

ON p -EMBEDDING PROBLEMS IN CHARACTERISTIC p

LIOR BARY-SOROKER AND NGUYỄN DUY TÂN

ABSTRACT. Let K be a valued field of characteristic $p > 0$ with non- p -divisible value group. We show that every finite embedding problem for K whose kernel is a p -group is properly solvable.

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1. INTRODUCTION

In the proof that every finite solvable group occurs as a Galois group over the rationals, Shafarevich studies the solvability of embedding problems with nilpotent kernel and solvable cokernel. To study the absolute Galois group $\text{Gal}(K)$ of a field K via embedding problems became a trend in recent papers, e.g. [BHH, Ha3, HS, Pa1, Po]. See also the upcoming book [Ja] and references therein.

In this work we consider a field K of characteristic $p > 0$ and the finite embedding problems for K whose kernels are p -groups which we call **finite p -embedding problems**. An obvious necessary condition to have a *proper* solution is to have a *weak* solution (see Section 3 for definitions). This latter condition is automatically satisfied in our case, since $\text{cd}_p(\text{Gal}(K)) \leq 1$, for a field of characteristic $p > 0$. We obtain a mild sufficient condition on K to have a proper solution of any finite p -embedding problem.

Theorem 1.1. *Let K be a field of characteristic p admitting a non- p -divisible valuation. Then every finite p -embedding problem for K is solvable.*

Some examples of fields satisfying this condition are the following. If R is a Noetherian domain or Krull domain of characteristic $p > 0$, then its fraction field K satisfies the hypothesis of Theorem 1.1. If R is an arbitrary domain of characteristic $p > 0$, then the fraction fields of the ring $R[x_1, \dots, x_n]$ of polynomials and of the ring of formal Taylor series $R[[x_1, \dots, x_n]]$ satisfy the hypothesis of Theorem 1.1, for any $n \geq 1$.

The proof of Theorem 1.1 is based on the following cohomological criterion of Harbater. A profinite group Π is called **strongly p -dominating** if $H^1(\Pi, P)$ is infinite for every nontrivial finite elementary p -group P on which Π acts¹.

Theorem 1.2 ([Ha2, Theorem 1b]). *Let Π be a profinite group. Assume that Π is strongly p -dominating and that $\text{cd}_p(\Pi) \leq 1$. Then every finite p -embedding problem for Π is properly solvable.*

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¹All actions, homomorphism, etc., in this work are assumed to be continuous.

Harbater's motivation for Theorem 1.2 is to show that every finite p -embedding problem for the étale fundamental group $\Pi := \pi_1(X)$ of an affine variety X over an arbitrary field K of characteristic $p > 0$ has a proper solution [Ha1].

We show that the converse of Theorem 1.2 also holds true, see Theorem 4.2. Moreover, to get the assertion of Theorem 1.2, one may suspect that the infinitude of $H^1(\Pi, \mathbb{Z}/p\mathbb{Z})$ suffices, where Π acts trivially on $\mathbb{Z}/p\mathbb{Z}$. This is true if both the kernel and cokernel are p -groups, but in general it fails, see [Ha2].

By Theorem 1.2, to prove Theorem 1.1 it suffices to show that $\text{Gal}(K)$ is strongly p -dominating. This is carried out by using that for every nontrivial finite elementary p -group P on which Π acts we have $H^1(K, P) = K/f(K)$, for some additive polynomial f (Lemma 3.1). Then using the non- p -divisible valuation of K we construct infinitely many $a \in K$ that are distinct modulo f .

We conclude the introduction with an example. Let K_0 be a field of characteristic $p > 0$ and $K = K_0((x))$ the field of formal Laurent series. Then by Theorem 1.1 every finite p -embedding problem is properly solvable. When K_0 is algebraically closed, Harbater proves this in [Ha2, Example 5] using a similar method. However, when K_0 is arbitrary Harbater invokes a theorem of Katz-Gabber in order to complete his proof (see [Ha2, Proposition 6]).

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2. VALUATION-THEORETIC LEMMAS

Let A be a ring. By a **valuation** of A we shall mean a map $v : x \mapsto v(x)$ of A onto a totally ordered commutative group Γ (written additively), together with an extra element ∞ , such that:

- (1) $\alpha + \infty = \infty$ and $\alpha < \infty$ for all $\alpha \in \Gamma$.
- (2) $v(x) = \infty$ if and only if $x = 0$.
- (3) $v(xy) = v(x) + v(y)$ for all $x, y \in A$.
- (4) $v(x + y) \geq \min\{v(x), v(y)\}$.

If A is a ring with a valuation v on A , we shall also say simply that A is a **valued ring**. The group Γ is called the **value group**.

Lemma 2.1. *Let Γ be a nontrivial totally ordered commutative group*

- (1) *For any element γ in Γ , there exists $\beta \in \Gamma$ such that $\beta < \gamma$.*
- (2) *Let $\gamma_1, \dots, \gamma_r$ be elements in Γ and let n_1, \dots, n_r be positive numbers. Then there exists an element γ_0 in Γ such that for all elements $\gamma < \gamma_0$, $\gamma \in \Gamma$, we have $n_i \gamma < \gamma_i$ for all i .*

Proof. 1) If $\gamma \geq 0$, then let $\beta < 0 \leq \gamma$ (such an element exists since Γ is nontrivial).

If $\gamma < 0$, one can take $\beta = 2\gamma < \gamma$.

2) We set

$$\gamma_0 := \min\{\gamma_1, \dots, \gamma_r, 0\}.$$

Now let γ be an arbitrary element such that $\gamma < \gamma_0$. Since $\gamma < \gamma_i$, $\gamma < 0$, it follows that $n_i\gamma < \gamma_i$, for all i . \square

Lemma 2.2. *Let A be a valued ring of characteristic $p > 0$ with nontrivial value group Γ . Let $f(T) = b_0T + \dots + b_mT^{p^m}$ be a p -polynomial in one variable with coefficients in A . Then there exists an element $\gamma_0 \in \Gamma$ such that if $a = f(a_1)$, $a_1 \in A$ and $v(a) < \gamma_0$ then $v(a) = v(b_m) + p^m v(a_1)$.*

Proof. By Lemma 2.1, there exists an element $\alpha \in \Gamma$ such that for all $\gamma < \alpha$ in Γ , we have

$$(p^m - p^i)\gamma < v(b_i) - v(b_m), \quad \forall 0 \leq i < m.$$

We set

$$\beta := \min\{v(b_i) + \alpha p^i \mid 0 \leq i \leq m\}.$$

Let γ_0 be any element with $\gamma_0 < \beta$. Now assume that $a = f(a_1)$ such that $v(a) < \gamma_0$ ($a_1 \in A$). Let s be an index such that

$$v(b_s a_1^{p^s}) = \min\{v(b_i a_1^{p^i}) \mid 0 \leq i \leq m\}.$$

Then

$$v(b_s) + p^s \alpha > \gamma_0 > v(f(a_1)) \geq v(b_s) + p^s v(a_1).$$

Thus $0 < p^s(\alpha - v(a_1))$ and hence $v(a_1) < \alpha$. By the choice of α , we have

$$v(b_i a_1^{p^i}) = v(b_i) + p^i v(a_1) > v(b_m) + p^m v(a_1) = v(b_m a_1^{p^m}), \quad \forall i < m.$$

Therefore $v(a) = v(b_m) + p^m v(a_1)$ as required. \square

Lemma 2.3. *Let Γ be a non- p -divisible totally ordered commutative group. Let α_0, γ_0 be elements in Γ . Then there exist infinitely many elements $\gamma_i \in \Gamma$ such that*

$$\gamma_0 > \gamma_1 > \dots > \gamma_i > \dots$$

and $\gamma_i \notin \alpha_0 + p\Gamma$, for all $i > 0$.

Proof. We first consider the case $\alpha_0 = 0$. Since Γ is not p -divisible, there is an element $a_0 \in \Gamma$ such that $a_0 \notin p\Gamma$. By Lemma 2.1 part (2), there exists an element δ_0 such that for all $\delta < \delta_0$, we have $p\delta < \gamma_0 = a_0$. By Lemma 2.1 part (1), there exists an infinite sequence

$$\delta_0 > \delta_1 > \dots > \delta_i > \dots$$

Set $\gamma_i := a_0 + p\delta_i$, for all $i > 0$. Then $\gamma_i \notin p\Gamma$, for all $i > 0$ and $\gamma_0 > \gamma_1 > \dots > \gamma_i > \dots$.

For the general case, applying the previous argument for $\gamma'_0 := \gamma_0 - \alpha_0$, we get an infinite sequence $\gamma'_0 > \gamma'_1 > \dots > \gamma'_i > \dots$ with $\gamma'_i \in \Gamma$ but $\gamma'_i \notin p\Gamma$. Setting $\gamma_i := \gamma'_i + \alpha_0$, we get a desired sequence of elements. \square

Proposition 2.4. *Let A be a valued ring of characteristic $p > 0$ with non- p -divisible value group Γ . Let $f(T) = b_0T + b_1T^p + \cdots + b_mT^{p^m}$ be a p -polynomial in one variable with coefficients in A with $m \geq 1$ and $b_m \neq 0$. Then $A/f(A)$ is infinite.*

Proof. Let γ_0 be as in Lemma 2.2. For any a in A such that $v(a) < \gamma_0$ and $v(a) \notin v(b_m) + p\Gamma$, Lemma 2.2 implies that a is not in $f(A)$. By Lemma 2.3 and noting that the valuation map v is onto, we may choose a sequence $\{a_i\}$ of elements from A such that $v(a_i) \notin v(b_m) + p\Gamma$ for all i , and $\gamma_0 > v(a_1) > v(a_2) > \cdots > v(a_i) > \cdots$. For every $i < j$, one has

$$v(a_i - a_j) = v(a_i) \notin v(b_m) + p\Gamma,$$

so $a_i - a_j \notin f(A)$ and hence a_i, a_j have different images in $A/f(A)$. Therefore, $A/f(A)$ is infinite. \square

3. PROOF OF THEOREM 1.1 AND A COROLLARY

An **embedding problem** \mathcal{E} for a profinite group Π is a diagram

$$\begin{array}{ccc} & & \Pi \\ & & \downarrow \alpha \\ \Gamma & \xrightarrow{f} & G \end{array}$$

which consists of a pair of profinite groups Γ and G and epimorphisms $\alpha : \Pi \rightarrow G$, $f : \Gamma \rightarrow G$.

A **weak solution** of \mathcal{E} is a homomorphism $\beta : \Pi \rightarrow \Gamma$ such that $f\beta = \alpha$. If such a β is surjective, then it is called a **proper solution**. We will call \mathcal{E} **weakly** (resp. **properly**) **solvable** if it has a weak (resp. proper) solution.

We call \mathcal{E} a **finite** embedding problem if the group Γ is finite.

The **kernel** of \mathcal{E} is defined to be $N := \ker(f)$. We call \mathcal{E} a **p-embedding problem** if N is a p -group.

We say \mathcal{E} is a **split** embedding problem if $f : \Gamma \rightarrow G$ has a group theoretical section, i.e., $f' : G \rightarrow \Gamma$ such that ff' is the identity map on G .

In this note, by a **K -group**, where K is a field, we mean an algebraic affine group scheme which is smooth ([Wa]). This notion is equivalent to the notion of a linear algebraic group defined over K in the sense of [Bo].

First we need the following lemma.

Lemma 3.1. *Let K be an infinite field of characteristic $p > 0$. Let P be a nontrivial finite commutative K -group which is annihilated by p . Then G is K -isomorphic to a K -subgroup of the additive group \mathbb{G}_a , of the form $\{x \mid f(x) = 0\}$, where $f(T) = T + b_1T^p + \cdots + b_mT^{p^m}$ is a p -polynomial with coefficients in K , $m \geq 1$ and $b_m \neq 0$.*

Proof. This is well known, see e.g. [CGP, Proposition B.1.13] or [Oe, Chapitre V, Proposition 4.1 and Subsection 6.1]. \square

We are now ready to prove Theorem 1.1.

Proof of Theorem 1.1. We have $\text{cd}_p(\text{Gal}(K)) \leq 1$ (see, e.g., [Se2, Chapter II, Proposition 3]). By Theorem 1.2 it suffices to prove that $\text{Gal}(K)$ is strongly p -dominating.

Indeed, let P be a non-trivial elementary p -group on which $\text{Gal}(K)$ acts. Consider P as a finite K -group. Then P is commutative and annihilated by p . Hence by Lemma 3.1, P is K -isomorphic to a subgroup of \mathbb{G}_a defined as the kernel of $f : \mathbb{G}_a \rightarrow \mathbb{G}_a$, where $f(T) = T + \cdots + b_m T^{p^m}$ is a p -polynomial in one variable with coefficients in K with $m \geq 1$ and $b_m \neq 0$. We have the following exact sequence of K -groups

$$0 \rightarrow P \rightarrow \mathbb{G}_a \xrightarrow{f} \mathbb{G}_a \rightarrow 0.$$

From this exact sequence we get the following exact sequence of Galois cohomology groups

$$H^0(K, \mathbb{G}_a) \xrightarrow{f} H^0(K, \mathbb{G}_a) \rightarrow H^1(K, P) \rightarrow H^1(K, \mathbb{G}_a).$$

By Hilbert 90 $H^1(K, \mathbb{G}_a) = 0$ (see e.g. [Se2, Chapter II, Proposition 1]), hence

$$H^1(K, P) \simeq H^0(K, \mathbb{G}_a)/\text{im}(f) = K/f(K).$$

The latter is infinite by Proposition 2.4. So we conclude that $H^1(K, P)$ is infinite, and hence $\text{Gal}(K)$ is strongly p -dominating. \square

We recall that a Hilbertian field is a field K having the irreducible specialization property: for every irreducible polynomial $f(T, X) \in k[T, X]$ that is separable in X , there exists $a \in K$ such that $f(a, X)$ is irreducible in $k[X]$ (we refer readers to [FJ, Chapters 12, 13] for more details about Hilbertian fields). In [DD], Dèbes and Deschamps give the following conjecture.

Conjecture 3.2 ([DD, 2.1.2]). *Let K be a Hilbertian field. Then every finite split embedding problem for $\text{Gal}(K)$ has a proper solution.*

An easy consequence of Theorem 1.1 is a simple proof of [MM, Theorem 8.3] which asserts that Conjecture 3.2 holds true whenever K is of characteristic $p > 0$ and if the kernel of the embedding problem is a p -group. Namely, we have

Corollary 3.3. *Let K be a Hilbertian field of characteristic $p > 0$. Then every finite p -embedding problem for $\text{Gal}(K)$ is properly solvable.*

Proof. Let $\mathcal{E} = (\alpha : \text{Gal}(K) \rightarrow A, f : B \rightarrow A)$ be a finite p -embedding problem for $\text{Gal}(K)$. Consider the finite p -embedding problem $\mathcal{E}_t := (\alpha \circ \text{pr}_t : \text{Gal}(K(t)) \rightarrow A, f : B \rightarrow A)$ for $\text{Gal}(K(t))$ obtained by composition with the restriction map $\text{Gal}(K(t)) \rightarrow \text{Gal}(K)$. Since $K(t)$ has discrete valuations, Theorem 1.1 gives a proper solution of \mathcal{E}_t , say $\theta_t : \text{Gal}(K(t)) \rightarrow B$. By the irreducible specialization property (applied to a polynomial a root of which generates the solution field of θ_t over $K(t)$) θ_t specializes to a proper solution θ of \mathcal{E} (see [FJ, Lemma 16.4.2]). \square

Remark 3.4. (1) Let G be a finite p -group, K a Hilbertian field of characteristic $p > 0$. By considering the finite (split) p -embedding problem $(\text{Gal}(K) \rightarrow \{1\}, G \rightarrow \{1\})$, Corollary 3.3 implies that G is realizable over K . In other words, this proposition shows that every finite p -group is realizable over an arbitrary Hilbertian field of

characteristic $p > 0$. This last statement is a special case of a theorem of Shafarevich, [FJ, Theorem 16.4.7].

- (2) Corollary 3.3 can also be derived from Ikeda's theorem ([FJ, Proposition 16.4.5]). Here we sketch the proof: one starts with a finite embedding problem for K corresponding to an exact sequence $1 \rightarrow P \rightarrow B \rightarrow A \rightarrow 1$, where P is a p -group and $B = \text{Gal}(L/K)$. We use the usual trick of decomposing this embedding problem to a series of embedding problems in order to assume that P is a minimal normal subgroup of B . In particular P is *abelian*. Since $\text{cd}_p(K) \leq 1$ we can replace this embedding problem by a bigger *split* embedding problem with the same kernel by taking the fiber product of B and the image of a weak solution. Now we use Ikeda's result that gives a *regular* solution over K , i.e., a solution over $K(t)$ with the extra condition that the solution field is regular over L . Then one uses Hilbertianity to reduce the solution to a solution over K .

Unfortunately, we do not know whether any finite p -embedding problem over a field of characteristic $p > 0$ has a regular solution.

- (3) For recent results concerning Conjecture 3.2, we refer readers to [BP, Pa1, Pa2, Po].

4. EMBEDDING PROBLEMS WITH p -KERNEL

In this section we show that the converse of Theorem 1.1 also holds true, see Theorem 4.2. Let

$$\mathcal{E} := \begin{array}{ccccccc} & & & & \Pi & & \\ & & & & \downarrow \alpha & & \\ 1 & \longrightarrow & P & \longrightarrow & \Gamma & \xrightarrow{f} & G \longrightarrow 1 \end{array}$$

be an embedding problem for Π with abelian kernel P . Since P is abelian, there is an induced conjugation action of G on P by choosing representatives in Γ . This in turn yields an action of Π on P via $\alpha : \Pi \rightarrow G$. Let $H^1(\Pi, P)$ be the corresponding Galois cohomology group.

Two weak solutions β and $\beta' : \Pi \rightarrow \Gamma$ of \mathcal{E} are defined to be equivalent, and denoted by $\beta \sim \beta'$, if there is an element p in P such that $\beta' = \text{inn}(p) \circ \beta$. (Here $\text{inn}(p) \in \text{Aut}(\Gamma)$ denotes left conjugation by p .) One can check that \sim is an equivalence on the set of weak solutions to \mathcal{E} . Denote by $\text{WS}(\mathcal{E})$ the set of weak solutions of \mathcal{E} modulo the equivalence relation \sim . We have a cohomological description of $\text{WS}(\mathcal{E})$.

Lemma 4.1. *With notation as above, assume that \mathcal{E} is weakly solvable. Then $\text{WS}(\mathcal{E})$ is a $H^1(\Pi, P)$ -torsor. In particular, any weak solution θ of \mathcal{E} induces a bijection*

$$\text{WS}(\mathcal{E}) \cong H^1(\Pi, P).$$

Proof. See [NSW, Proposition 9.4.4]. □

Next we prove the converse of Theorem 1.2. For future reference we formulate it as an if and only if theorem.

Theorem 4.2. *Let Π be a profinite group. Then every finite p -embedding problem for Π has a proper solution if and only if $\text{cd}_p(\Pi) \leq 1$ and Π is strongly p -dominating.*

Proof. (\Leftarrow): This is Theorem 1.2.

(\Rightarrow): It suffices to prove that Π is strongly p -dominating. Let P be a nontrivial elementary abelian p -group on which Π acts continuously. We have to show that $H^1(\Pi, P)$ is infinite.

Since the action of Π on P is continuous, it factors via a finite quotient. I.e., there is a map $\alpha: \Pi \rightarrow G$ and an action of G on P that induces the action of Π on P . Let Γ be the semidirect product of P and G . We get the following split embedding problem with elementary abelian p -kernel

$$\mathcal{E} := \begin{array}{ccccccc} & & & & \Pi & & \\ & & & & \downarrow \alpha & & \\ 1 & \longrightarrow & P & \longrightarrow & \Gamma & \xrightarrow{f} & G \longrightarrow 1 \end{array}$$

For any $n > 0$ let Γ_G^n be the n -th fold fiber product of Γ over G , i.e.,

$$\Gamma_G^n = \{(\gamma_1, \dots, \gamma_n), \gamma_i \in \Gamma, \text{ and } f(\gamma_1) = \dots = f(\gamma_n) \in G\}.$$

We have a map $f_n: \Gamma_G^n \rightarrow G$, defined by $f_n((\gamma_i)_{i=1}^n) = f(\gamma_1)$.

We have an embedding problem \mathcal{E}_n for Π corresponding to the exact sequence

$$1 \longrightarrow P^n \longrightarrow \Gamma_G^n \xrightarrow{f_n} G \longrightarrow 1.$$

By assumption, there is a proper solution β to \mathcal{E}_n . By composing β with the projections $\text{pr}_i: \Gamma_G^n \rightarrow \Gamma$, we get n proper solutions β_1, \dots, β_n .

We show that these β_i are pairwise non-equivalent (and in particular distinct). Indeed, if $\beta_i \sim \beta_j$, for some $1 \leq i < j \leq n$, then there is a element $p \in P$ such that $\beta_i(s) = p\beta_j(s)p^{-1}$, for all $s \in \Pi$. Since P is a non-trivial group, we can take two different elements q, q' from P . Set $x = (1, \dots, q, \dots, q', \dots, 1) \in \Gamma^n$, where q, q' are in i -th and j -th entry, respectively and 1 is in all other entries. Then $x \in \Gamma_G^n$. Since β is a *proper* solution, there exists s in Π such that $\beta(s) = x$. We then have

$$q = \beta_i(s) = p\beta_j(s)p^{-1} = pq'p^{-1} = q',$$

a contradiction.

Therefore, we get that $\text{WS}(\mathcal{E})$ is infinite, and by Lemma 4.1, $H^1(\Pi, P)$ is infinite, as needed. \square

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INSTITUT FÜR EXPERIMENTELLE MATHEMATIK, UNIVERSITÄT DUISBURG-ESSEN, ELLERNSTRASSE 29, D-45326 ESSEN, GERMANY

E-mail address: lior.bary-soroker@uni-due.de

UNIVERSITÄT DUISBURG-ESSEN, FB6, MATHEMATIK, 45117 ESSEN, GERMANY, AND INSTITUTE OF MATHEMATICS, 18 HOANG QUOC VIET, 10307, HANOI - VIETNAM.

E-mail address: duy-tan.nguyen@uni-due.de