Unit Commitment under Wind Power and Demand Uncertainties

V.S. Pappala, I. Erlich, and S.N. Singh

Abstract—This paper addresses a multistage stochastic model for the optimal operation of wind farm, pumped storage and thermal power plants. The output of the wind farm and the electrical demand are considered as two independent stochastic processes. The evolution of these processes over time is modeled as a scenario tree. Considering all possible realizations of stochastic process, leads to a huge set of scenarios. These scenarios are reduced by a particle swarm optimization based scenario reduction algorithm. The scenario tree modeling transforms the cost model to a stochastic model. The stochastic model can be used to estimate the operation costs of the hybrid system under the influence of the uncertainties. The stochastic model is solved using adaptive particle swarm optimization.

Index Terms — Economic Model, Evolutionary Programming, Multi-stage Scenario tree, Particle Swarm Optimization, Random Process, Stochastic Programming

I. INTRODUCTION

THE drastic changes in environment and climate can be avoided by replacing fossil energy sources with clean and fuel free energy generation. The growing concern for environment has asked for rapid developments in wind power generation technology. Wind energy has a special importance in German energy planning. By 2020, 20% of the power consumed in Germany will be supplied by wind generation. Due to the stochastic nature of the wind, the output of the wind farm can not be predicted accurately. With the increased penetration of the wind energy, there will be huge fluctuation in the power generation. Therefore storage devices such as pumped storage are necessary. The pumped storage is used to level the mismatch between power generation and demand. They store the excess generation from wind farms and also the excess generation by the base load generation plants during off-peak periods for later use. This will enable efficient utilization of the base-load generation units and to smooth the peak loads. The pumped storage can also be used to provide reserve during off-peak period so that no other unit is committed just for providing the reserve.

This paper presents a particle swarm optimization approach for the optimal operation of the thermal, wind and pumped storage units under stochastic load and wind generation for 24 hours planning horizon. The objective is to utilize the total wind generation, smooth the peak loads and reduce the operation cost of the thermal power plants. Many researchers [1], [2] have already estimated the operating costs of such a hybrid system. But they have all considered the electrical demand and wind generation as deterministic quantities which are in real, stochastic. These cost models ignore the influence of these stochastic parameters. A minor change of these stochastic parameters leads to huge changes in operating cost of the power system and may lead to huge losses for the utility company. Hence there is a risk involved in using such models for planning and operation of the power system. In [3], [4] probabilistic model for wind power generation uncertainties were discussed. These models develop a cumulative distribution function (CDF) for each wind turbine generator (WTG) and then convolute these individual CDFs to obtain the output CDF of the wind farm. The main disadvantage of these models is their dependency on the individual WTGs. This paper models the stochastic nature of the wind power generation by the wind farm using scenario tree analysis.

The output power of the wind farm and the deviations of the electrical demand cannot be forecasted accurately. The error associated with the forecast of these parameters increases with time. Hence these parameters are considered as two independent random processes. The evolution of these random processes which represents the future realizations of the uncertainties is modeled as a suitable scenario tree [5]. So this tree gives the complete information of the uncertainties prevailing in the cost model. The better the scenario tree, the better will be the stochastic solution for the cost model. A new method is proposed to improve the quality of the scenario tree, to reduce the modeling error and also to improve the stochastic solution. The scenario tree is then embedded into the cost model which transforms it into a multistage nonlinear stochastic cost model. The aim of this stochastic model is to minimize the average operating costs over this scenario tree.

The main contributions of this paper include 1) significant improvements to the scenario reduction algorithm proposed in [6] 2) model the stochastic nature of the wind power generation of the wind farm using the forecast error and the confidence interval, 3) evaluate the effects of wind power and electrical demand on the operation cost of the power system.
II. UNCERTAINTY MODELING

In order to solve the cost model, the underlying stochastic electrical demand and wind generation have to be modeled in an appropriate form. The uncertainty is assumed to increase with time. The increase in uncertainty of electrical demand and wind generation are as shown in Fig. 1. The bold curve

delays in the uncertainty of electrical demand and wind generation have to be modeled in appropriate forms. The uncertainty is thus incorporated with a weight in the cost model.

This leads to a multi-stage scenario tree with 96 branching stages. The stochasticity of the uncertainties at each time stage is analyzed at every 15 minutes of a 24 hour planning horizon. Complete stochastic uncertainties have to be modeled for the entire search space until it finds the best set of scenarios. The reduction technique is formulated as an optimization problem and is solved using the particle swarm optimization method (PSO).

III. SCENARIO REDUCTION ALGORITHM USING PSO

Particle Swarm Optimization (PSO) is a population-based searching algorithm. It consists of a group of particles called swarm which explore the entire search space until it finds the global solution. The particle represents a solution to a given problem. The particles fly in the entire search space with a certain velocity in search of an optimal solution. In the process they try to refine their performance by interacting with its neighboring particles. The velocity of the particle is influenced by its previous velocity, its previous best performance and also by the performance of the best particle in the swarm. The particles trace the optimal solution by cooperation and competition among the particles.

For scenario reduction technique, the objective of the optimization is not to find the global optimal solution but to find a solution that maximizes the fitness of each particle. Here each particle represents a scenario. For a scenario tree with 96 branching stages, the particle consists of 96 dimension vector representing the random variable. For N_0 number of random variables, the dimension of the particle is 96*N_D. The process of scenario tree reduction using particle swarm optimization is as follows:

**Step 1: Generation of initial swarm**: The particles position x and velocity v are randomly initialized within the allowable range. Each dimension of the position vector is then approximated to its corresponding discrete point. The vector representing the best performance of the particle (pbest) is initialized to the current position x.

**Step 2: Evaluate node probabilities**: Each node of the scenario tree has five successor nodes. If any of these successors are missing then the probability of the missing successor is added to its nearest neighbor.

**Step 3: Evaluate the fitness**: The fitness of each particle is the minimum weighted Euclidean distance from the other particles in the swarm.

\[
\text{fitness} \left( \text{particle}^m \right) = \min_{m 
eq p} \left\{ \frac{1}{N_{D}} \sum_{i=1}^{N_{D}} \left( \frac{\sum_{j=1}^{N_{T}} (A_{i,j}^m - A_{i,j}^p)^2}{N_{i}} \right)^{\frac{1}{2}} \right\}
\]

Where N_p is the total number of particles in the swarm, N_T is the total number of branching stages, N_D is the number of random variables, \(\pi_m\) is the total Probability of particle “m”, N_i is the normalizing factor for the distance corresponding to random variable i and A_{i,j}^m is the wind power output and electrical demand corresponding to particle m at stage j for i=1 and 2 respectively. For instance, if the swarm has six particles of equal probability as in Fig. 2, the fitness of particle 1 is the

![Fig. 1. The evolution of electrical demand and wind generation uncertainty with time.](image-url)
shortest distance with the remaining five particles in the swarm. Since particle 4 happens to be the nearest neighbor to particle 1, the fitness of particle 1 is \(d_{14}\). If the fitness value is better than \(p_{best}\), then \(p_{best}\) is replaced with the current value.

Fig. 2. Euclidean distance of particle 1 with the other particles in the swarm.

**Step 4:** Update the searching point of each particle: The nearest neighbor \(g_{min}\) of each particle is identified. The particle’s new velocity and position are evaluated by using the update equations (2) and (3). The standard PSO update equations are slightly modified as in (2) so that the particles move apart from each other to increase their individual fitness.

\[
v_{jk}(t+1) = \chi(wv_{jk}(t)+c_1r_1(p_{best_k}-x_{jk})-c_2r_2(g_{min_k}-x_{jk}))
\]

(2)

\[
x_{jk}(t+1) = x_{jk}(t) + v_{jk}(t+1)
\]

(3)

The particle’s velocity is guided by its previous velocity \((v_{jk})\), its previous best performance \((p_{best_k})\) and the performance of its nearest neighbor. The particle moves in a direction opposite to its nearest neighbor \((g_{min_k})\) so as to improve its fitness. If the distance from the nearest neighbor happens to be zero then the position of the particle is randomly initialized. The calculated velocity is then added to the current position to obtain the new position for the particle.

**Step 5:** Check for termination condition: The termination criterion is usually the maximum iteration number. If the condition is satisfied then exit else go to step 2.

During the optimization process the particles traverse the whole search space to obtain an optimal fitness value. Since the fitness value is the distance with the other particles, the particles fly so as to move farther away from each other. Towards the end of the process, the particles are scattered in the whole search space with significant distance from its neighbors. Since the particles represent the scenarios, we are left with only the distinct scenarios. The initial scenario tree with 5^{56} scenarios is reduced to 10 scenarios as shown in Fig. 3.

**IV. COST MODEL**

The hybrid system consists of thermal power plants, pump storage devices, nuclear power plant and a wind farm.

**4.1 Thermal Unit**

The fuel cost \([9]\) of unit \(i\) is a quadratic function of the generator output power:

\[
FC_i = a_i + b_i p_i + c_i p_i^2
\]

(4)

\[
SCI = \sigma_i + \delta_i \left(1 - \exp\left(-\frac{T_{off,i}}{\tau_i}\right)\right)
\]

(5)

Where \(a_i, b_i, c_i\) represent the cost coefficients. The generator start-up cost depends on the time the unit has been off prior to start up. The start-up cost is given by the following exponential cost curve:

The unit constraints include:
- The minimum and maximum rated unit capacities
- Ramp rates
- Minimum up/down time limits of the units
- The initial states of the units must be taken into account

**4.1 Pump Storage**

The constraints for pump storage \([10]\) are as follows:
- Pond level dynamics

\[
PL(t+1) = PL(t) - PD(t) m \eta \rho
\]

(6)

\[
PL(t+1) = PL(t) - PD(t) m \eta \rho
\]

(7)

- Pond level limits

\[
PL_{min} \leq PL(t) \leq PL_{max}
\]

(8)

- Initial and final pond level

\[
PL(0) = PL_0
\]

(9)

\[
PL(T) = PL_f
\]

(10)

- Generation and pumping level constraints

\[
PD_{min} \leq PD(t) \leq PD_{max}
\]

(11)
where \( PL(t) \) and \( PL(t+1) \) are the pond level at the beginning and the end of interval \( t \) respectively, \( PD(t) \) is the power generated (positive value) or power used for pumping (negative value) by pumped storage unit at time \( t \), \( \eta_p \) and \( \eta_g \) are the pumping and generation efficiencies respectively, \( PL_{min} \) and \( PL_{max} \) are the minimum and maximum pond level limits, \( PL_0 \) and \( PL_f \) are initial and final pond levels, \( PD_{g_{min}} \) and \( PD_{g_{max}} \) are the minimum and maximum generation levels of pump storage unit.

The overall objective function is to minimize the operation cost (OC) of the thermal units.

\[
OC = \sum_{i=1}^{T} \sum_{t=1}^{4} (FC_{i,t}u_{i,t} + SC_{i,t}u_{i,t}^2 (1-u_{i,t-1}))m
\]  

(13)

Apart from the unit constraints the cost model is subjected to a set of system constraints as shown below:

- System hourly power balance. Total power generation must equal the load demand, \( P_D \), in all time steps

\[
\sum_{i=1}^{I} p_{i,t} + PD(t) + WG(t) = P_D(t) \quad \forall t \in T
\]  

(14)

- Spinning reserve requirements \( R \) at each time step must be met.

\[
\sum_{i=1}^{I} P_{i,t}^{max} u_{i,t} + PL(t+1) \cdot \eta_g / m \geq P_D(t) + R(t) \quad \forall t \in T
\]  

(15)

Where \( u_{i,t} \) is the commitment of unit \( i \) at time \( t \), \( WG(t) \) is the wind power generation at time \( t \) and \( m \) is the time step duration.

V. STOCHASTIC COST MODEL

Once the uncertainties are modeled as a scenario tree, the cost model has to be transformed to a stochastic multistage model [11]. The scenario tree formulation of the objective function is as shown below:

\[
\min \sum_{m=1}^{4} \sum_{n=T}^{N} \left\{ FC_{i,n} u_{i,n}^p + SC_{i,n} u_{i,n}^p (1-u_{i,n-1}) \right\}
\]  

(16)

Where \( n \) corresponds to a node of the scenario tree, \( \pi_n \) is the node probability. The unit and system constraints are also transformed into a similar structure. The resulting stochastic model is solved using stochastic programming approach. The objective of the multistage stochastic optimization process is to minimize the expectation of the daily operating cost (OC) subjected to the unit and system constraints. An adaptive particle swarm optimization (APSO) algorithm [12] is used for solving the stochastic model. No parameter tuning is required for this algorithm. It can find the best parameters on its own and hence is independent of the problem. The swarm size is also adaptive. The optimal number of particles required to explore the search space is determined by the algorithm itself. Hence, few iterations are required for the optimization. The algorithm is therefore fast and robust.

VI. RESULTS

The cost model consists of four thermal power plants (2*1000MW+2*500MW), one nuclear power plant (5000MW) serving as the base load generation, wind farm (326 MW) and six pump storage (PS) units (pump:6*190MW, Generation:6*175MW) serving a mean load of 6523MW. The objective of the optimization problem is to determine the commitment, the start-up, shutdown times and the power output levels of all the units at each time step of 15 minutes, over a scheduling period of 24 hours, so that the total operating costs are minimized subjected to system and unit operating constraints. The cost of nuclear power is assumed to be 0.4 cents/kWh. The cost model is solved using adaptive particle swarm optimization. The demand curves and wind power generation data is obtained from [12]. The reserve capacity is assumed to be 5% of the electrical demand. The cost coefficients for the 1000MW unit are \( a_i = 1500 \), \( b_i = 27.74 \), \( c_i = 0.00712 \), \( \sigma_i = 7500 \), \( \delta_i = 7500 \), \( \tau_i = 10 \) and for 500MW unit are \( a_i = 750 \), \( b_i = 39.1 \), \( c_i = 0.0097 \), \( \sigma_i = 5000 \), \( \delta_i = 5500 \), \( \tau_i = 5 \). The pumping and generation efficiency of the pumped storage is assumed to be 0.8448 and 0.871 respectively.

The results of the deterministic optimization are shown in Fig. 4, 5 and Fig. 6. Fig. 4(a) shows the power generated by the wind farm. Fig. 4(b) shows the scheduling of the pump storage units. In Fig. 4(c) the peak electrical demand observed to be from 10:00a.m. to 2:00p.m. amounts to a maximum of 7800MW. The use of pump storage reduces this peak demand. This is shown by the curve "electrical demand with PS". The power used by the pump storage is added to the electrical demand to account for the net electrical demand of the power system. When the pump storage operates in generation mode, it is supplying a part of the demand whereas when it is in pumping mode, it adds an additional demand which has to be supplied by the conventional units. During the peak hours the pump storage operates in generation mode to reduce the peak demand which amount to a maximum of 7500MW. Fig. 4(d) depicts the power delivered by the thermal and nuclear units. The presence of pump storage and wind farm in the power system reduces the net power during the peak hours to be supplied by the conventional units by nearly 8%. The pumping and generation modes of the pump storage are optimally controlled by APSO so as to reduce the peaks in the demand distribution. Usually, the power system operator has to switch on additional units to supply the peak demand for a few hours and then run these units in minimum generation levels to satisfy the minimum up time constraint. The use of pump storage eliminates this additional switching of the units and hence reduces the operation cost of the power system. In Fig. 5 the reserve capacity and the contribution by the pump unit and thermal plant are shown. During the peak load period, no additional thermal unit is switched on only to supply the reserve. The pump storage unit also supports the system by supplying reserve power. This reduces the operating costs of...
the whole system. Fig. 6 shows the generation by the four thermal units. Unit1 (1000MW) is always “on” to supply the load whereas the other units are optimally scheduled to meet the demand and reserve constraints. Unit3 and unit4 each of 500MW capacity have high fuel costs compared to the larger units. Hence these units have to be optimally utilized to reduce the overall operation costs. This ensures that unit3 to be “off” all the time and unit4 is used only for 14 hours. The overall operating cost amounts to 1,546,701euro. These results correspond to the deterministic cost model. The effect of uncertainties on the cost model can only be realized by using the stochastic model. Fifty scenarios are considered for solving the stochastic cost model. The operating costs of the stochastic model with a common unit commitment schedule for the whole set of scenarios amounts to 1,807,717 euro. The solution provided by the deterministic model is optimal only to a particular scenario and is not optimal for the scenario that may actually occur. The solution obtained by the stochastic model is optimal over all the possible scenarios. The stochastic solution may not be a global optimal solution to the individual scenarios but it a robust solution over all the scenarios. Hence the expectation of the operating costs corresponding to all these scenarios is high compared to the deterministic cost model. However the risk involved in using this model for the operation and planning is quite low and is therefore more preferred than the deterministic model. The unit commitment schedule for the four generating units for deterministic and stochastic model is given in Table I and Table II respectively. By defining a common unit commitment schedule for all possible realizations of the uncertainties will help the power system operator to decide the operation of the generators one day in advance irrespective of the evolution of

### TABLE I

**UC SCHEDULE FOR DETERMINISTIC MODEL**

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### TABLE II

**UC SCHEDULE FOR STOCHASTIC MODEL**

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the uncertainties. This enables better planning and operation of the power system.

VII. CONCLUSION

This paper presented a solution for a day-ahead operation of a system with thermal, nuclear, wind and pump storage units considering the demand and wind generation uncertainties. A new method for modeling the uncertainties in the cost model has been successfully implemented using particle swarm optimization. The improved update equations will enhance PSO to generate better quality scenario trees.

The nonlinear mixed integer multistage stochastic cost model was solved using the adaptive particle swarm optimization. The robust solution provided by APSO will enable power system operator to plan the operation of the power system under the influence of demand and wind generation uncertainties. Although the costs involved with the resulting stochastic model happens to be high, this model provides low risk scheduling solution to the power system.

VIII. REFERENCES

[11] V.S Pappala, M.Wilch, S.N.Singh, I.Erlich (2007): Reactive power management in offshore wind farms by adaptive PSO, 14th International conference on Intelligent system application to power system, Kaohsiung, Taiwan. This paper has won the best paper award

IX. BIOGRAPHIES

Venkata Swaroop Pappala (1981) received the B.E degree in electrical engineering from Faculty of Electrical engineering, S.V.H College of engineering, India in 2002, and M.Sc. degree in electrical engineering with emphasis on power and automation from University Duisburg-Essen, Germany in 2005. He is currently a Ph.D. student at the University of Duisburg-Essen, Germany. His research interests include stochastic optimization under uncertainty using evolutionary algorithms.

Istvan Erlich (1953) received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen/Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is a member of VDE and senior member of IEEE.

S. N. Singh (SM’02) received the Ph.D. degree in Electrical Engineering from the Indian Institute of Technology Kanpur, India, in 1995. Currently, he is an Associate Professor in the Department of Electrical Engineering at the Indian Institute of Technology Kanpur. He was with the Department of Electrical Engineering at the Hong Kong Polytechnic University, Hong Kong, China, and was Assistant Professor with the Energy Program at Asian Institute of Technology (AIT) Bangkook, Thailand. His research interests include power system restructuring, flexible ac transmission systems (FACTS), power system optimization and control, security analysis, and power system planning. He has received several awards including Young Engineer Award 2000 of Indian National Academy of Engineering, Khosala Research Award, and Young Engineer Award of CBIP New Delhi (India). Dr. Singh is a Fellow, Institution of Engineers (India), Fellow IETE (India), senior member IEEE and member IET (UK).