Identification of the Interactions among the
Power System Dynamic Voltage Stability
Controllers using Relative Gain Array

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Abstract—This paper concerns the use of relative gain array (RGA) for analyzing the interactions among the power system dynamic voltage stability controls. RGA analyses for both static and dynamic voltage stability controls are carried out based on the multi-input multi-output (MIMO) transfer function matrix \( J_v(s) \). By means of the singular value and RGA, the dynamic voltage stability and the controller interactions are analyzed. Then, the dynamic voltage stability control loop can be selected. A multi-machine power system with different working conditions is simulated for the demonstration of the proposed approach. Also, limitations and advantages of the use of singular value and RGA in the dynamic voltage stability analysis and control are discussed. The proposed approach is simple and can be easily implemented into large power system dynamic voltage stability control.

Index Terms—Controller design, Dynamic voltage stability, Multi-variable feedback control, Residue, RGA, Singular value analysis, Static voltage stability

I. INTRODUCTION

In [1], a novel approach for power system dynamic voltage stability analysis is presented based on the multi-input multi-output (MIMO) transfer function. Considering the bus voltage magnitudes as outputs and possible control variables as inputs, the MIMO transfer function matrix of multi-machine power system is analyzed by means of singular value decomposition (SVD). The singular values, input and output singular vectors are calculated for the power system critical modes of oscillation frequencies. The output singular vectors provide an overview of the most critical buses that affected by the dynamic voltage stability problem. The magnitudes of the input singular vectors indicate that which input has the largest effect on the dynamic voltage stability control.

In this paper, the interaction among the dynamic voltage stability controllers and the controller design will be discussed.

In power system voltage stability control, there are many possible control loops. How to select the proper feedback control signal and control loop which provides little interactions with the other control loops is still a problem. In this paper, by means of the relative gain array (RGA), the controller interactions are analyzed for different frequencies and thus the most suitable control loops can be selected.

This paper is organized as follows: Following the introduction, the RGA analysis for the dynamic voltage stability control is described in section II. The simulation results of the RGA analyses are given in section III. Finally, brief conclusions are deduced.

II. RGA ANALYSIS FOR VOLTAGE STABILITY CONTROL

The RGA was introduced as a steady state measure of interactions for multivariable, decentralized control [2]. It was later extended to the frequency domain [3,4]. In MIMO control system, the RGA is mainly used to analyze the interactions among different controllers. It allows to match up input-output variables that have the biggest effect on each other, without having undesirable effects on the others [3]. First, the definition and properties of RGA will be introduced in this section.

A. Definition of RGA

For a multivariable system, the RGA is defined as the matrix of relative gains [3].

If a multivariable system is defined by the matrix equation

\[
y(s) = J_v(s)u(s)
\]

where

\( J_v(s) \) is the transfer function matrix for the dynamic voltage stability analysis defined in [1]. In the following analyses, the \( J_v(s) \) is assumed as a non-square transfer function matrix with \( m \) input and \( n \) output variables.

\( u(s) \) and \( y(s) \) are vectors that contain the selected input and output variables.
The relative gain $\gamma_{ji}(s)$ for an input $u_j(s)$ and an output $y_i(s)$ is then defined by the ratio of the uncontrolled gain and the controlled gain, as shown in (2) [3,4].

$$
\gamma_{ji}(s) = \frac{\text{Controlled gain between } y_i(s) \text{ and } u_j(s)}{\text{Uncontrolled gain between } y_i(s) \text{ and } u_j(s)}
$$

$$
\gamma_{ji}(s) = \frac{y_i(s)}{u_j(s)}_{u_j(s) = 0, k \neq j} = \frac{y_i(s)}{u_j(s)}_{v_i(s) = 0, k \neq j}
$$

The uncontrolled gain is the gain between the input $u_j(s)$ and the output $y_i(s)$ when all outputs are uncontrolled, as shown in Fig. 1. This gain is the element $g_{ji}(s)$ of multivariable transfer function matrix $J_{ij}(s)$ [3,4].

The controlled gain is the gain between the input $u_j$ and the output $y_i$ when perfect controls are performed on the rest of the outputs (i.e., their control deviation is zero), as given in Fig. 2 [3,4].

![Fig. 1 Uncontrolled gain](image1)

![Fig. 2. Controlled gain](image2)

Hence, for the input $u_j(s)$ and the output $y_i(s)$, RGA element $\gamma_{ji}(s)$ measures the effect that the control of the rest of the variables has on gain between $y_i(s)$ and $u_j(s)$ [3,4].

**B. Calculation of RGA**

A direct or uncontrolled gain of input $u_j(s)$ and output $y_i(s)$ $g_{ji}$ is given by the elements of the multivariable transfer function $J_{ij}(s)$ [3,4].

The controlled gain $\hat{g}_{ji}(s)$ of input $u_j(s)$ and output $y_i(s)$ is the $ji$th element of $J_{ji}^{-1}(s)$ [4]. In this paper, since the $J_{ij}(s)$ is a non-square transfer function matrix, $J_{ij}^{-1}(s)$ represents the pseudo-inverse of the $J_{ij}(s)$. Hence, the relative gain for input $u_j$ and output $y_i$, is [4]:

$$
\gamma_{ji}(s) = \frac{\hat{g}_{ji}(s)}{\hat{g}_{ij}(s)} = \left[J_{ij}(s)\right]_{ji} \times \left[J_{ji}^{-1}(s)\right]_{ij}
$$

and the matrix of all relative gains is:

$$
\Gamma(s) = J_{ij}(s) \times \left[J_{ji}^{-1}(s)\right]^T
$$

where $\times$ is the element-by-element multiplication or Schur product [3,4].

RGA is calculated based on the multivariable frequency response. In frequency domain, it involves inversion of complex matrix whose size is determined by the number of the inputs and the outputs to the system [3].

**C. Main properties of RGA**

The main algebraic and control properties of the RGA are given in [3,4]. Here, only the most important control properties associated with the dynamic voltage stability control are selected:

1. The RGA depends only on plant model and not on the controller. It is calculated based on the analysis of open loop system.
2. RGA is independent on the input and the output scaling, i.e., it is not influenced by the choice of units.
3. The negative inverse of the RGA element is equal to the ratio of relative change in the element of $J_{ij}(s)$ and its transpose inverse [3,12]:

$$
\frac{d[J_{ji}^{-1}(s)]_{ij}}{[J_{ji}^{-1}(s)]_{ij}} = -\gamma_{ji}(s) \frac{dg_{ij}(s)}{g_{ij}(s)}
$$

4. If the elements of RGA are large, the elements of the inverted matrix $J_{ij}^{-1}(s)$ must sensitive to changes in the elements of $J_{ij}(s)$. Therefore, plants having large RGA elements are very sensitive to model uncertainty.

The most important property of the RGA for voltage stability analysis is that RGA is independent on scaling, it depends only on the plant model and not on the controller and that the RGA elements measure their own sensitivity to uncertainty.
D. Control loop pairing using RGA

As shown in the definition and the properties of RGA, it provides a measure of control loops interactions and gives an indication of control loop pairings. The interpreting of the results of RGA is given as follows:

1. \( \gamma_{ij}(s) = 1 \)
   Uncontrolled gain and controlled gain are identical and interaction does not affect the pairing of the input \( u_j \) and the output \( y_i \).

2. \( \gamma_{ij}(s) = 0 \)
   Uncontrolled gain is zero, i.e. \( u_j \) has no effect on \( y_i \).

3. \( 0 < \gamma_{ij}(s) < 1 \)
   Other control loops interaction increases gain. The interaction is most severe when \( \gamma_{ij}(s) = 0.5 \).

4. \( 1 < \gamma_{ij}(s) < 10 \)
   Other control loops interaction reduces gain. Higher controller gains are required.

5. \( \gamma_{ij}(s) \gg 10 \)
   The pairing of variables with large RGA elements is undesirable. It can indicate a system sensitive to small variations in gain and possible problems applying model based control techniques.

6. \( \gamma_{ij}(s) < 0 \)
   Controlled gain is in opposite direction from the uncontrolled gain.

Therefore, considering the above interpretations, the objective is to pair the input and output variables where \( \gamma_{ij}(s) \) is nearest to 1 while avoiding the \( \gamma_{ij}(s) \) which are zero or negative. Therefore, the following control input and output pairings should be avoided:

1. Small elements: a zero element means a lack of input and output relationship.
2. Negative elements: this implies a change of sign in gain between open and closed loop.
3. Very large elements: this means that closed loop sensitivity is much less than open loop and mean that there will be substantial interaction among the control loops.

III. SIMULATION RESULTS

A typical four-machine two-area power system model [5,6], as shown in Fig. 3, is used for demonstration of the proposed method. The corresponding dynamic model consists of generators with 6th order model, governors, static exciters, power system stabilizers (PSS). Induction motor loads are modeled at Bus 4 (deep bar and leakage inductance saturation) and Bus 9 (double cage and leakage inductance saturation). For the reactive compensation, a SVC (Static Var Compensator) is applied on Bus 11.

![Fig. 3 Four-machine two-area power system model](image)

The purpose of SVC is to maintain the bus voltage by means of reducing the effect of power system disturbances. It is a fast acting device and always be used when mechanically switched capacitors are too slow to prevent voltage collapse. The SVC dynamic model is given in Fig. 4 [6].

![Fig. 4 Dynamic model of SVC](image)

where

- \( u_{ref} \) is the reference voltage magnitude of SVC
- \( u_{SVC} \) is the terminal voltage magnitude of SVC
- \( svc\_sig \) is supplementary control signal into the reference input of SVC
- \( K_r \) is the regulator gain
- \( T_r \) is the regulator time constant
- \( B_{SVC\_min} \) is the minimum susceptance of SVC
- \( B_{SVC\_max} \) is the maximum susceptance of SVC
- \( B_{SVC} \) is the controlled susceptance of SVC

A. SVC location

As mentioned in [6], the three possible locations for SVC are Bus 3, 8 and 11. Unlike the allocation method used in [3], where the main task of SVC is to damp power system oscillations, the location of SVC is selected on the mid-point of the tie-line between area 1 and area 2 in this research. The main purpose of SVC is to provide reactive support under high power transfer [8,9,10].
B. **RGA analyses**

Two different simulations are carried out for the RGA analyses:

1. **RGA analyses with different operating conditions.**

   150 different operating conditions of the test system are simulated. The whole trend is that the load at Bus 4 is increasing and load at Bus 9 is decreasing, i.e. the power flow from Bus 9 to Bus 4 on the tie-line 4-9 is increasing. For each operating point, RGA analysis is carried out.

2. **RGA analyses with different load modulations.**

   There are altogether 9 PQ buses in the test system. In the original case [5,6], there are only two load modulations at Bus 4 and Bus 9. By means of load modulation, the active and reactive power of the load can be considered as an input control signal to the dynamic voltage stability controller. In order to analyze the RGA with the increasing of load modulations on different buses, additional loads are added to the other PQ buses step by step.

   For the above mentioned two simulations, the RGA for steady state (s=0), inter-area mode, local mode 1 and exciter mode 1 of oscillation frequencies are analyzed. The selected input controls are given in Table V-VI in Appendices.

1) **Steady state voltage stability analysis (s=0)**

   a) **Steady state RGA with different operating conditions**

   Steady state RGAs are analyzed with different operating conditions. Based on the singular value analysis mentioned in [1], buses 3, 4, 8, 9 and 11 are the most critical ones associated with the steady state voltage stability. Since the Bus 11 is the most critical one, it is selected to demonstrate the simulation result, as shown in Fig. 5.

   ![Fig. 5 Steady state RGA of Bus 11 voltage magnitude for different operating conditions](image)

   **b) Steady state RGA with different load modulations**

   Focusing on one basic operating condition of the test system [5,6], the numbers of load modulations are increased. At first, there are only two load modulations at Bus 4 and Bus 9. Then the load modulations are added to all PQ-Buses step by step. With the increase of the load modulations, the RGA will also be changed. According to [1], Bus 11 is selected to demonstrate the simulation result. The simulation results are demonstrated in Fig. 6.

   The inputs number and there corresponding control variables are given in Table V-VI in Appendices.

   ![RGAs of Bus Voltage 11](image)

   **Fig. 6 Steady state RGA with different load modulations**

   From Fig. 6 (4-loads), it can be seen that the control number 1 (SVC control signal) and control number 4 (the active power control of generator 3 (Bus 6)) are all with RGA close to 1. However, the generator active power control is not selected as the suitable control signal for the steady state voltage stability control, because:

   1. Though the generator active power control has the least interaction with the other control loops, this active power control is not a suitable for the voltage control.
   2. Furthermore, this control signal is not a local signal for the Bus 11 voltage control.

   Therefore, the SVC control is the most suitable local control signal for the Bus 11 voltage magnitude control in the steady state.

2) **Voltage stability analysis of inter-area mode**

   a) **Inter-area mode RGA with different operating conditions**

   For the inter-area mode of oscillation frequency, RGAs are analyzed with different operating conditions. Based on the singular value analysis mentioned in [1], buses 3, 4, 10 and 11
are the most critical ones associated with the inter-area mode dynamic voltage stability. Since the Bus 11 is the most critical one, it is selected to demonstrate the simulation result. The simulation results are given in Fig. 7.

Fig. 7 RGA with different operating conditions for the inter-area mode of oscillation frequency

b) Inter-area mode RGA with different load modulations

Focusing on one basic operating condition, the numbers of load modulations are increased step by step. According to [1], Bus 3 is selected to demonstrate the simulation result, as shown in Fig. 8.

Fig. 8 RGA with different load modulations for the inter-area mode of oscillation frequency

3) Voltage stability analysis of local mode 1

a) RGA with different operating conditions

For the local mode 1 of oscillation frequency, RGAs are analyzed with different operating conditions. Based on the singular value analysis mentioned in [1], buses 2, 3, 4 and 10 are the most critical ones associated with the inter-area mode dynamic voltage stability. Since the Bus 2 is the most critical one, it is selected to demonstrate the simulation result. The simulation results are shown in Fig. 9.

Fig. 9 RGA with different operating conditions for the local mode 1 of oscillation frequency

b) RGA with different load modulations

Focusing on one basic operating condition, the numbers of load modulations are increased step by step. According to [1], Bus 2 is selected to demonstrate the simulation result, as given in Fig. 10.

Fig. 10 RGA with different load modulations for the local mode 1 of oscillation frequency

4) Voltage stability analysis of exciter mode 1

a) RGA with different operating conditions

With different operating conditions, RGA analyses are performed for the exciter mode 1 oscillation frequency. Based on the singular value analysis mentioned in [1], buses 6, 7, 12
and 13 are the most critical ones associated with the inter-area mode dynamic voltage stability. Since the Bus 7 is the most critical one, it is selected to demonstrate the simulation result, as shown in Fig. 11.

From the simulation results given in Fig. 5, Fig. 7, Fig. 9 and Fig. 11, it is obvious that with the variation of operating point, the RGA will also be changed. Therefore, for every new operating point, the RGA must be recalculated to select the proper control loop pairing for dynamic voltage stability control.

1. With the variation of the operating point, only the RGAs of some special operating points is in the range of $[0.5, 5]$, i.e. Fig. 5 $(U_{11}, SVC, U_{11}, Q_1)$. In this situation, instead of control loop pairing, the RGA can be used as a measure to remove the control signals which have the severe interactions with the other outputs.

2. From Fig. 5, Fig. 7, Fig. 9 and Fig. 11, it is clear that with the variation of operating point, RGA will also be changed. Therefore, for every new operating point, the RGA must be recalculated to select the proper control loop pairing for dynamic voltage stability control.

3. Unlike the RGA analysis performed in [3], the input and output signals are selected for the dynamic voltage stability analysis. Therefore, the RGA plots in this research do not reveal the critical modes of oscillation frequencies. However, the critical modes of oscillation frequencies can be obtained from the maximal singular value plot, as described in Fig. 5 and Fig. 6 in [1].

4. As shown in Fig. 5, Fig. 7, Fig. 9 and Fig. 11, the RGAs related with generator active power control $P_1$, $P_3$, $P_4$ and $P_7$ are quite large in every situation and every operating point. Therefore, the generator active power controls must have severe interaction with the other dynamic voltage stability control loops. Furthermore, as shown in Fig. 8, 10 and 12 in [1], the input singular vectors associated with them are quite small. This means the generator active power controls have little influence on the dynamic voltage stability control. In a word, the generator active power controls are not suitable for the dynamic voltage stability control.

5. The RGAs related with active load modulation at $P_4$ and $P_9$ are smaller than the generator active power control. However, they are still large in every situation and every operating point, as shown in Fig. 5, Fig. 7, Fig. 9 and Fig. 11. Furthermore, as shown in Fig. 8, 10 and 12 in [1], the input singular vectors associated with them are also relatively small. This means that the active load modulations are also not suitable for the dynamic voltage stability control.

6. The RGAs associated with the reactive power controls (generation reactive power controls $Q_1$, $Q_2$, $Q_6$, $Q_7$ and load reactive power controls $Q_4$, $Q_{13}$) vary in a range of $[-5, 5]$. Furthermore, Fig. 7, 9 and 11 in [1] show that the input singular vectors associated with the generation reactive power controls are relatively large. Therefore, the generator reactive power controls are suitable for the dynamic voltage stability control. Furthermore, they have also fewer interactions with the other control loops. However, in case those RGAs are less than 0, these control signals should not be selected.

From the simulation results given in Fig. 6, Fig. 8, Fig. 10 and Fig. 12, it is obvious that with the increase of load modulations, the RGA will also be changed. It follows that:
With the increase of load modulations, the RGAs of some input and output pairs are tended to 1. This means that with the increase of load modulations, the voltage magnitude of each node may be controlled decentralized using the local or nearby feedback signal. The most suitable control loop pairings are shown in Table I-IV. The inputs number and there corresponding control variables are given in Table V-VI in Appendices.

### TABLE I
**SUITABLE CONTROL INPUTS FOR STEADY STATE**

<table>
<thead>
<tr>
<th>Output Voltage</th>
<th>Number of loads</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input No.</td>
<td>4.6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Input No.</td>
<td>-</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Input No.</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>Input No.</td>
<td>10</td>
<td>14</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Where "-" means that no suitable control signal can be selected based on the RGA analyses. In other words, "-" means all control pairings have sever interactions.

### C. SVC dynamic voltage stability controller design

As mentioned before, the dynamic voltage stability is closely associated with the dominant modes of oscillations. Since the PSSs are well designed for the damping of local and inter-area mode of oscillations, it is not necessary to develop an additional dynamic voltage stability controller associated with the exciter.

The SVC is located on the mid-point of the tie-line. The dynamic voltage stability problem that affects the SVC and nearby bus voltages are closely related with the inter-area mode of oscillations. Therefore, the SVC dynamic voltage stability controller can be designed for the inter-area mode of oscillation damping control. The design method is given in [11].

### D. The advantages and disadvantages of the proposed method

By means of the input singular vector and the RGA analysis, the interactions among the control loops of the dynamic voltage stability control can be clearly analyzed. Therefore, together with the controller design method, i.e. residue method introduced in [1], the most suitable control loop for the corresponding mode of dynamic voltage stability control can be selected.

Furthermore, the residue method can be used for feedback signal selection and the dynamic voltage stability controller design. However, in order to verify the stability of the whole system, modal analysis must also be used.

### IV. CONCLUSION

This paper concerns the use of RGA for analyzing the interactions among the power system dynamic voltage stability control loops. RGA analyses for both static and dynamic voltage stability controls are carried out. A typical multi-machine power system with different working conditions is simulated for the demonstration of the proposed approach. Also, limitations and advantages of the use of RGA in the dynamic voltage stability analysis and control are also proposed. This approach takes the advantages of the classical static voltage stability analysis and the modern multi-variable feedback control theory. Since the input and output signal can be limited to a small range, this approach can be easily implemented into large power system dynamic voltage stability analyses and control.
V. REFERENCES


VI. APPENDICES

TABLE V
The Input Numbers and Their Corresponding Input Variables for the SVC and Generation Modulations

<table>
<thead>
<tr>
<th>Input Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVC</td>
<td>P_1</td>
<td>P_2</td>
<td>P_3</td>
<td>P_4</td>
<td>Q_1</td>
<td>Q_2</td>
<td>Q_3</td>
<td>Q_4</td>
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</table>

TABLE VI
The Input Numbers and Their Corresponding Input Variables for the Active and Reactive Load Modulations

<table>
<thead>
<tr>
<th>Input Number</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
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<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Variable</td>
<td>P_5</td>
<td>Q_5</td>
<td>P_6</td>
<td>Q_7</td>
<td>P_1</td>
<td>Q_8</td>
<td>P_3</td>
<td>Q_9</td>
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<td></td>
</tr>
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</table>

VII. BIOGRAPHIES

Li-Jun Cai was born in 1970. He received the B.-Eng. and M.-Eng. degrees from the Electrical Engineering Department, North China Electrical Power University, China, in 1992 and 1997, respectively, and the Ph.D. degree in electrical engineering from the University of Duisburg-Essen, Essen, Germany, in 2004. From 2004 to 2006, he worked as a post-doctoral fellow in the University of Duisburg-Essen, Germany. Presently, he is with the Vattenfall Europe Transmission GmbH, Berlin, Germany.

His research interest is in the power system planning, optimal location, multi-objective coordinated control of FACTS devices and power system dynamic stability analyses and control.

István Erlich (M’99) was born in 1953. He received the Dipl.-Ing. degree in electrical engineering in 1976 and the Ph.D. degree in 1983, both from the University of Dresden, Dresden, Germany.

After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems, University of Dresden. During 1991–1998, he worked with the consulting company EAB, Berlin, Germany, and the Fraunhofer-Institute IITB, Dresden, respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he has been a Professor and Head of the Institute of Electrical Power Systems at the University of Duisburg-Essen, Essen, Germany.

His major scientific interest is focused on power system stability and control, modeling, and simulation of power system dynamics, including intelligent system applications.

Dr. Erlich is a member of VDE and Senior Member of IEEE.