathcal{B}\otimes\mathcal{M}^\nu)=0$ for $i=1,\dots,n$ and $\nu\geqq 0$. Let $\mathcal{B}'=\mathcal{B}\otimes\mathcal{H}\otimes O_{Y'}$ and consider the long exact cohomology sequence of

$$0\to\mathcal{B}\otimes\mathcal{M}^\nu\to\mathcal{B}\otimes\mathcal{H}\otimes\mathcal{M}^\nu\to\mathcal{B}'\otimes\mathcal{M}^\nu\to 0.$$

One obtains $h^i(Y,\mathcal{B}\otimes\mathcal{M}^\nu)\leqq h^{i-1}(Y',\mathcal{B}'\otimes\mathcal{M}^\nu)$ for $i=1,\dots,n$ and hence (3. 1) follows for $i\geqq 1$. However $\chi(Y,\mathcal{B}\otimes\mathcal{M}^\nu)$ behaves like a polynomial of degree at most n, and we obtain (3. 1) for all i.

Corollary 3. 2. *If $\mathcal{M}$ is as in Theorem I, then $\kappa(\mathcal{M})=n$.*

Proof. $\chi(Y,\mathcal{B}\otimes\mathcal{M}^\nu)$ is a polynominal whose highest term is $a\cdot c_1(\mathcal{M})^n\cdot\nu^n$ for some $a>0$ (see for example [5] appendix A).

(3. 3) *Proof of Theorem I.* From now on we use the notations and assumptions of Theorem I. Let H be any ample divisor. The exact sequence

$$0\to H^0(Y,\mathcal{M}^\nu\otimes O_Y(-H))\to H^0(Y,\mathcal{M}^\nu)\to H^0(H,\mathcal{M}^\nu\otimes O_H)$$

shows that for $N'\gg 0$ we have an inclusion of $\mathcal{H}=O_Y(H)$ into $\mathcal{M}^{N'}$. In fact $h^0(Y,\mathcal{M}^\nu)$ is increasing like ν^n and $h^0(H,\mathcal{M}^\nu\otimes O_H)$ at most like ν^{n-1}.

Write $\mathcal{M}^{N'}=\mathcal{H}\otimes O_Y(\sum\varrho_k F_k)$. Choose a blowing up $\tau\colon Y'\to Y$, such that $\tau^*(\sum\nu_j E_j)+\tau^*(\sum\varrho_k F_k)$ is a divisor with regular components intersecting transversally. Allowing the coefficients to be zero, we may write

$$\tau^*(\sum\nu_j E_j)=\sum\nu'_j E'_j\quad\text{and}\quad\tau^*(\sum\varrho_k F_k)=\sum\varrho'_k E'_k.$$

Let $\mathcal{M}'=\tau^*\mathcal{M}$, $\mathcal{H}'=\tau^*\mathcal{H}$ and $\mathcal{L}'=\tau^*\mathcal{L}$. For any $N''>0$ we know that $\mathcal{H}\otimes\mathcal{M}^{N''}$ is ample, and hence a power of $\mathcal{H}'\otimes\mathcal{M}'^{N''}$ is generated by its global sections. Applying Theorem II to

$$\mathcal{L}'^{N(N'+N'')}=\mathcal{M}'^{N''}\otimes\mathcal{H}'\otimes O_{Y'}(\sum(\varrho'_j+(N'+N'')\nu'_j)E'_j),$$

we have that for given $i>0$, $p>0$ and all $N''>0$

$$H^p\left(Y',\mathcal{L}'^i\otimes\omega_{Y'}\otimes O_{Y'}\left(-\sum\left[\frac{\varrho'_j+(N'+N'')\nu'_j}{N(N'+N'')}\cdot i\right]E'_j\right)\right)=0.$$

However for $N''\gg 0$ we have for all j with $\varrho'_j\neq 0$ that $\dfrac{\varrho'_j\cdot i}{N(N'+N'')}<\left[\dfrac{\nu'_j\cdot i}{N}\right]-\dfrac{\nu'_j\cdot i}{N}+1$. Hence $\left[\dfrac{\nu'_j\cdot i}{N}\right]=\left[\dfrac{\varrho'_j+(N'+N'')\nu'_j}{N(N'+N'')}\cdot i\right]$ for all j, and

$$H^p\left(Y',\mathcal{L}'^i\otimes\omega_{Y'}\otimes O_{Y'}\left(-\sum\left[\frac{\nu'_j\cdot i}{N}\right]E'_j\right)\right)=0.$$

Proposition 2. 3 shows that the Leray spectral sequence for τ degenerates in our case and hence that

$$H^p(Y,\mathcal{L}^{(i)}\otimes\omega_Y)=0\quad\text{for}\quad p>0.$$

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