In-Situ Gain Characterisation of Low Noise Amplifiers using Differential Thermal Noise Analysis

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***Abstract*—A novel in-situ methodology for determining the voltage gain of low noise amplifiers (LNAs) without the use of vector network analysis is presented. The approach evaluates the differential thermal noise spectra of constant impedances at two distinct temperatures. By calculating the difference, the method excludes the LNA's inherent noise contribution. Comparison between the calculated voltage gain and independently acquired reference data demonstrates strong agreement between the gain values.**

***Keywords*—** *Low Noise Amplifier (LNA), In-Situ Gain Characterisation, Thermal Noise Analysis, RF Metrology*

I. Introduction

Essential for radio frequency (RF) front-end design is an accurate characterisation of Low Noise Amplifiers (LNAs), as it directly influences receiver sensitivity and dynamic range. Traditionally, gain measurements are performed using Vector Network Analysers (VNAs) [1]. However, in many practical implementations, particularly within integrated or field-deployed systems, disconnecting the LNA for VNA-based measurements is impractical [2]. Such interventions may shift reference planes, alter thermal conditions, and introduce uncertainties associated with connectors [3]. To mitigate these sources of error, a methodology for in-situ determination of LNA voltage gain is introduced, based on the analysis of differential thermal noise power spectra. This approach eliminates the requirement for active VNA measurements during characterisation.

II. Theory and Methodology

The proposed method relies on the principle that the total noise power at the amplifier output consists of both amplified source noise and the low noise amplifier's intrinsic noise. By differentiating the output signals under two distinct thermal states of the source impedance, invariant parameters can be mathematically eliminated [2]. The subsequent sections provide a detailed description of the individual components of the measured signal and the calculation of the differential.

A. Analysis of the measured voltage

Fig. 1 gives a schematic of a noisy resistor, represented by $\underline{Z}_\mathrm{x}$ and a temperature dependant voltage source (V_T). This resistor is connected with an LNA with an input impedance of $\underline{Z}_\mathrm{LNA}$. The LNA´s output voltage V_a is calculated from a noise power measurement with a spectrum analyser, measuring the voltage square. The portion of the noise voltage (1) dropping across the voltage divider with $\underline{Z}_\mathrm{x}$ on the amplifier´s side is increased by a voltage gain of G_{volt}.

$$V_T = \sqrt{4k_B \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_\mathrm{x}\} \cdot T} \quad (1)$$

Here, k_B denotes the Boltzmann constant and Δf is the measurement bandwidth.

The intrinsic noise voltage square ${V_{int}}^2$ of the LNA is added to this signal composition [4]. The correlating and non-correlating intrinsic noise voltages and currents are simplified within V_{int}. The noise voltage square at the spectrum analyser can be written as

$$V_\mathrm{o}^2 = G_\mathrm{volt}^2 \cdot \left|\frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA}+\underline{Z}_\mathrm{x}}\right|^2 \cdot \left(4k \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_\mathrm{x}\} \cdot T + {V_\mathrm{int}}^2\right). \quad (2)$$

B. The Differential Approach

The output noise voltage is measured at two temperatures, T_1 and T_2, for a constant source impedance $\underline{Z}_\mathrm{x}$. Calculating the difference, ΔV_o^2 (${V_{\mathrm{o}2}}^2 - {V_{\mathrm{o}1}}^2$), cancels the amplifier's internal noise and other constant offsets. The resulting expression for the differential noise voltage square is

$$\Delta V_\mathrm{o}^2 = G_\mathrm{volt}^2 \cdot \left|\frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA}+\underline{Z}_\mathrm{x}}\right|^2 \cdot \left(4k \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_\mathrm{x}\}(T_2 - T_1)\right). \quad (3)$$

In this approach, $\underline{Z}_{LNA}$ is assumed to be known, as determined by a secondary, published in-situ characterisation technique. This enables the extraction of the gain G from purely passive noise measurements.

III. Experimental Setup

A. Hardware Configuration

The experimental setup is designed to ensure a repeatable and stable temperature gradient. The test impedance is implemented as a high-precision 50 Ω terminator, connected to the LNA through a low-loss RF relay. The measurement reference plane is established at the relay-terminator interface.

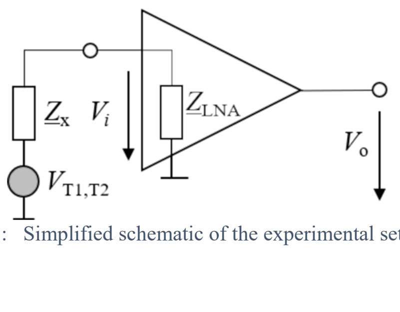

Fig. 1: Simplified schematic of the experimental setup.

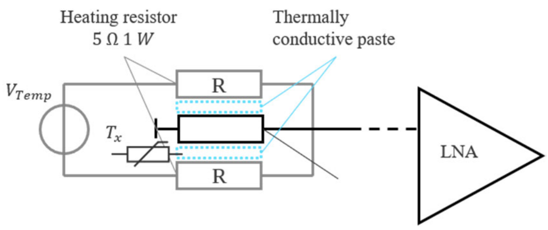

Fig. 2: Simplified schematic of the used heating structure. V_{Temp} applies a constant DC voltage across two resistors R (5 Ω). Thermally conductive paste connects the resulting heat to the 50 Ω terminator. Its Temperature is measured by a sensor T_x.

The relay's transmission characteristics are treated as a constant transfer function, characterised once using a VNA during the initial manufacturing of the test bed.

A thermal shift is induced using two 5 Ω lumped resistors as heating elements, powered by a regulated DC source. The terminator temperature is monitored in real time by a sensor positioned in close proximity (Fig. 2). The entire assembly, including the terminator, heaters, and sensor, is enclosed in a thermally insulated housing to maintain thermal stability and prevent environmental interference.

B. Measurement Procedure

First, the noise power spectrum is measured at ambient temperature T_1, of roughly 293 K. The heating elements are subsequently activated to achieve a stable temperature increase of approximately 40 K, after which the hot-state spectrum is recorded. Data acquisition is performed using a spectrum analyser, isolated by a -10 dB precision attenuator to suppress port reflections. Equation (3) is used to determine the values for ${G_{volt}}^2$.

$$G_{\mathrm{volt}}^2 = \Delta V_{\mathrm{o}}^2 \cdot \left|\frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA}+\underline{Z}_{\mathrm{x}}}\right|^{-2} \cdot \left(4k_B \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_{\mathrm{x}}\}(T_2 - T_1)\right)^{-1} \quad (4)$$

IV. Results and Discussion

The validity of the method is demonstrated by comparing gain data extracted from noise differentials with traditional VNA measurements. The voltage gain is calculated from S-parameters via

$$G_{volt} = \frac{\underline{S}_{21}}{1+\underline{S}_{11}} \quad (5)$$

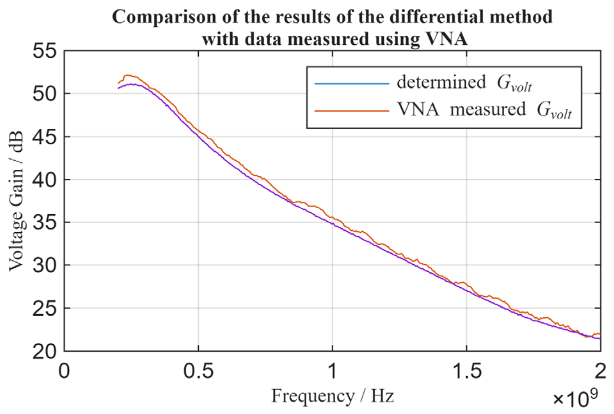

Fig. 3: Comparison of the voltage gain (G_{volt}) determined via the differential temperature method with the reference measurement from a VNA up to 2 GHz.

These data are presented in Fig. 3 in decibel. The VNA measured reference results show agreement, with the differential noise method, reliably determining the frequency-dependent gain profile of the LNA. Despite the high degree of agreement, the remaining discrepancies can be explained by the physical limitations of the measurement. The extremely low thermal noise levels lead to minimal disturbances influencing the result. Combined with the high sensitivity of the difference method to the determination of the impedances $\underline{Z}_{\mathrm{x}}$ and $\underline{Z}_{\mathrm{LNA}}$, the observed deviations occur.

The elimination of the amplifier's noise floor through the ΔV_{o}^2 calculation ensures robustness, even for LNAs with substantial intrinsic noise figures.

V. Conclusion

This study demonstrates a reliable, VNA-free approach for in-situ LNA gain determination. By employing controlled thermal states and a differential mathematical model, the method preserves the integrity of operational reference planes and improves the characterisation process in complex RF environments. The use of pre-characterised relay paths and external $\underline{Z}_{LNA}$ data enables highly accurate gain extraction, providing a practical alternative to active instrumentation for long-term monitoring and field-test applications.

Acknowledgment

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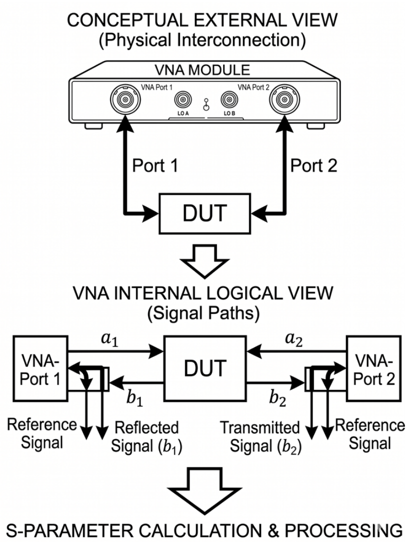
The Problem with Traditional LNA Characterisation



Standard Approach
- Gain measurements traditionally require Vector Network Analysers (VNAs)
- Professional calibrated noise sources
- Precise but impractical in integrated or field-deployed systems

Why It Falls Short
- Disconnecting the LNA shifts reference planes
- Alters thermal conditions
- Introduces connector-related uncertainties

NoCRaFT

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The Solution: In-Situ Differential Thermal Noise Method

No VNA Required
Gain is extracted from passive thermal noise measurements alone.

Differential Approach
Noise spectra at two temperatures cancel the LNA's intrinsic noise contribution.

In-Situ Integrity
Operational reference planes remain undisturbed throughout characterisation.

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Theoretical Foundation

The total output noise power combines **amplified source noise** and the LNA's **intrinsic noise**. The output voltage squared is:

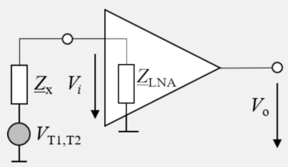


Fig. 2: Simplified schematic of the experimental setup.

$$V_o^2 = G_{volt}^2 \cdot \left| \frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA} + \underline{Z}_x} \right|^2 \cdot \left(4k \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_x\} \cdot T + V_{int}{}^2\right)$$

Source Noise
Thermal noise from resistor $\underline{Z}_x$ at temperature T

Voltage Divider
Impedance ratio between LNA and source impedance

Intrinsic Noise
eliminated by differencing

k_B: Boltzmann constant
Δf: measurement bandwidth
V_{int}^2 : LNA's intrinsic noise voltage squared

The Differential Calculation

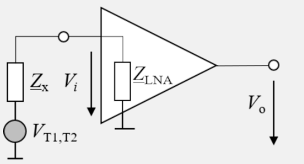


Fig. 2: Simplified schematic of the experimental setup.

Measuring output noise at two temperatures **T₁** and **T₂** with constant source impedance $\underline{Z}_x$, the difference ΔV_o^2 ($= {V_{o2}}^2 - {V_{o1}}^2$) cancels all constant terms, including the LNA noise floor:

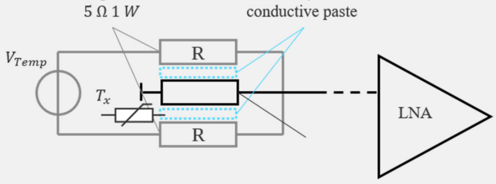
$$\Delta V_o^2 = G_{volt}^2 \cdot \left| \frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA} + \underline{Z}_x} \right|^2 \cdot \left(4k_B \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_x\} \cdot (T_2 - T_1) + \cancel{({V_{int}}^2 - {V_{int}}^2)}\right)$$

Rearranging yields the voltage gain directly:

$$G_{volt}^2 = \Delta V_o^2 \cdot \left| \frac{\underline{Z}_{LNA}}{\underline{Z}_{LNA} + \underline{Z}_x} \right|^{-2} \cdot \left(4k_B \cdot \Delta f \cdot \mathrm{Re}\{\underline{Z}_x\} \cdot (T_2 - T_1)\right)^{-1}$$

k_B: Boltzmann constant
Δf: measurement bandwidth
$\underline{Z}_x$: chosen impedance
$\underline{Z}_{LNA}$: known LNA input impedance

Experimental Setup



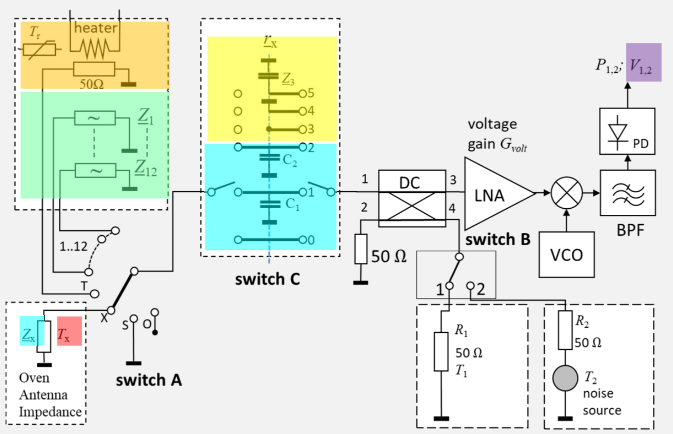
50 Ω Terminator
High-precision terminator connected to the LNA via a low-loss RF relay. Relay transfer function pre-characterised once by VNA.

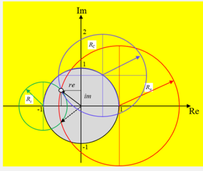
Thermal Control
Two 5 Ω heating resistors powered by a regulated DC source induce a stable ~40 K temperature rise above ambient (~293 K).

Thermal Insulation
Entire assembly enclosed in thermally insulated housing to maintain stability and prevent environmental interference.

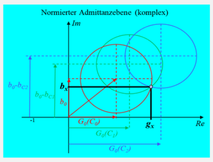
Setup

$$T_x = \frac{V_o^2 \cdot \left|\frac{Z_x + Z_{LNA}}{Z_x}\right|^2 - |V_n|^2 - |I_n|^2 \cdot |Z_x|^2 - 2 \cdot \mathrm{Re}\{Z_{ant}\} \cdot \mathrm{Re}\{I_n \cdot V_n^*\} - 2\,\mathrm{Im}\{Z_x\} \cdot \mathrm{Im}\{I_n \cdot V_n^*\}}{G_{volt}^2 \cdot 4k\Delta f\ \mathrm{Re}\{Z_x\}}$$

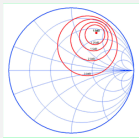




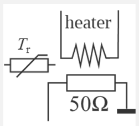
6 sweeps
⇒ Z_{LNA}; V_x



6 sweeps
⇒ Z_{ant}



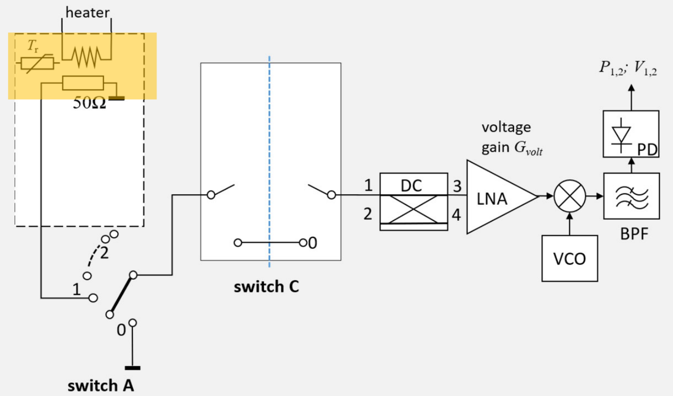
6n sweeps: (n = no. of noise impedances)
⇒ $|I_n|^2$; $\mathrm{Re}(I_n V_n^*)$; $\mathrm{Im}(I_n V_n^*)$;

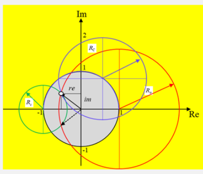


2 sweeps
@ 2 temperatures
⇒ G_{volt}, V_n

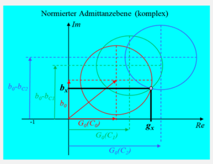
Setup

$$T_x = \frac{V_o^2 \cdot \left|\frac{Z_x + Z_{LNA}}{Z_x}\right|^2 - |V_n|^2 - |I_n|^2 \cdot |Z_x|^2 - 2 \cdot \mathrm{Re}\{Z_{ant}\} \cdot \mathrm{Re}\{I_n \cdot V_n^*\} - 2\,\mathrm{Im}\{Z_x\} \cdot \mathrm{Im}\{I_n \cdot V_n^*\}}{G_{volt}^2 \cdot 4k\Delta f\ \mathrm{Re}\{Z_x\}}$$

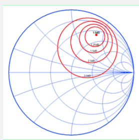




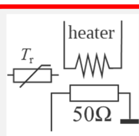
6 sweeps
⇒ Z_{LNA}; V_x



6 sweeps
⇒ Z_{ant}



6n sweeps: (n = no. of noise impedances)
⇒ $|I_n|^2$; $\mathrm{Re}(I_n V_n^*)$; $\mathrm{Im}(I_n V_n^*)$;



2 sweeps
@ 2 temperatures
⇒ G_{volt}, V_n

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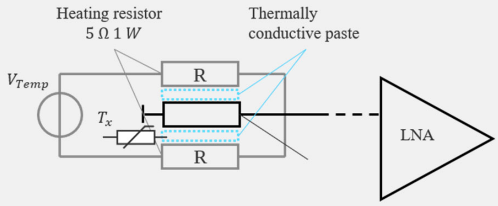
Measurement Procedure



Record Cold → Activate Heater → Record hot → Compute ΔV_o^2

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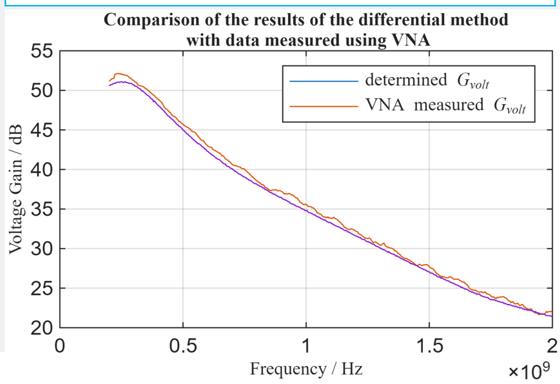
Results: Gain Comparison

Voltage gain from S-parameters (VNA reference) is calculated as:

$$G_{volt} = \left| \frac{s_{21}}{1 + s_{11}} \right|$$

Voltage gain from differential approach is calculated as:

$$G_{\mathrm{volt}} = \sqrt{\Delta V_o^2 \cdot \left| \frac{Z_{LNA}}{Z_{LNA} + Z_{\mathrm{x}}} \right|^{-2} \cdot \left(4k_{\mathrm{B}} \cdot \Delta f \cdot \mathrm{Re}\{Z_{\mathrm{x}}\} \cdot (T_2 - T_1) \right)^{-1}}$$



Strong Agreement
Noise-derived gain closely tracks VNA measurements across the measured frequency range.

Noise Floor Robustness
The ΔV_o^2 differencing ensures robustness even for LNAs with substantial intrinsic noise figures.

Residual Deviations
Minor discrepancies arise from extremely low thermal noise levels and sensitivity to impedance determination and deviating RBW (SA settings)

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Conclusion

In-Situ Differential Thermal Noise Method

No VNA Required
Gain is extracted from passive thermal noise measurements alone.

Differential Approach
Noise spectra at two temperatures cancel the LNA's intrinsic noise contribution.

In-Situ Integrity
Operational reference planes remain undisturbed throughout characterisation.

Practical Applications
Suited for long-term monitoring and field-test environments in complex RF systems.

Outlook
Generate a statistically relevant amount of measurements.
Further investigations into the deviation

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Thank you for your attention… again

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