Magnetic Vortex Dynamics in Spherical Objects

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Abstract— Topological magnetic monopoles, also known as Bloch points, are 3D non-local spin textures and have been recently experimentally observed in chiral magnets, vortex rings and vortex cores [1]. Switching of magnetic vortex cores involves a topological transition characterized by the presence of the Bloch points. The goals of this work are: a) to study the modes excited inside a non-homogeneously magnetized sphere and investigate scattering of linearly or circularly polarized fields; b) to use the Bloch point to modify the scattering fields. We use the scattered field as a spectroscopic probe of the magnetic state and monitor the polarization of the scattered wave.

To study this problem numerically, we apply the rigorous coupled wave approach taking into account the exchange energy between the magnetic dipoles, the Landau and the demagnetization energies. We have already successfully applied this method to dispersion characteristics in multiferroic heterostructures [2]. Unlike the previous study [2], the present structure has a spherical symmetry [3]. Firstly, linearizing an equation of motion for the magnetic dipoles in the external magnetic field, we derive an expression of the magnetic permeability tensor, which is a function of the spherical angle θ and a distance from the center of the sphere and the observation point. In our analysis we neglect field dependence on the azimuthal ϕ angle. Next, we include the expression for the magnetic permeability into Maxwell equations and derive a closed system of the first-order differential equations in the spherical coordinate system. The sphere is divided into several thin shells and the fields in each shell inside the inhomogeneous region (i.e., in the region with the magnetic monopoles), are expanded into the eigenvectors and the eigenvalues of the matrix constructed from the Fourier components of the elements of the first-order nonhomogeneous differential equation. The fields in the homogeneous regions (outside the sphere in the form of the outgoing wave, or inside the sphere around the origin in the from of the standing wave) are expressed through the Schelkunoff spherical Bessel and Hankel functions, respectively. Applying the boundary conditions for the ϕ and θ components of the fields as between the boundaries of thin spherical layers, as well as between the sphere and the surrounding homogeneous region, the reflection and transmission matrices are rigorously derived and the fields in each region are calculated with high precision.

ACKNOWLEDGMENT

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3. Jarem, J. M., "Rigorous coupled-wave-theory analysis of dipole scattering from a three-dimensional, inhomogeneous, spherical dielectric, and permeable system," *IEEE Trans. Microw. Theory Tech.*, Vol. 45, 1193–1203, 1997.

Magnetic Vortex Dynamics in Spherical Objects

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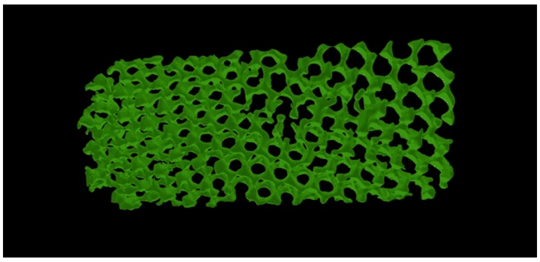
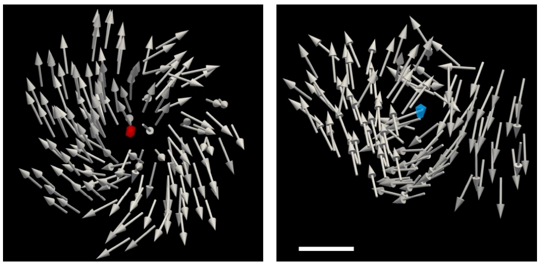
INTRODUCTION (1)





nature nanotechnology

Letter https://doi.org/10.1038/s41565-022-01311-0

Three-dimensional topological magnetic monopoles and their interactions in a ferromagnetic meta-lattice

Topological Magnetic Monopoles, known as Bloch points (BP), are 3-D spin textures. We may call the Bloch points topological point defects with vanishing local magnetization.
In the animation stable magnetic texture in ferromagnetic meta-lattice at room temperature is reported.

BP is formed in **3D magnetic textures** such as **vortex cores** in confined geometries (like nanospheres or nanowires).

Material is synthesized of silicon nanospheres embedded in nickel (usually Permalloy – nickel-iron magnetic alloy is used). It has **high magnetic permeability** and **low coercivity**, which means domain walls and non-uniform structures (like vortices) can form easily.

The design and experimental realization of the magnetic textures is well established but have not been yet exploited for photonic studies.
To drive a controllably BP is challenging, mainly due to their vanishing magnetization.

INTRODUCTION (2)

Bloch-point (BP) singularity can be viewed as a magnetic monopole and can be surrounded by a hedgehog-type structure (figure left) or other types of non-collinearities.
BPs are ***hedgehog***s types, when they are surrounded by magnetization vectors that are oriented outward and ***twisted*** type. The Bloch point is **sub-nanometer scale** — roughly the **exchange length** (a few nm or less).

On the microscopic level the formation of BP is dominated by the exchange interaction and hence, the magnetic response should be much faster than the typical GHz magnonic response in homogeneous magnets.

So far, functionalizing topological spin texture and spin dynamics for photonics has not been addressed. We believe that the reported novel findings will trigger immediate research activities both in the field of spintronics and nanophotonics [1].

[1] V. Jandieri, R. Khomeriki, D. Erni, N. Tsagareli, Q. Li, D. H. Werner, and J. Berakdar, "*Photonics of topological magnetic textures*" (submitted).

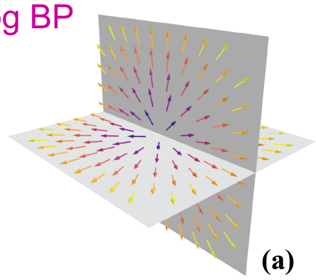
INTRODUCTION (3)

We will set up a coupled, self-consistent magnetic-photonic scheme for the case involving a large photon number (classical EM fields) and the length (energy) scales of relevance are much larger (smaller) than the atomistic ones.
Under these conditions it is reasonable to work within *classical field theories* and incorporate the atomistic electronic information, as usual in a dielectric/permeability tensors.

Using the developed formalism, which is very general, we study the scattering of structured electromagnetic (EM) waves, for example with a polarization distribution which is spatially structured.

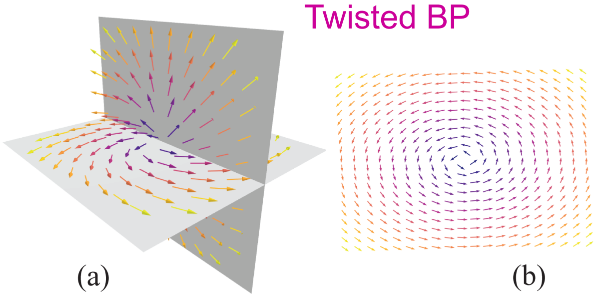
An important issue to clarify is how the singular magnetic texture responds to external EM fields and how the fields are modified by the magnetic texture, when the EM fields frequencies are in the range of magnetic excitations.

Hedgehog BP



(a)

Twisted BP



(a) (b)

Formulation of the Problem (1)

The total magnetic field originates from the external magnetic field $\boldsymbol{H_{ext}}$ and the demagnetizing field $\boldsymbol{H}$ caused by the spontaneous magnetization. The total magnetization can be written as a sum of the spontaneous magnetization $\boldsymbol{M}$ and the magnetization $\boldsymbol{M_H}$ induced by the magnetic field $\boldsymbol{H_{tot}}$, i.e. $\boldsymbol{M_H} = \chi \boldsymbol{H}$.

Hence, for the magnetic flux density (similar as for ferroelectrics [1]):

$$\mathbf{B}(\mathbf{r}) = \mu_0 \boldsymbol{H_{tot}}(\mathbf{r}) + \mu_0 \boldsymbol{M}(\mathbf{r}) + \mu_0 \cancel{\chi \boldsymbol{H_{tot}}}(\mathbf{r}) = \mu \boldsymbol{H_{tot}}(\mathbf{r}) + \mu_0 \boldsymbol{M}(\mathbf{r})$$

Free energy term U(**M**) as a function of **M**:

$$U(\mathbf{M}) = \int_V dv \left[\underbrace{\frac{A_s}{2}(\nabla \mathbf{M})^2}_{\text{Exchange energy}} + (\underbrace{aM^2 + bM^4}_{\text{Landau energy density}}) - \frac{\mu_0}{2} \underbrace{\mathbf{M}\cdot(\mathbf{H}+\mathbf{H}_{ext})}_{\text{Demag. energy}} \right] \qquad a(T) < 0$$

$$\frac{dU(\mathbf{M})}{d\mathbf{M}} = \mathbf{H}_{eff}$$

$$\Downarrow$$

*Even though the BP is nanoscale, the **excitation frequency** is only a few–tens of GHz (the frequency is defined by **effective magnetic field** driving its dynamics).*

$$\mathbf{H}_{eff} = \mathbf{H} + \mathbf{H}_{ext} + \mathbf{m}(\theta)\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2}{r^2}\right)M(r) + 2\nu \mathbf{m}(\theta)\left[M(r) - \beta M^3(r)\right]$$

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THE EUROPEAN PHYSICAL JOURNAL B

Regular Article

Magnetization structure of a Bloch point singularity

[1] R. Khomeriki, V. Jandieri, K. Watanabe, D. Erni, D. H. Werner, M. Alexe and J. Berakdar, "Photonic Ferroelectric Vortex Lattice," *Physical Review B, vol.* 109, 045428, 2024.

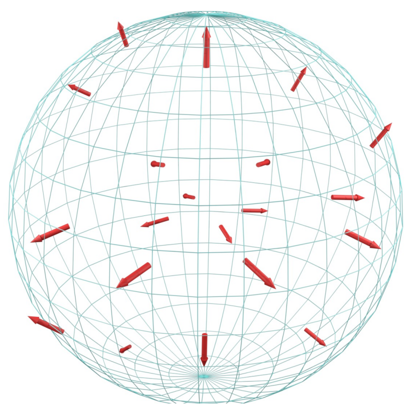
Unit vector components of magnetic moment ***m***:

$$m_x = \cos(\varphi+\gamma)\sin\theta$$
$$m_y = \sin(\varphi+\gamma)\sin\theta$$
$$m_z = \cos\theta$$

$\Downarrow$ *γ is a local rotation angle*

$$m_r = \sin^2\theta\cos\gamma + \cos^2\theta = \frac{1}{2}(1+\cos\gamma) + \frac{1}{2}(1-\cos\gamma)\cos 2\theta$$
$$m_\theta = \cos\theta\sin\theta(\cos\gamma - 1) = -\frac{1}{2}\sin 2\theta(1-\cos\gamma)$$
$$m_\varphi = \sin\gamma\sin\theta$$

$\gamma \neq 0$ - Twisted Bloch point; $\gamma = 0$ - Hedgehog Bloch point.



$\mathbf{M}_0(\mathrm{r},\theta)$

Magnetized sphere with dipole moments $\mathbf{M}_0$(r,θ) (our model).

Free energy term U(**M**) as a function of **M**:

$$U(\mathbf{M}) = \mu_0 \int_V dv \left[\frac{\ell^2}{2}(\nabla\mathbf{M})^2 + \nu \underbrace{\left(-M^2 + \frac{\beta}{2}M^4\right)}_{f(\mathbf{M})} - \frac{1}{2}\mathbf{M}\cdot(\mathbf{H}+\mathbf{H}_{ext}) \right]$$

The equilibrium distribution of the magnetic potential is determined from the variation of the free energy U(**M**) with respect to the magnetization vector:

$$\boxed{-\Delta\mathbf{M} + \nu\frac{\partial f(\mathbf{M})}{\partial \mathbf{M}} - \mathbf{H}_{eff} = 0}$$

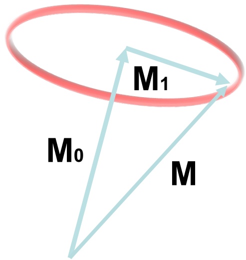
In the spherical coordinate system: $\Delta\mathbf{M} = \mathbf{m}(\theta)\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\right)M(r)$

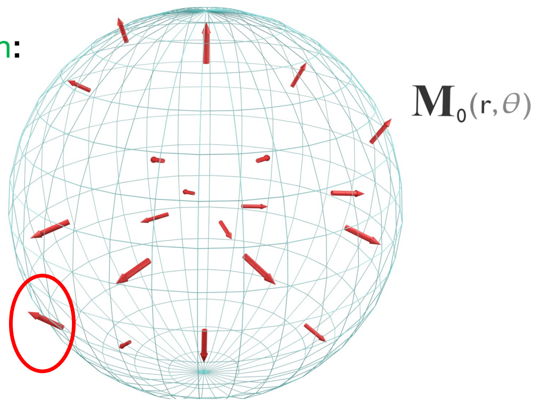
Linearization (seek solution as φ – **independent**)
Normalization by max. of saturation magnetization:

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_1$$

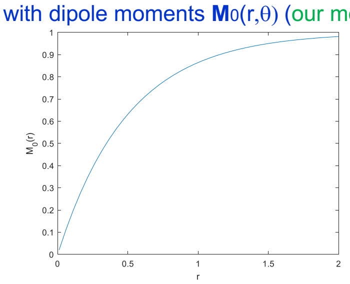
$\mathbf{M}_0 = M_0(r)\mathbf{m}_0(\theta)$ - 0th approximation

$\mathbf{M}_1 = M_0(r)\mathbf{m}_1(\theta)$ - 1st approximation





Magnetized sphere with dipole moments $\mathbf{M}_0(r,\theta)$ (our model).



Initial are equation of motion (Landau-Lifshitz-Gilbert equation) coupled with the Maxwell equation:

$$\frac{\partial \mathbf{m}_1}{\partial t} = -g\mu_0 M_s\left[\mathbf{m}_0 \times \mathbf{H}_{eff}\right]$$

$$\mathbf{H}_{eff} = \mathbf{H}_{ext} + \mathbf{H} + \left[\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)M_0(r) + 2\nu\left(M_0(r) - M_0^3(r) - \frac{2}{r^2}M_0(r)\right)\right]\mathbf{m}_1(\theta)$$

$$\nabla \times (\mathbf{H} + \mathbf{H}_{ext}) = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

V. Jandieri, R. Khomeriki, L. Chotorlishvili, K. Watanabe, D. Erni, D.H. Werner and J. Berakdar, "Photonic Signatures of Spin-Driven Ferroelectricity in Multiferroic Dielectric Oxides," *Physical Review Letters,* vol. 127, 127601, 2021.

Magnetic permeability tensor is rigorously derived:

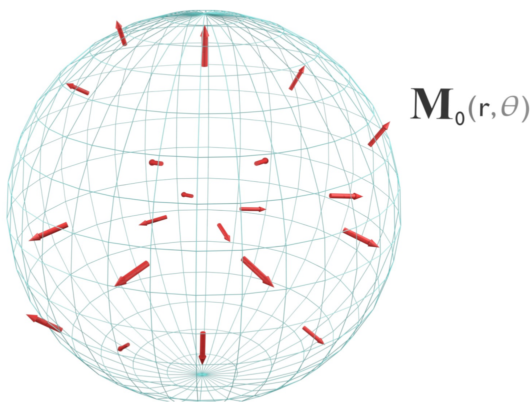
$$\mathbf{m}_{1,i} = \bar{\bar{\mu}}_{ij}(\theta, r)\mathbf{H}_j; \qquad \mathbf{m}_{1,i} = (m_r, m_\theta, m_\varphi)^T$$

$$\bar{\bar{\mu}}_{ij}(\theta, r) = \begin{pmatrix} \mu_{11}(\theta, r) & \mu_{12}(\theta, r) & \mu_{13}(\theta, r) \\ [\mu_{12}(\theta, r)]^* & \mu_{22}(\theta, r) & \mu_{23}(\theta, r) \\ [\mu_{13}(\theta, r)]^* & [\mu_{23}(\theta, r)]^* & \mu_{33}(\theta, r) \end{pmatrix}$$

As an example, only *two components* of the permeability tensor are presented:

$$\mu_{11}(\theta, r) = 1 + \frac{\left(\frac{g\mu_0 M_s}{\omega}\right)^2\left([m_\theta^0]^2 + [m_\varphi^0]^2\right)\left[\hat{O}M_0(r)\right]}{1 - \left(\frac{g\mu_0 M_s}{\omega}\right)^2\left[\hat{O}M_0(r)\right]^2}$$

$$\mu_{12}(\theta, r) = \frac{-\left(\frac{g\mu_0 M_s}{\omega}\right)^2 m_r^0 m_\theta^0\left[\hat{O}M_0(r)\right] - j\left(\frac{g\mu_0 M_s}{\omega}\right)m_\varphi^0}{1 - \left(\frac{g\mu_0 M_s}{\omega}\right)^2\left[\hat{O}M_0(r)\right]^2}$$



Magnetized sphere with dipole moments $\mathbf{M}_0(r,\theta)$ (our model).

$$\hat{O} = \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\right)^2 + 2\nu(1 - M_0^2)$$

[1] V. Jandieri, R. Khomeriki, D. Erni, N. Tsagareli, Q. Li, D. H. Werner, and J. Berakdar, "*Photonics of topological magnetic textures*" (submitted).

Field components in the spherical coordinate system:

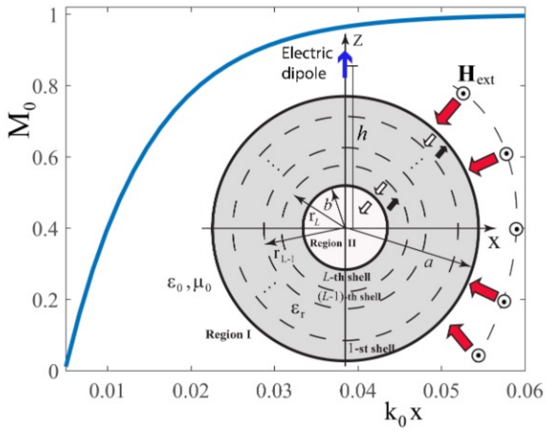
$$S_\theta = rE_\theta, \quad S_\phi = r\sin\theta E_\phi$$

$$U_\theta = \sqrt{\frac{\mu_0}{\varepsilon_0}} rH_\theta, \quad U_\phi = \sqrt{\frac{\mu_0}{\varepsilon_0}} r\sin\theta H_\phi$$

Fourier representation of the fields:

$$\Psi(r,\theta) = \sum_{p=-N}^{N} \Psi_p(r) e^{jp\pi\kappa}$$

$$\Psi = \{S_\theta, S_\varphi, U_\theta, U_\varphi\}$$



Schematic 2-D view. Numerical model of our problem

$$\boxed{\frac{d}{dr}\begin{pmatrix} \mathbf{s}_\theta \\ \mathbf{s}_\varphi \\ \mathbf{u}_\theta \\ \mathbf{u}_\varphi \end{pmatrix} = jk_0[\mathbf{\Phi}]\cdot\begin{pmatrix} \mathbf{s}_\theta \\ \mathbf{s}_\varphi \\ \mathbf{u}_\theta \\ \mathbf{u}_\varphi \end{pmatrix} = jk_0\begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{pmatrix}\cdot\begin{pmatrix} \mathbf{s}_\theta \\ \mathbf{s}_\varphi \\ \mathbf{u}_\theta \\ \mathbf{u}_\varphi \end{pmatrix}}$$

A linear system is applied only to inhomogeneous region.

Φ matrix components are expressed through the Fourier coefficients (Toeplitz matrices).

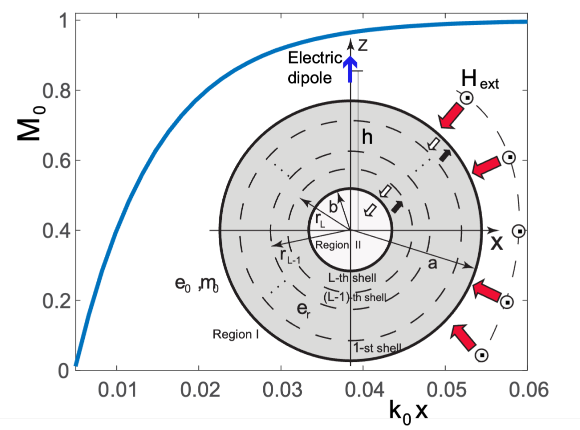
J. M. Jarem, "Rigorous Coupled-Wave-Theory Analysis of Dipole Scattering from a Three-Dimensional, Inhomogeneous, Spherical Dielectric, and Permeable System," IEEE Trans. Microw. Theory Tech., 45, 1997, pp. 1193–1203.

Some Important Remarks about the Developed Method

1) Spherical structure $e^{ji\pi\kappa}$ is analogous to the $e^{jk_x x}$ Floquet harmonic *x*-propagation factor used in the planar diffraction analysis [1] .

2) The inhomogeneous region is divided into a number of ***thin*** *radial layers* (produces in each layer a set of State Variable Equations, whose matrices are constant with respect to the radial coordinate) [1, 2].

3) Eigenmatrix analysis is used to solve the State Variable Equations in each radial layer for the *inward* and *outward* propagating and non-propagating (evanescent) EM eigenmodes.



Schematic 2-D view. Numerical model of our problem

Incident field is decomposed into TE*r* and TM*r* :

$$S_\theta^{inc}(r,\kappa) = j\,\hat{j}_1'(k_0 r)\sum_{i=-N}^{N} S_{\theta,i}^{inc}\, e^{ji\pi\kappa} \qquad U_\phi^{inc}(r,\kappa) = \hat{j}_1(k_0 r)\sum_{i=-N}^{N} U_{\phi,i}^{inc}\, e^{ji\pi\kappa}$$

$$S_{\theta,i}^{inc} = \frac{1}{2}\int_{-1}^{1} d\kappa\sqrt{1-\kappa^2}\,e^{-ji\pi\kappa} \qquad U_{\phi,i}^{inc} = \frac{1}{2}\int_{-1}^{1} d\kappa(1-\kappa^2)e^{-ji\pi\kappa}$$

Fields in homogeneous region (an incident field region) are expressed by Schelkunoff Bessel and Hankel functions.

[1] V. Jandieri, R. Khomeriki, L. Chotorlishvili, K. Watanabe, D. Erni, D.H. Werner and J. Berakdar, "Photonic Signatures of Spin-Driven Ferroelectricity in Multiferroic Dielectric Oxides," *Physical Review Letters,* vol. 127, 127601, 2021.

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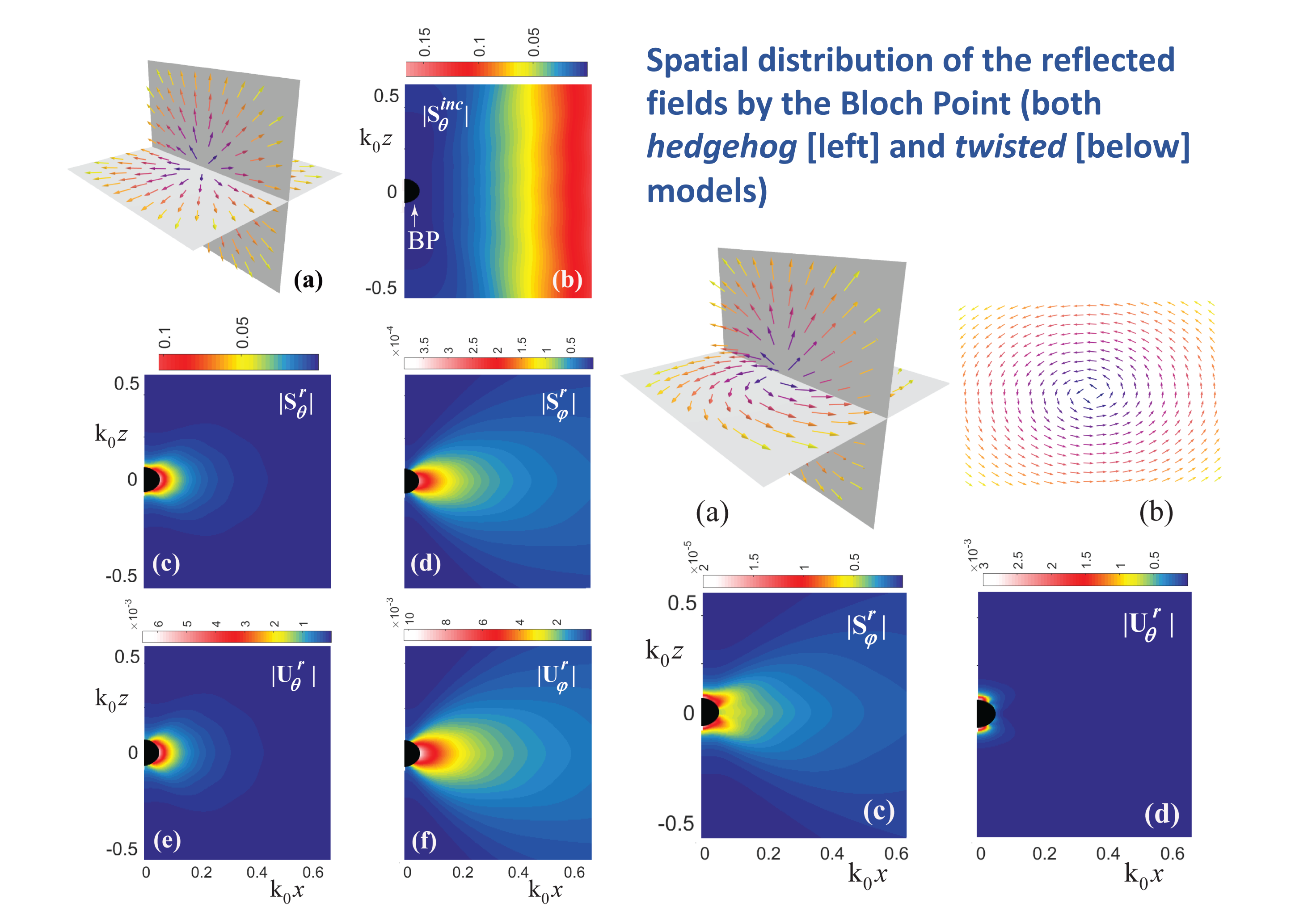

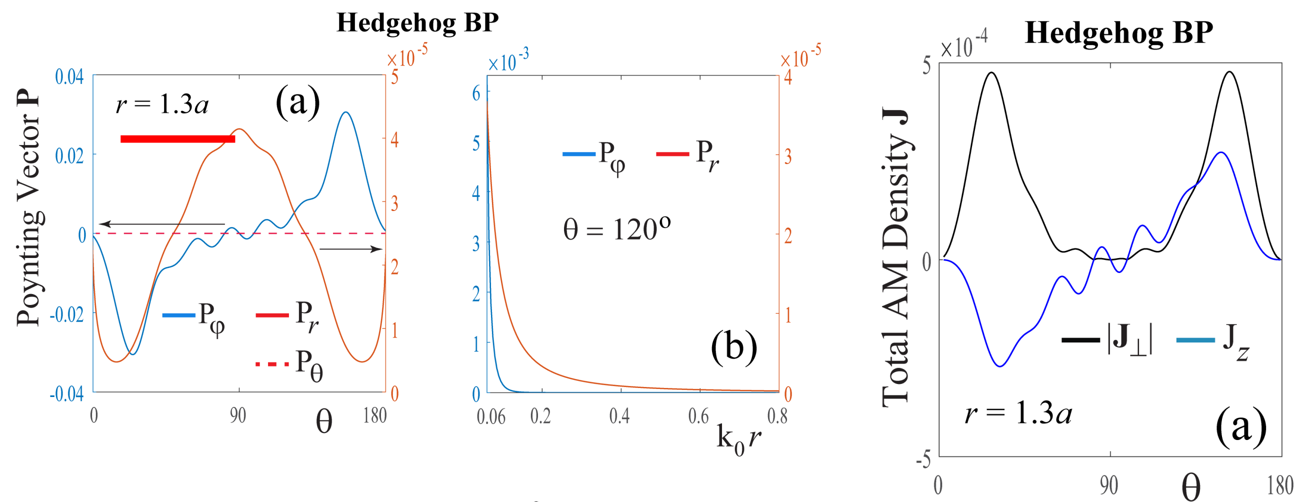

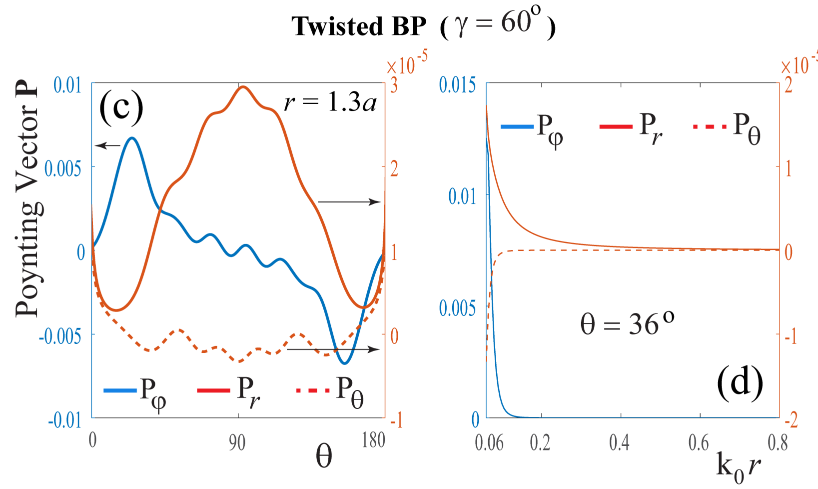

$$P_r = \frac{1}{2\eta^2\sqrt{1-\kappa^2}}\Re(S_\theta U_\varphi^* - S_\varphi U_\theta^*),$$

$$P_\theta = \frac{1}{2\eta^3\sqrt{1-\kappa^2}}\Re\left(\frac{U_\varphi^*}{j}\frac{\partial U_\varphi}{\partial\kappa} + S_\varphi\left(\frac{1}{j}\frac{\partial S_\varphi}{\partial\kappa}\right)^*\right),$$

$$P_\varphi = \frac{1}{2\eta^3\sqrt{1-\kappa^2}}\Re\left(-\frac{U_\theta^*}{j}\frac{\partial U_\varphi}{\partial\kappa} - S_\theta\left(\frac{1}{j}\frac{\partial S_\varphi}{\partial\kappa}\right)^*\right)$$

$$|\boldsymbol{J}_\perp| = \sqrt{\kappa^2 J_\theta^2 + J_\varphi^2}, \quad J_z = -\sqrt{1-\kappa^2}J_\theta$$

with $J_\theta = -rP_\varphi$ and $J_\varphi = rP_\theta$

Concluding Remarks

The work presents an efficient and self-consistent approach for the analysis of EM wave scattering by hedgehog and twisted BPs. The goal was to investigate how non-collinear magnetic textures respond to EM fields in the frequency range of magnetic excitations and how to use BPs as photonic elements that modify the scattered (reflected) fields.

The formalism is general and can be applied to various excitation sources and spin configurations apart from BPs. Special attention is paid to field components generated solely by an interaction of EM waves and BPs and to their modifications when transforming from a hedgehog to a twisted configuration.

The field's spatial distributions, chirality density, Poynting vector, and total angular momentum density are numerically evaluated and analyzed.

Finally, it is important to mention that our EM simulations are carried out for a particular size of the BP, the extent to which depends on the temperature.

Thank You!