A Hybrid Method for Voltage Stability Constrained Optimal Reactive Power Dispatch

Worawat Nakawiro, Student Member, IEEE and Istvan Erlich, Senior Member, IEEE

Abstract—This paper presents a hybrid method for solving voltage stability constrained optimal reactive power dispatch (VSCORPD) problem. The objective of VSCORPD is to minimize the energy loss at the current time interval and the cost of adjusting discrete control devices while maintaining system security and voltage stability constraints within the permissible limits. An artificial neural network is trained to approximate the voltage stability margin during the optimization process. A modified ant colony optimization is used to solve the VSCORPD. The numerical results have demonstrated that the proposed method can effectively reduce energy losses, avoid excessive control device adjustments and simultaneously prevent the voltage collapse problem.

Index Terms—Artificial neural network, ant colony optimization, reactive power dispatch, voltage stability, optimal power flow

I. INTRODUCTION

One of the important tasks for power system operation nowadays is to manage all available reactive power sources such optimally that the active power transmission losses are minimum while satisfying a number of system and equipment constraints such as bus voltages and transmission line loadings. This problem is known as optimal reactive power dispatch (ORPD) which is a subclass of optimal power flow (OPF) problems [1]. The available reactive sources in a power system are: shunt reactors and capacitors connected to busbars, transformer tap positions, generator reactive power, etc. Sometimes FACTS devices such as SVC, STATCOM or UPFC are also considered in ORPD [2].

In the past two decades, several blackouts throughout the world have been found to be associated with voltage collapses. It has been well demonstrated that one of the major causes of voltage instability is due to the lack of reactive power supports. Therefore, reactive power sources should be properly and continuously controlled not only to maintain system security limits but also to guarantee a sufficient loading margin to the voltage collapse point. This task can be achieved by solving the voltage stability constrained optimal reactive power dispatch (VSCORPD) problem.

The VSCORPD problem [3] has been formulated as a nonlinear and mixed-integer optimization problem because some control variables, such as shunt capacitors and transformer tap positions are of discrete characteristics. To achieve a practical VSCORPD solution, the discrete control variables should not be be frequently adjusted because such a control action increases maintenance costs and shortens life time expectancy of the equipment. Therefore, the number of control actions should be simultaneously minimized along with the transmission losses.

The ACO algorithm was originally proposed by Dorigo aiming at searching for the optimal path in a graph [4]. The first ACO versions were intended to handle only combinatorial optimization problems. However, the development during the past decade elevates its capabilities for handling continuous complex optimization problems. Among these methods, the method proposed by Socha and Dorigo in [5] is an effective tool in dealing with global optimization problems. Such an algorithm has been modified and applied to solve a few power system problems with promising successes [6]-[7]. This paper applies the same algorithm as in [7] as an optimization tool.

The rest of this paper is organized as follows. The proposed method is conceptually discussed in section II. The procedures for estimating voltage stability margin are described in section III. The VSCORPD problem is formulated in section IV. Section V presents the ACO algorithm and the major steps in the VSCORPD implementation. Numerical results are discussed in section VI. Finally, conclusions are drawn in section VII.

II. THE PROPOSED METHOD

In this paper, two computational intelligence methods are applied to solve the VSCORPD problem. Fig.1 illustrates the conceptual implementation of our proposal. It is well accepted that the voltage stability margin (VSM) found from the continuation power flow (CPF) [8] provides meaningful information to the power system operator. However, solving CPF for a single operating condition normally involves a number of power flow calculations. Therefore, it might be very time-consuming if CPF is directly included in the OPF program solved by any heuristic optimization methods, such as ACO applied in this paper. To resolve this problem, an artificial neural network (ANN) is developed to estimate VSM and CPF is bypassed during the ACO iterative process. The ANN is trained based on measurement and/or offline
perform accurately when the loading level and system parameters are changed. To simulate this environment, daily active and reactive power demand profiles with the forecast error are considered. The forecast error is assumed to be linearly increasing from 2% to 5% from the beginning toward the end of the day. This is due to the accumulative error of the preceding time steps. The nominal load profile with upper and lower bands of forecast error define the probable operating region in the daily operation as shown in Fig.2. The red bold line is the nominal profile while the dotted upper and lower lines correspond to maximum and minimum boundaries, respectively. A demand level at time $t$ is modeled as a random normal distribution with mean of nominal demand at time $t$ and standard deviation of the corresponding forecast error in the physical unit (MW or MVar). The sampling time used in this study is 15 minutes. Based on three scenarios plotted (in different colors) in the Fig.2, it is possible to generate 288 ($96 \times 3$) operating conditions with different total active and reactive power demand levels. These operating conditions are used for training an ANN to estimate VSM.

$$\begin{align*}
    P_{Di}^j &= \sum_j P_{Dk}^j \cdot k \\
    Q_{Di}^j &= \sum_j Q_{Dk}^j \cdot k
\end{align*}$$

where $\sum_j P_{Dk}^j \cdot$ is the total active power demand of the operating state $j$; $\sum_j Q_{Dk}^j \cdot$ is the total reactive power demand of the operating state $j$; $k = [k_1 \ k_2 \ \ldots \ k_{npq}]^T$ is a vector of...

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**Simulations.** A state of power system operation can be characterized by demand and generation patterns and dependent variables of power flow such as bus voltages and power flows over transmission lines. This set of information constitutes a set of input features. With the growing size of power systems, the dimension of input features increases significantly. This affects the learning capability of the ANN. Therefore, feature reduction techniques should be applied to reduce dimensionality of input features by linear or nonlinear combinations [9]. Some other additional inputs can be also given to the ANN to enhance the estimation accuracy. Once the ANN is well trained, the estimated VSM is treated as one of VSCORPD constraints. ACO is responsible solving the VSCORPD for the optimal control variables resulting in minimum total active power losses whereby all concerning system variables are maintained within their allowable limits.

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**III. ESTIMATING THE VOLTAGE STABILITY MARGIN**

Historical information regarding VSM can be obtained from offline studies as well as online recordings. Based on this knowledge, it is possible to explore the influence of changes in power systems variables on the VSM. To conduct such a study, a set of rules was developed in this research.

**A. Test System**

The IEEE 30 bus is the test system for all simulations presented in this paper. The original test system is slightly modified by adding reactive power sources to buses 15,16,17,18,20,22,23,25 and 30. The test system, parameters and initial bus data are discussed in detail in [10]. The limits of control variables and one-line diagram are given in Appendix. The PSAT software package [11] is used for all power system simulations in this paper.

**B. Random Adjustment of Power System Variables**

Power system equipments are normally operated within allowable upper and lower limits, such as generator reactive power generation, transformer tap positions or settings of reactive compensation devices. These boundary conditions are established such that the power system operation can be maintained within realistic operating limits and system capability.

The starting point of developing the ANN for VSM estimation is to generate a large number of power flow cases. This dataset serves as the knowledge for ANN learning. The developed ANN should be such generalized that it can...

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**Fig. 1. Conceptual diagram of the proposed method**

**Fig. 2. Daily active (a) and reactive (b) power demand profiles with probable operating regions**
normalized MVNRND and $\sum_{i=1}^{npg} k_i = 1$ and $npg$ is the number of load buses.

Fig. 3 shows the scatter plot of $P_D$ of two buses at different correlation coefficients (R). It can be observed from Figs. 3 (a) and (b) that $P_D$ at bus 5 has positive correlation with $P_D$ at bus 15 while negative correlation exists between $P_D$ at buses 5 and 12. There is relatively less correlation in case of Figs. 3 (c) and (d).

Fig. 3. Correlation of load demands

Random changes of reactive power control variables are also considered in this study. Shunt capacitors and transformer tap positions have discrete characteristics. Discrete normal probability distribution function (PDF) is used. Generator reactive power and shunt inductor are modeled by uniform PDF. Random variation of all control variables are maintained within their respective limits. For all defined operating conditions, power flow is solved to ensure that it is a feasible one. The unsolvable power flow cases are simply rejected. Voltage stability analysis is conducted by using CPF and the unsolvable cases are simply rejected. Voltage stability analysis is conducted by using CPF and the unsolvable cases are simply rejected. In this paper, $\lambda$ is adopted as VSM and the two terms are interchangeably used.

C. ANN for VSM estimation

Artificial neural network (ANN) is a mapping tool used to analyze the complex relation between a set of input variables and corresponding outputs. It is not the scope of this paper to discuss ANN in details. Additional information on this topic can be found from [12]. There are various types of ANN where different networks are appropriate for different applications. Feed-forward neural network (FFNN) is very suitable for function approximation problems. FFNN learns and gains the knowledge through the training session. Back propagation is a method to teach the FFNN to perform a given task. The weights are adjusted over iterations until the minimum error is obtained. A two-hidden layer FFNN is developed for VSM estimation in MATLAB neural network toolbox [13].

The input vector describing an operating state and corresponding outputs is

$$\mathbf{x}_i = [sf_{i1}, sf_{i2}, \ldots, sf_{im}, u_{i1}, u_{i2}, \ldots, u_{im}, \lambda_i]^T$$

where $sf_{ik}$ is the MVA power flow over line $k$ for state $i$; $u_j$ is the voltage at bus $j$ for state $i$; $m$ is the number of lines and $n$ is the number of buses excluding the slack, $\lambda_i$ is the VSM of state $i$ and the target of FFNN estimation. Line flows and bus voltages can reasonably characterize a power system state because the changes in loading conditions and control variables influence bus voltages. Power flow over a transmission line is a function of two bus voltages and line admittance. Therefore, this quantity can reflect the change in network topology. Principal component analysis [14] is applied to extract the useful input features and improve the learning capability of FFNN. The training session uses 15,000 operating conditions generated offline based on the procedures discussed in section III.B.

Fig. 4. Statistics of FFNN testing (a) histogram of 5000 testing conditions (b) cumulative probability distribution of estimation error

After the proper training, the VSM of new 5000 operating conditions were estimated by the FFNN. The estimation error is defined by the percentage of absolute difference between estimated and actual values relative to the actual one. Histogram in terms of percentage of occurrence is shown in Fig. 4(a). Two vertical lines represent mean and three-standard deviation, respectively. The latter is set as the threshold for outlier analysis. The samples with estimation error greater than this value are considered as outliers. From the statistical analysis, the average error is 3.72%, the standard deviation is 3.07% and the maximum error is 34.02%. According to the criterion discussed, there are 59 outliers accounting for 1.18% of the entire samples. The cumulative probability distribution function (CPDF) of the estimation error is plotted as shown in Fig. 4(b). It can be observed that accurate results can be obtained with acceptably high probability. For example, the probability of estimation error of FFNN is less than or equal to 5% is 0.7171.

An additional test for VSM estimation by FFNN has been also carried out. This experiment aims at verifying if the developed FFNN still performs well under the condition of fixed power demand level but varying control variable settings. This is precisely the condition that FFNN will have to perform when incorporated in the ACO process. An hourly demand profile for a day was randomly generated. The
demand and generation profiles at each hour are fixed. At every time step, 100 different samples are generated based on the rule that only control variables are allowed to be changed within the respective limits. The corresponding VSM is calculated for each operating conditions. Estimation errors as defined recently are calculated and box plots at each hour are shown in Fig.5. It is demonstrated that acceptable accuracy has been obtained with average error throughout the day is less than 8%. Even there are some outliers at each hour; however the number of those points is relatively small.

Fig. 5 Box-plot of testing for variation of control settings

IV. PROBLEM FORMULATION

In this paper, the objective voltage stability constrained optimal reactive power dispatch (VSCORPD) is to minimize the cost of energy losses and the cost of adjusting discrete control devices. The latter cost is defined by the multiplication the cost of energy losses and the cost of adjusting discrete variables. The superscripts min and max in (5) to (9) represent minimum and maximum limits of the respective quantities. In (10) based on the input shown in (3), the VSM is determined by the FFNN developed in section IV and it has to be maintained to be greater than the required level $VSM_{lim}$.

V. ANT COLONY OPTIMIZATION

Ant colony optimization (ACO) is applied to solve the VSCORPD. The basics of ACO algorithm used in this paper is summarized in section V.A and the major steps of VSCORPD implementation are discussed in section V.B.

A. Basics of ACO algorithm

A colony of ants represents candidate solutions of the problem with the matrix $X_t$ of size $n_{ant} \times n_{var}$, where $n_{ant}$ is the number of ants and $n_{var}$ is the number of variables. Quality of each ant is assessed by a fitness value. Ants move in the search space based on the normal PDF. Therefore, two parameters namely mean and standard deviation must be determined. The former is equivalent to the search direction while the latter indicates the distance of the ant movement. The search experience of ants is stored in the so-called solution archive $X_{t}$ as a matrix of size $n_{arch} \times n_{var}$, where $n_{arch}$ is the number of archive solutions. This dataset is important for guiding the ants of subsequent generations to explore the better results. For each ant, the mean is selected applying the roulette wheel selection (RWS) to the vector of probability $\mathbf{p}_r$. Each element of $\mathbf{p}_r$ is calculated in such a way that the better solution has greater probability. The function RWS returns the rank of archive solution $i_{sel}$ that is selected as the mean for the ant $i$ as;

$$ i_{sel} = RWS(\mathbf{p}_r); \quad \text{where } \mathbf{p}_r = \{p_{ri} | \forall i = 1,2,...,n_{arch}\} \quad (11) $$

for $i = 1,2,...,n_{ant}$

The position of the ant $i$ in the search space at the iteration $k$ is then updated based on;

$$ \sigma_{j} = \frac{\xi}{n_{arch}} \sum_{i=1, i \neq i_{sel}}^{n_{arch}} \left| x_{ij}^{k}(i_{sel},j) - x_{ij}^{k}(i,j) \right| \quad (12) $$

$$ x_{ij}^{k}(i,j) = x_{ij}^{k}(i_{sel},j) + \sigma_{j}/N(0,1) \quad (13) $$

where $\sigma_{j}$ is the standard deviation (SD) of the variable $j$, $N(0,1)$ is a normal PDF with mean of zero and SD of one; $\xi$ is the pheromone evaporation coefficient. $X_t$ is update over
iterations. It is also sorted in the descending order of feasibility and quality of the solutions.

In our ACO implementation, we have modified the original ACO\(_R\) to handle constrained optimization problems. The constraints are handled using adaptive penalty function approach [15]. This penalty function technique automatically adapts itself to the requirement of the population at different stages of the search. When there is no feasible solution in the population the penalized objective function is modeled to minimize constraint violations. If the population is in the optimal search region, the objective is to minimize the objective value as well as constraint violations. A prominent feature of such a technique is that the penalty term is self-adapted during the search process. Therefore, no parameter tuning is required. It is also flexible to any optimization problems without the need to restructure the constraint handling method.

### B. Implementing ACO for VSCORPD

Let \( x_a \) and \( x_c \) are any row vectors which are members of \( X_t \) and \( X_a \), respectively.

Step 0: Read the power system data and store ACO parameter settings.

Step 1: At time \( t \leq t_{\text{max}} \) (\( t_{\text{max}} \) is the number of time step in the optimization horizon), set demand and generation levels. Then, set the generation counter \( \text{gen} = 0 \).

Step 2: If \( \text{gen} = 0 \), randomly initialize \( X_t \) within the respective limits.

Step 3: For each \( x_t \), run power flow program to assess the corresponding state.

Step 4: For each \( x_t \), apply the inputs in (3) to the developed FFNN to estimate the VSM.

Step 5: Calculate the vector of constraint violations by subtracting (8)-(10) from the designated boundaries for every \( x_t \).

Step 6: Evaluate the penalized fitness values for \( X_t \).

Step 7: Sort \( X_t \) based on their feasibilities and fitness values.

Step 8: To move ants in the search space, set \( \text{gen} = \text{gen} + 1 \).

Step 9: For every \( x_a \), apply (11) to (13).

Step 10: Repeat steps 3 to 6 on \( X_a \).

Step 11: Find the generation (local) best \( x_{\text{local}} \) and global best \( x_{\text{global}} \) based on the following criteria:
   
   a) Any feasible solution is preferred to any infeasible solution;
   
   b) Between two feasible solutions, the one having better objective value is preferred;
   
   c) Between two infeasible solutions, the one having smaller fitness value (smaller constraint violation) is preferred.

Step 12: Store \( x_{\text{local}} \) and \( x_{\text{global}} \)

Step 13: Update the archive by replacing the worse \( x_t \) by the better \( x_t \) while keeping \( n_{\text{arch}} \) unchanged.

Step 12: If \( \text{gen} < \text{gen}_{\text{max}} \) go to step 8, else store \( x_{\text{global}} \) and go to step 1. If \( \text{gen} < \text{gen}_{\text{max}} \) and \( t = t_{\text{max}} \) go to step 13.

Step 13: Stop the VSCORPD

Two stopping criterions are set up. The first criterion is the maximum number of allowable generations. The second one is the condition when there is no improvement of solution quality in \( X_t \) during ten successive generations. The ACO generations at time \( t \) stops if one of them is met.

### VI. NUMERICAL RESULTS

The effectiveness of the proposed method for solving VSCORPD was tested on IEEE 30 bus system. The active and reactive load profile for a day with 15 minute sampling interval is shown in Fig. 6. The generators are assumed to supply the load with a fixed participation factor throughout the daily operation. All transformers are assumed to have 21 discrete tap sets between \( \pm 10\% \) and all capacitors have 13 discrete switching steps with different power ratings as given in Table A.II of the Appendix. There is one inductor connected to bus 22 and it is assumed to be a continuous variable. The upper and lower limit of bus voltages are 1.1 and 0.9 pu, respectively. Power flow over a line is restricted due to the thermal limit represented by 1 pu of the respective rating. It is the requirement that 6% of voltage stability margin has to be guaranteed at every operating state \( \text{VSM}_{\text{limit}} = 1.06 \). The unit operating cost of transformer of all transformers \( C_{\text{T}} \) is assumed to be equal to $10/change where all capacitors have the identical unit operating cost \( C_{\text{C}} \) of $3/change. The electricity price \( C_W \) is assumed to be 6 cent/kWh.

Fig. 6 Daily load profile (15-minute sampling interval)

To demonstrate effectiveness of the proposed algorithm, three control schemes namely ACO #1 to ACO #3 are set up where different objective functions are minimized whereby the same set of constraints listed from (5) to (10) is considered. The ACO #1 is the proposed model where the objective function is (4). In ACO #2, the traditional model of loss minimization is applied. The objective function of ACO #2 is the first term of (4). The number of discrete control actions is minimized in ACO #3 where the objective function can be
represented by the last two terms of (4). The ACO parameters are set up as follows: \( n_{arch} = 20, n_{ant} = 10; \) \( gen^{max} = 300. \)

Following the given demand and generation patterns, CPF analyses reveal that six operating conditions at 14:00, 14:30, 15:00, 15:30, 17:15 and 17:30 have VSMs less than the VSMlimit. All three control schemes ACO #1 to ACO #3 were applied to solve for the optimal control scheme at every operating time interval. VSMs based on the optimal control settings determined from each control scheme are shown in Fig. 7. It can be observed that all three control schemes are successful in improving the VSM to be greater than VSMlimit as it is shown that there is no operation condition with VSM less than 1.06.

The total operation times of transformer taps in 24 hours determined by each method are summarized in Table I. Similarly, Table II summarizes the total operation times of capacitors in 24 hours. It is not surprising that ACO #3 result in the fewest operation times because the cost of adjustment is only the objective function. The operation times found from ACO #1 are slightly greater than ACO #3 but still significantly less than ACO #2 in both cases of transformer and capacitors.

Due to the limitation, only some of optimal control settings of discrete control devices within 24 hour operation can be shown in Figs. 8 and 9. In Fig. 8, the optimal tap positions of transformers 1 and 4 are shown. Obviously, ACO #2 results in a relative larger number of device operations because the cost of adjustment is ignored. Such excessive operation significantly reduces the life time expectancy of the device. On the other hand, ACO #1 and ACO #3 can help reduce the number of operations by considering costs in the objective function. The similar observation can be obtained from the operation of two capacitors as shown in Fig. 9. It can be observed that capacitor settings change comparatively more frequently that transformer taps because of the cheaper unit cost of operation.
Based on the results discussed so far, it seems that ACO #3 outperforms the others. Costs due to energy losses ($C_{\text{loss}}$) over the period of 6 hours before and after three control schemes are shown in Fig. 10. It can be observed that in every time step $C_{\text{loss}}$ of ACO #3 is relatively equal or even higher than the $C_{\text{loss}}$ of the initial operating condition (before ACO). Interestingly, $C_{\text{loss}}$ of ACO #1 and ACO #2 are slightly different. The summation of $C_{\text{loss}}$ at each time step results in the cost of energy losses for a daily operation $\Sigma C_{\text{loss}}$. Table III gives $\Sigma C_{\text{loss}}$ before the optimization and results of all control schemes. Percentage of cost reduction with respect to the case without optimization is shown in the second row of Table III. ACO #1 and ACO #2 have nearly identical performance in reducing optimization is shown in the second row of Table III. ACO #1 and ACO #2 have nearly identical performance in reducing optimization problem with the objective to minimize energy losses and cost of adjusting control devices while maintaining the system security and voltage stability constraints. An ANN is trained offline by the dataset simulated to emulate probable operating conditions during the daily power system operation. The trained ANN is responsible for estimating the voltage stability margin (VSM) during the optimization process. The VSCORPD is solved by ACO. The proposed hybrid method demonstrates the ability to decrease cost of energy losses, avoid excessive operations of control devices and simultaneously provide the sufficient VSM. Further studies can be undertaken on ANN training to overcome difficulties of estimating VSM with the network topology changes. It would be also more interesting to investigate the performance of the proposed method with larger power systems.

### Table III

<table>
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<th>Before ACO</th>
<th>ACO #1</th>
<th>ACO #2</th>
<th>ACO #3</th>
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<td>4882.1512</td>
<td>4678.2525</td>
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<tr>
<td>%</td>
<td>-</td>
<td>+30.17</td>
<td>+33.08</td>
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**VII. Conclusion**

A hybrid method between artificial neural network (ANN) and ant colony optimization (ACO) has been successfully applied to solve the voltage stability constrained optimal reactive power dispatch (VSCORPD) problem. The VSCORPD was formulated as a nonlinear mixed integer optimization problem with the objective to minimize energy losses and cost of adjusting control devices while maintaining the system security and voltage stability constraints. An ANN is trained offline by the dataset simulated to emulate probable operating conditions during the daily power system operation. The trained ANN is responsible for estimating the voltage stability margin (VSM) during the optimization process. The VSCORPD is solved by ACO. The proposed hybrid method demonstrates the ability to decrease cost of energy losses, avoid excessive operations of control devices and simultaneously provide the sufficient VSM. Further studies can be undertaken on ANN training to overcome difficulties of estimating VSM with the network topology changes. It would be also more interesting to investigate the performance of the proposed method with larger power systems.

### Table IV

<table>
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<tr>
<th>CPU Time Used for Each Method (s)</th>
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<tr>
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<tr>
<td>ACO #2</td>
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<td>ACO #3</td>
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### References


Worawat Nakawiro received his B.Eng. degree in electrical engineering from Faculty of Engineering, Thammasat University, Thailand in 2002 and M.Eng. in electric power system management from Asian Institute of Technology, Thailand in 2004. He is currently a Ph.D. student at the University of Duisburg-Essen, Germany with financial support from DAAD. His research interests include computational intelligence with emphasis on heuristic optimization and its applications in power systems and voltage stability problems.

Istvan Erlich received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen/Germany. His major scientific interest is focused on power system stability and control, modeling and simulation of power system dynamics including intelligent system applications. He is a member of VDE and IEEE.