Power System Dynamic Voltage Stability Analysis for Integration of Large Scale Wind Parks

Li-Jun Cai, Jens Fortmann, Roman Bluhm and István Erlich

Abstract— This paper presents a method for analyzing the influence of large wind park integrations and other voltage controllers on power system dynamic voltage stability behavior. REpower wind turbine and its wind park voltage control strategy are applied as an example of the wind park voltage control. The system for dynamic voltage stability analysis is modeled as a multi-input multi-output (MIMO) control system and the singular value analysis is applied. The proposed approach takes the advantages of the classical static voltage stability technique and the modern multi-variable feedback control theory. The dynamic voltage stability criterion, controllability and participation indices can be obtained. Simulation results show the efficiency of the proposed approach.

Index Terms-- Doubly-fed induction generator (DFIG), Wind turbine voltage control, Wind park voltage control, Dynamic voltage stability, Singular value analysis, Modal analysis, Dynamic voltage stability index, Dynamic voltage controllability index.

I. INTRODUCTION

Since the wind energy is renewable and environmental natural resource, the utilization of wind power plant increased quickly [1,2]. In Germany, the installed wind power capacity in 2009 reached 25 GW and 50 GW is expected till 2020. In the future, the development of wind power utilization will focus on large offshore wind parks [1]. Usually, these large offshore wind parks will be connected to the 400-kV network. Wind power constitutes the renewable generation technology which has experienced the fastest growing among all types of renewable generation technologies currently investigated [2].

Plenty of large modern wind turbines employed doubly-fed induction generators (DFIGs). These machines are collectively referred to as variable speed machines and they possess important advantages such as reactive power control capabilities, smaller and cheaper converter compared with a full size one. Furthermore, the voltage control of the wind turbine could reinforce the system voltage and dynamic stability [2].

Because of the increasingly integration of large wind parks, their impacts on power system static and dynamic behavior must be analyzed. Also, their interaction with conventional power plants must be taken into consideration.

This paper provides a method to analyze the influence of wind park voltage control and other power system controllers on the power system dynamic voltage stability. The singular value and singular vector analyses are employed. Furthermore, measures for enhancing power system dynamic voltage stability are also provided.

This paper is organized as follows: Following the introduction, REpower DFIG structure and its voltage control strategy will be described. In section III, the classical voltage stability analysis is summarized. In section IV and V, singular value method for dynamic voltage stability analysis is introduced in detail. Then system studies are given in Section VI. Finally, brief conclusions are deduced.

II. VOLTAGE CONTROL OF DOUBLY-FED INDUCTION GENERATOR

In order to support the power system voltage stability, REpower developed a new advanced two levels voltage control strategy which has very fast and accurate control behavior [3]. The structure of the DFIG and the REpower voltage control strategy will be introduced in this section.

A. Structure of DFIG

The basic example structure of DFIG is shown in Fig. 1. As a general approach, the space-phasor coordinates with orthogonal direct (d) and quadrature (q) axes are used. The choice of the stator voltage as the reference frame enables the decoupled control of \( P \) (active power, \( d \) control channel) and \( Q \) (reactive power, \( q \) control channel) [1,2].
B. Wind park layout and control signals

According to Grid Codes, active and reactive power should be controlled at the point of common coupling (PCC), which is usually located at medium voltage (MV) or high voltage (HV) side of the HV transformer. Therefore the wind park controller can be installed at the PCC, as shown in Fig. 2 [3].

C. Voltage control

The basic function of a voltage controller is to calculate the setpoint for reactive power depending on the voltage. A general block diagram for a proportional characteristic is shown in Fig. 3. The voltage measurement \( U_{\text{meas}} \) is subtracted from the voltage setpoint \( U_{\text{set}} \) to calculate the voltage deviation \( \Delta U \). This deviation is multiplied by the proportional control factor \( K_{VC} \) to calculate the reactive power setpoint \( Q_{\text{set}} \). Depending on the desired control characteristic, a reactive current setpoint \( I_{q\text{set}} \) can be used instead of the reactive power setpoint [3].

Fig. 3. Proportional voltage control block diagram

The voltage control can be implemented in both turbine level (local) and the wind park level (central) and the detailed description can be found in [3]. A combination of the central and local voltage control can combine their favorable characteristics with benefit for grid stability. The continuous voltage control at the turbine level delivers a very fast response to deep voltage drops and also to small voltage deviations inside the standard operation range. The combination with voltage control at the wind park level ensures an exact adjustment of the required reactive power value at the grid connection point. A stable control of the combined controller can be guaranteed because the time constant of the subordinate local control is more than 10 times faster than the time constant of the wind park controller. Settings such as slope or response time of the combined voltage control can be easily adapted to achieve the desired characteristics required at different connection points or in different countries [3].

The above mentioned wind park voltage control characteristic will be applied for analyzing its influence on the power system dynamic voltage stability.

III. CLASSICAL DYNAMIC AND STATIC VOLTAGE STABILITY ANALYSES AND CONTROL

A. Dynamic voltage stability analysis

The mathematical model for the dynamic stability study of a power system comprises first order differential equations and a set of algebraic equations [4,7]:

\[
\begin{align*}
\dot{x} &= f(x, y) \\
0 &= g(x, y)
\end{align*}
\]

with a set of known initial conditions \((x_0, y_0)\), where

\begin{align*}
x &\quad \text{state vector of the system} \\
y &\quad \text{bus voltage, stator current, injection of real and reactive power vector}
\end{align*}

Equations (1) and (2) can be solved in time-domain by means of the numerical integration and power flow analysis methods [11].

The steady state values equilibriums of the dynamic
system states can be evaluated by setting the derivative in Equation (1) to zero. In the neighborhood of an equilibrium point, Equations (1) and (2) can be linearized as [7,11]:

$$\frac{d\Delta x}{dt} = \Delta x + B\Delta y$$  \hspace{1cm} (3)

$$0 = C\Delta x + D\Delta y$$  \hspace{1cm} (4)

By eliminating $\Delta y$, the linearized equation (3) is [7,11]:

$$\frac{d\Delta x}{dt} = (A - BD^{-1}C)\Delta x = \hat{A}\Delta x$$  \hspace{1cm} (5)

The static bifurcation will occur when $\det(D) = 0$. For the dynamic bifurcation phenomenon, it is always assumed that $\det(D) \neq 0$ and $D^{-1}$ exists [7,11].

By analyzing the eigenvalues of $\hat{A}$, dynamic voltage stability can be achieved.

B. Static voltage stability analysis

The static approach captures snapshots of system conditions at various time frames along the time-domain trajectory. At each of these time frames, time derivatives of the state variables in Equation (1) are assumed to be zero and the state variables take on values appropriate to the specific time frame. Consequently, the overall system equations reduce to purely algebraic equations allowing the use of static analysis techniques [4,11].

The algebraic equation (2) can be expressed in the following linearized form [4]:

$$\begin{bmatrix} \Delta P_{pq} \\ \Delta Q_{pq} \\ \Delta \theta \\ \Delta V_{pq} \end{bmatrix} = \begin{bmatrix} J_{pq} \\ J_{q0} \\ J_{0q} \\ J_{00} \end{bmatrix} \cdot \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta \theta \\ \Delta V \end{bmatrix}$$  \hspace{1cm} (6)

where

- $\Delta P_{pq}$: incremental change in bus real power
- $\Delta Q_{pq}$: incremental change in bus reactive power
- $\Delta \theta$: incremental change in bus voltage angle
- $\Delta V$: incremental change in bus voltage magnitude

Power system voltage stability is affected by both real and reactive power. Keeping real power as constant at each operating point, the $Q-V$ analysis can be achieved by assuming $\Delta P_{pq, PV} = 0$ in Equation (6) [4,10]:

$$\Delta Q_{pq} = \begin{bmatrix} J_{q0} - J_{0q} \cdot J_{pq}^{-1} \cdot J_{pq} \end{bmatrix} \cdot \Delta V_{pq} = J_{pq}^{-1} \cdot \Delta V_{pq}$$  \hspace{1cm} (7)

and

$$\Delta V_{pq} = J_{pq}^{-1} \cdot \Delta Q_{pq}$$  \hspace{1cm} (8)

Based on the analysis of $J_{pq}^{-1}$, which is the reduced $V-Q$ Jacobian matrix, the $Q-V$ modal analysis can be performed. Therefore, the bus, branch and generator participation factors can be obtained. Moreover, the stability margin and the shortest distance to instability can also be determined [4,10,11].

As discussed in [8,9], the singular value analysis can also be applied to the static voltage stability analysis.

C. Voltage stability control

In order to prevent the voltage collapse, different measures can be applied [4,11]. The reactive power compensation, under-voltage load shedding and the control of transformer tap-changers are the most important controls for enhancing the static voltage stability [4,11,12].

For the dynamic voltage stability, the reactive power control and generator voltage control are important measures. Furthermore, with the development of wind turbine and wind park voltage control capabilities, they should also be considered as an important measure for the dynamic voltage stability control.

IV. DYNAMIC VOLTAGE STABILITY MODELING

The proposed approach in this paper considers power system dynamic models for analyzing the dynamic voltage stability:

1. The generators with 6th order, and
governors, static exciters, power system stabilizers (PSS) and
3. non-linear load modulations.

Since the dynamic voltage stability is associated with the local, inter-area and controller modes of oscillation frequencies, the tap-changer is relative too slow and it is not considered in this research.

The voltage stability problems are always associated with the generator outputs, changes in real and reactive loads. In this study, a multi-input multi-output (MIMO) system transfer function is employed by using the generation and load modulation as the input signals. The voltage magnitudes at each bus are the output signals, as shown in Fig. 4.
Therefore, the MIMO transfer function can be obtained as:

\[
\begin{bmatrix}
\Delta U_1 \\
\Delta U_2 \\
\vdots \\
\Delta U_n
\end{bmatrix} = \mathbf{J}_f(s) \begin{bmatrix}
\Delta P_1 \\
\Delta P_2 \\
\vdots \\
\Delta P_n
\end{bmatrix} + \begin{bmatrix}
\Delta Q_1 \\
\Delta Q_2 \\
\vdots \\
\Delta Q_n
\end{bmatrix}
\]

\[
\Delta U = \mathbf{J}_f(s) \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

(9)

and

\[
\mathbf{J}_f(s) = \begin{bmatrix}
f_{P1,1}(s) & f_{P1,2}(s) & \ldots & f_{P1,n}(s) \\
f_{P2,1}(s) & f_{P2,2}(s) & \ldots & f_{P2,n}(s) \\
\vdots & \vdots & \ddots & \vdots \\
f_{Pn,1}(s) & f_{Pn,2}(s) & \ldots & f_{Pn,n}(s) \\
f_{Q1,1}(s) & f_{Q1,2}(s) & \ldots & f_{Q1,n}(s) \\
f_{Q2,1}(s) & f_{Q2,2}(s) & \ldots & f_{Q2,n}(s) \\
\vdots & \vdots & \ddots & \vdots \\
f_{Qn,1}(s) & f_{Qn,2}(s) & \ldots & f_{Qn,n}(s)
\end{bmatrix}
\]

(10)

In order to analyze this typical non-square MIMO system, the singular value analysis of the transfer function \( \mathbf{J}_v(s) \) is applied.

V. SINGULAR VALUE ANALYSIS FOR DYNAMIC VOLTAGE STABILITY

A. Singular value decomposition (SVD)

As discussed in [9,15], for every fixed frequency where the \( \mathbf{J}_v(s) \) is a constant input-output \( n_{\text{output}} \times n_{\text{input}} \) complex matrix and it can be decomposed into its singular value decomposition:

\[
\mathbf{J}_f(s) = \mathbf{U} \Sigma \mathbf{V}^T
\]

(11)

where

\( \Sigma \) is a \( n_{\text{output}} \times n_{\text{output}} \) matrix with \( k = \min \{l,m\} \) non-negative singular values, \( \sigma_i \), arranged in descending order along its main diagonal; the other entries are zero. The singular values are the positive square roots of the eigenvalues of \( \mathbf{J}_f^*(s) \times \mathbf{J}_f(s) \), where \( \mathbf{J}_f^*(s) \) is the complex conjugate transpose of \( \mathbf{J}_f(s) \) [15].

\[
\sigma_i(\mathbf{J}_f(s)) = \sqrt{\lambda_i(\mathbf{J}_f^*(s) \times \mathbf{J}_f(s))}
\]

(12)

\( \mathbf{U} \) is a \( n_{\text{Output}} \times n_{\text{Output}} \) unitary matrix of output singular vectors, \( \mathbf{u}_i \)

\( \mathbf{V} \) is an \( n_{\text{Input}} \times n_{\text{Input}} \) unitary matrix of input singular vectors, \( \mathbf{v}_i \)

B. Singular vectors

The column vectors of \( \mathbf{U} \), denoted \( \mathbf{u}_i \), represent the output directions of the researched power system. They are orthogonal and of unit length (orthonormal). Likewise, the column vectors of \( \mathbf{V} \), denoted \( \mathbf{v}_i \), are orthogonal and of unit length, and represent the input directions. These input and output directions are related through the singular values [15].

For dynamic voltage stability analysis, the singular values and their associated directions vary with frequency, and for control purposes it is usually the frequency range corresponding to the closed-loop bandwidth which is of main interest [15]. In this research, the frequency range of inter-area modes, local modes and controller modes of oscillations are considered.

By analyzing the maximum singular values \( \mathbf{J}_v(s) \) and their related input and output singular vectors, the relationship between input and output can be obtained at each frequency:

1. The output singular vector shows at which bus the voltage magnitude is the most critical.
2. Furthermore, the input singular vector shows which input has the greatest influence on the corresponding output. Therefore, the dynamic voltage stability can be analyzed.

C. Dynamic voltage stability index

As explained in (16), \( \mathbf{J}_v(s) \) represents a relationship between voltage magnitude and active/reactive inputs. The dynamic voltage stability index can be considered as the maximal singular value of \( \mathbf{J}_v(s) \). However, it is difficult to define a stability index by means of the maximal value. Therefore, the pseudo inverse of \( \mathbf{J}_v(s) \) is applied in this paper:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \left[\mathbf{J}_f(s)\right]^{-1} \cdot \Delta U
\]

(13)

The singular value of \( \left[\mathbf{J}_f(s)\right]^{-1} \) determines the weakness of the nodal voltage. If the singular value is zero, the respective node will have a voltage collapse, because any change in active or reactive power will cause an infinite nodal voltage.

Therefore, the minimal singular value of \( \left[\mathbf{J}_f(s)\right]^{-1} \) will be considered as the dynamic voltage stability index.

VI. SYSTEM STUDIES

A. Power System Model

As shown in Fig. 5, the typical four-machine two-area power system model is applied for the dynamic voltage stability studies.
stability research [4,5]. The dynamic model consists of generators described by 6th order model, governors, static exciters and nonlinear voltage and frequency dependent loads. The detailed generator, controller and load models can be found in [8,9]. Between Bus 3, 101 and 13, there is a parallel transmission line for enhancing the power transfer capability [9].

![Four-machine two-area power system](image)

Fig. 5. Four-machine two-area power system

Based on this power system model, the dynamic voltage stability assessment and control can be achieved as the following steps:

1. Static voltage stability analysis to choose the most critical node for voltage collapse.
2. Using the modal analysis to find the critical modes of oscillations.
3. Analysis of the $J_V(s)$ at the corresponding local and inter-area oscillation frequencies.
4. Find the critical bus which have the most severe dynamic voltage stability problem.
5. Find the corresponding input signals and design of the suitable controller for the enhancement of dynamic voltage stability.

The detailed system model is given in Appendix A.

B. Static voltage stability analysis – singular value approach and classical approach

Based on the theory provided in [5,8,9], the static voltage stability is analyzed by means of the singular value analysis of $J_R^{-1}$, as explained in Equation (8).

The simulation result is given in Fig. 6. The dominant singular value is 0.136 and the corresponding maximum output singular vector is 0.581. It associates with Bus 101 (internal bus number: 11).

In order to compare the results of singular value analysis and traditional modal analysis, the classical static voltage stability is also analyzed and the result is given in Fig. 7. The dominant eigenvalue is 0.136 and the corresponding maximum eigenvector is 0.591. It also associates with Bus 101.

The classical static voltage stability analysis verifies the proposed singular value approach.

![Magnitude of the output singular vector](image)

Fig. 6. Magnitude of the output singular vector

![Magnitude of the output eigenvector](image)

Fig. 7. Magnitude of the output eigenvector

C. Dynamic voltage stability analysis – modal analysis and singular value approach

1) Modal analysis

![Critical eigenvalues of the power system](image)

Fig. 8. Critical eigenvalues of the power system

- Eigenvalues with PSS controller
- Eigenvalues without PSS controller
First, based on the selected operating point, the linearized power system model is obtained. Then the modal analysis is employed and the critical eigenvalues is shown in Fig. 8. It is clear that, with the PSS controller, all modes of oscillations are well damped. However, the PSS controller has negative influences on some exciter modes. Furthermore, the exciter mode 1 and the inter-area mode have relative low damping ratios and may have influences on the dynamic voltage stability.

The exciter mode 1 has a damping ratio of 11.93% and the frequency is 2.82Hz. The inter-area mode has a damping ratio of 13.63% and the corresponding frequency is 0.57Hz. Singular value analyses are performed on the two critical frequencies.

2) Singular value analysis

Using the dynamic voltage stability model discussed in section IV, the transfer function \( J_v(s) \) can be obtained. Then the singular value analysis is performed for analyzing the two critical oscillation mode of frequencies. The critical output singular vector associated with the exciter mode 1 and inter-area mode are given in Fig. 9 and Fig. 10.

The detailed results of the singular value analysis are given in Table I.

### TABLE I
RESULTS OF THE SINGULAR VALUE ANALYSIS

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq (Hz)</th>
<th>Min. singular value of ([J_v(s)]^{-1})</th>
<th>Output singular vector</th>
<th>Input singular vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exciter Mode 1</td>
<td>2.82</td>
<td>0.1388</td>
<td>0.323</td>
<td>101</td>
</tr>
<tr>
<td>Inter-area Mode</td>
<td>0.57</td>
<td>0.37</td>
<td>0.520</td>
<td>101</td>
</tr>
</tbody>
</table>

D. Measures for enhancing the static and dynamic voltage stability

In order to improve the dynamic voltage stability behavior, following measures could be made:

1. Wind park voltage control
2. Synchronous generator voltage control
3. Reactive compensation

As shown in Table I, the voltage control at the following buses can enhance the power system dynamic voltage stability:

1. For the exciter mode 1: Bus 12 and Bus 2
2. For the inter-area mode: Bus 2 and Bus 1

Since the wind park voltage control strategy has the same behavior as the conventional generator voltage control, the control of wind park voltage will also be helpful in enhancing the power system dynamic voltage stability.

For the voltage controller design, methods introduced in [4,6] could be applied.

VII. CONCLUSION AND FUTURE WORKS

This paper proposed an approach for the dynamic voltage stability assessment and control in multi-machine power system. REpower wind turbine and its wind park voltage control strategy are applied as an example of the wind park voltage control. Based on the modal analysis and singular value analysis of the MIMO system, the dynamic voltage stability of multi-machine power system has been carried out. The proposed approach takes the advantages of the classical static voltage stability theory and the modern multi-variable feedback control theory. The dynamic voltage stability, controllability and participation indices are obtained. Simulation results show that this approach is efficient in analyzing the power system dynamic voltage stability.
A. Power System Data

Values are per unit and the base MVA is 100MVA.

<table>
<thead>
<tr>
<th>Bus</th>
<th>$U_0$</th>
<th>$zU$</th>
<th>$P_0$</th>
<th>$Q_0$</th>
<th>$P_1$</th>
<th>$Q_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.03</td>
<td>18.5</td>
<td>7.2249</td>
<td>2.0208</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.01</td>
<td>8.1528</td>
<td>7</td>
<td>2.6235</td>
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<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.95376</td>
<td>-7.3071</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0.94709</td>
<td>-10.404</td>
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<td>0</td>
<td>9.761</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1.0641</td>
<td>11.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
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<td>1.03</td>
<td>-6.3527</td>
<td>7.16</td>
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</tr>
<tr>
<td>7</td>
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</tr>
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<td>9</td>
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<td>0</td>
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<td>10</td>
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</tr>
</tbody>
</table>


IX. REFERENCES


X. BIOGRAPHIES

Li-Jun Cai (1970) received the B.-Eng. and M.-Eng. degrees from the Electrical Engineering Department, North China Electrical Power University, China, in 1992 and 1997, respectively, and the Ph.D. degree in electrical engineering from the University of Duisburg-Essen, Germany, in 2004. Following the Ph.D study, he worked as a post-doc fellow from 2004 to 2006 in the University of Duisburg-Essen, Germany. From 2006 to 2009 he worked with Vattenfall-Europe Transmission and implementation of new technologies for improved grid compatibility of wind turbines like voltage control and ride-through of grid faults. He is the head of the FGW working group that specifies the modeling and model validation guideline TR4.

Jens Fortmann (1966) received his Dipl.-Ing. degree in electrical engineering from the Technical University Berlin, Germany, in 1996. From 1995 to 2002 he worked on the simulation of the electrical system and the control design of variable speed wind turbines at the different wind turbine manufacturers. Since 2002 he is with REpower Systems AG, Germany presently as team leader of model and system development for the simulation and implementation of new technologies for improved grid compatibility of wind turbines like voltage control and ride-through of grid faults. He is the head of the FGW working group that specifies the modeling and model validation guideline TR4.

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István Erlich (1953) received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his Ph.D. degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen/Germany. His major scientific interest is focused on power system stability and control, modeling and simulation of power system dynamics including intelligent system applications.