Problem sheet 6

Due date: Nov. 20, 2018.

Problem 25
Let $X$ be an irreducible topological space, $E$ a set, and $\mathcal{F}$ the constant sheaf on $X$ associated with $E$. Show that $\mathcal{F}(U) = E$ for every non-empty open set $U \subseteq X$.

Problem 26
Let $X$ be a topological space, and let $\mathcal{F}$, $\mathcal{G}$ be sheaves of abelian groups on $X$. For every open $U \subseteq X$, denote by Hom$(\mathcal{F}|_U, \mathcal{G}|_U)$ the abelian group of morphisms $\mathcal{F}|_U \to \mathcal{G}|_U$ of sheaves of abelian groups on $U$. This defines a presheaf in a natural way. Show that this presheaf is a sheaf.

Problem 27
Let $X$ be a topological space, $U \subseteq X$ open, and denote by $j: U \to X$ the inclusion map. Let $\mathcal{F}$ be a sheaf of abelian groups on $U$. Denote by $j_!(\mathcal{F})$ the sheaf associated with the following presheaf on $X$:

$$ V \mapsto \begin{cases} \mathcal{F}(V) & \text{if } V \subseteq U, \\ 0 & \text{otherwise,} \end{cases} \quad \text{for } V \subseteq X \text{ open} $$

Compute the stalks of $j_!(\mathcal{F})$ and the restriction $j_!(\mathcal{F})|_U$. It is easy to define $j_!$ on sheaf morphisms, so that $j_!$ is a functor. Find a functor which is right adjoint to $j_!$.

Problem 28
Let $(X, \mathcal{O}_X)$ be a locally ringed space which is not connected. Prove that there exists a non-trivial idempotent element in $\Gamma(X, \mathcal{O}_X)$. (Cf. Problem 9.)