

In-Situ Input Impedance Determination of Low Noise Amplifiers in Radiometric Applications

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Abstract—With radiometric thermometry the temperature of weakly conductive objects placed in closed cavities can be determined by measuring the emitted Planck radiation captured by an antenna. In order to calibrate the system, the complex electrical coupling coefficient between object and a low noise amplifier (LNA) must be known. This study introduces a mathematical extension to a former presented self-calibrating radiometry method. The previous approach assumed a constant $50\ \Omega$ input impedance for the LNA used. To address variations in the LNA's input impedance, an in-situ method is introduced to determine the complex reflection coefficient of the amplifier. Therefore, a three-impedance calibration technique, utilizing short-circuit, open-circuit, and capacitive loads, is developed. By resolving resulting circular equations in the complex plane of the LNA's reflection coefficient, the impedance of the LNA can be determined, thereby enabling the functionality of the self-calibrating radiometer design.

Keywords—amplifiers, low noise amplifiers, calibration, electromagnetic reflection, microwave radiometry, reflection coefficient, temperature measurement, wide-band radiometer

I. INTRODUCTION

An extension of the approach published in [1] for non-invasive temperature measurement of weakly conductive objects in closed cavities, based on thermal radiation and Planck's law [2], is described. The method employs various combinations of reference impedances, capacitances, noise sources and ground connections, to achieve self-calibration and to determine parameters such as system gain, the reflection coefficient of the object in the cavity, and its impedance. These parameters are essential for calculating the object's absolute temperature. A major advantage of this method is its use of a directional coupler (DC) instead of a circulator, which enables a wide-band operation and requires only simple measurement electronics, distinguishing it from conventional radiometer applications [3, 4].

However, the design assumes a low noise amplifier (LNA) with an ideal $50\ \Omega$ input impedance across the measurement bandwidth. This impedance varies with frequency, resulting in a voltage divider ratio between the object and the LNA that changes according to the LNA's input impedance. Since the object temperature is proportional to its emitted noise power, accurate determination of the voltage divider ratio is essential to calculate the emitted thermal noise power from the noise power measured at the LNA output.

Here, a mathematical approach for in-situ determination of the reflection coefficient and, thus, the input impedance of the LNA is incorporated into the concept in [1]. This approach accounts for additional signal reflections at the LNA input. Initially, the signal reflections are mathematically modelled.

The mathematical extension for in-situ determination of the LNA input impedance is described subsequently.

II. DETERMINATION OF LNA INPUT IMPEDANCE

The frequency-dependent input impedance of the LNA and the resulting signal reflections at its input are incorporated into the design of a self-calibrating radiometer [1]. A method for calculating the LNA's reflection coefficient based on three known impedances connected to the LNA input is presented. Therefore, the concept is expanded to include three additional loads at ports 3, 4 and 5 of switch C as shown in Fig. 1. Further details regarding the structure are available in [1].

To determine the LNA's reflection coefficient, two noise sources (switch B) are used. Their noise power levels simulate both temperatures $T_{1,2}$. According to the equation for thermal noise and taking into account the attenuation factor β of the coupled path of the DC, initial waves $a_{1,2}^0$ (1) propagate from the noise sources into the reference plane (blue line) [2].

$$a_{1,2}^0 = \beta \cdot \sqrt{T_{1,2} \cdot k_B \cdot \Delta f} \quad (1)$$

Here, k_B denotes the Boltzmann constant and Δf is the measurement bandwidth.

In the reference plane the reflection coefficients are r_x towards the object dependent antenna impedance Z_x and r_{LNA} towards the LNA. Due to impedance mismatch between the LNA input and Z_x , the LNA receives the two superimposed waves $b_{1,2}^{\text{tot}}$ consisting of the initial reflection of $a_{1,2}^0$ from Z_x and an infinite series of subsequent reflections between Z_x and the LNA (2).

$$b_{1,2}^{\text{tot}} = a_{1,2}^0 \cdot \left| \sum_{n=1}^{\infty} r_x^n \cdot r_{LNA}^{n-1} \right| = a_{1,2}^0 \cdot \left| \frac{r_x}{1 - r_x \cdot r_{LNA}} \right| \quad (2)$$

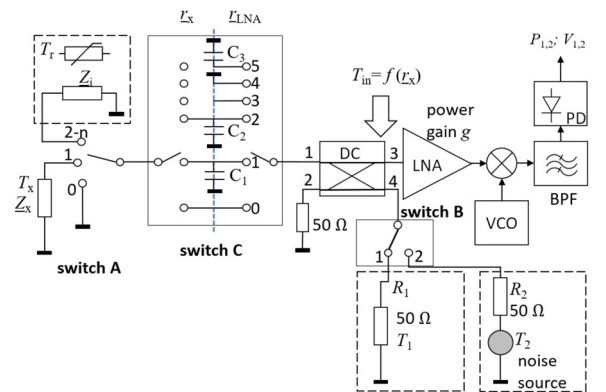


Fig. 1. Schematic of the self-calibrating radiometer

Subtracting the uncorrelated voltages $V_{1,2}$ measured at the LNA output yields (3), where V_x is an unknown parameter with units of volts, that contains the LNA's gain and the waves from (1). In both measurements the same source impedance applies to the LNA input, so the LNA's intrinsic noise remains constant and its influence is eliminated in (3). The same holds for the noise of the impedance to the left of the reference plane.

$$\Delta V_{\text{out}} = \sqrt{V_2^2 - V_1^2} = V_x \cdot \left| \frac{r_x}{1 - r_x \Gamma_{\text{LNA}}} \right| \quad (3)$$

To determine the reflection coefficient Γ_{LNA} , three known calibration impedances are successively connected to the reference plane at switch C, thereby defining r_x in (3). These impedances are selected to maximise reflection of the noise signal, enables improvements in measuring the differential voltage ΔV_{out} . The first impedance applied at switch C port 3 is an open circuit ($r_x = 1$). After rearrangement of (3), the measured differential voltage ($\Delta V_{\text{out}} = V_o$) yields a circular equation in the complex plane of Γ_{LNA} with the radius R_o (4).

$$R_o^2 = \left(\frac{V_o}{V_x} \right)^2 = |1 - \Gamma_{\text{LNA}}|^2 = (1 - \text{Re}\{\Gamma_{\text{LNA}}\})^2 + \text{Im}\{\Gamma_{\text{LNA}}\}^2 \quad (4)$$

When a short circuit ($r_x = -1$) is applied at switch C port 4, the measured differential voltage ($\Delta V_{\text{out}} = V_s$) provides a second circular equation with the radius R_s (5).

$$R_s^2 = \left(\frac{V_s}{V_x} \right)^2 = |1 + \Gamma_{\text{LNA}}|^2 = (1 + \text{Re}\{\Gamma_{\text{LNA}}\})^2 + \text{Im}\{\Gamma_{\text{LNA}}\}^2 \quad (5)$$

The intersection points of these two circles yield two potential solutions for Γ_{LNA} . However, since V_x remains unknown, the exact radii of the circles are undefined, leaving the true reflection coefficient undetermined. To resolve this ambiguity, the capacitance C_3 at switch C port 5 is used as a third, highly reflective impedance. With a normalized susceptance ($b_{C_3} = 50 \Omega \cdot \omega C_3$), r_x can be calculated as described in [5]. Substituting r_x into (3) yields the differential voltage ($\Delta V_{\text{out}} = V_{C_3}$) and a third circular equation with the radius R_{C_3} :

$$R_{C_3}^2 = \left(\frac{V_{C_3}}{V_x} \right)^2 = (X_{C_3} - \text{Re}\{\Gamma_{\text{LNA}}\})^2 + (Y_{C_3} - \text{Im}\{\Gamma_{\text{LNA}}\})^2 \quad (6)$$

With

$$X_{C_3} = \frac{1 - b_{C_3}^2}{1 + b_{C_3}^2}, Y_{C_3} = \frac{2b_{C_3}}{1 + b_{C_3}^2}. \quad (7)$$

The circles corresponding to the three circle equations (4), (5) and (6) are presented in Fig. 2. An examination of the circles defined in (4) and (5) confirms that they are insufficient to determine Γ_{LNA} . They intersect at two points, such that the imaginary part cannot be uniquely determined. The inclusion of the third circle (6) enables, by varying V_x and the resulting adjustment of the radii of all circles to achieve a single, common intersection point in the complex plane of Γ_{LNA} . V_x then satisfies all circle equations, and Γ_{LNA} is determined by the coordinates of this point.

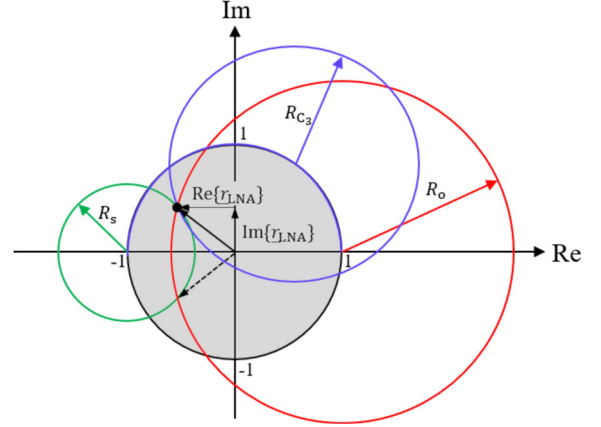


Fig. 2. Visualization of the three circular equations in the complex plane of the LNA's reflection coefficient. Open (red) centered at 1 on the real axis, Short (green) centered at -1 on the real axis and Capacitance (blue) centered on the unit circle in the upper complex half plane

Once Γ_{LNA} is mathematically characterised, the frequency-dependent input impedance of the amplifier Z_{LNA} can be calculated (8). This enables compensation for the voltage divider effect when calculating the object temperature.

$$Z_{\text{LNA}} = 50 \Omega \cdot \frac{1 + \Gamma_{\text{LNA}}}{1 - \Gamma_{\text{LNA}}} \quad (8)$$

III. CONCLUSION

Assuming an ideal 50Ω LNA input impedance introduces a significant error in the targeted wideband radiometry application. The proposed mathematical extension provides an in-situ method to dynamically quantify the LNA's complex reflection coefficient through the use of a three-impedance Open-Short-Capacitance calibration method and resolving the resultant intersecting circles in the complex plane. This advancement enables the accurate determination of the temperature of an object in a cavity, even when the LNA exhibits a frequency-dependent, non-ideal input impedance.

ACKNOWLEDGMENT

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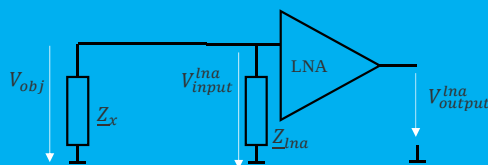
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Radiometric Thermometry



- Temperature information is contained in the noise voltage

$$V_{obj} = \sqrt{4k_B T_{obj} Re\{Z_x\} \Delta f} \quad (1)$$

- Voltage at the LNA input:

$$V_{input}^{Lna} = \left| \frac{Z_{Lna}}{Z_{Lna} + Z_x} \right| \cdot V_{obj} \quad (2)$$

- Voltage at the LNA output:

$$V_{output}^{Lna} = \left| \frac{S_{21}^{Lna}}{1 + S_{11}^{Lna}} \right| \cdot V_{input}^{Lna} \quad (3)$$

k_B : Boltzmann's constant
 Δf : resolution bandwidth
 T_{obj} : Temperature within the object
 Z_{ant} : Impedance of the system
 Z_{Lna} : Input impedance of the LNA
 Z_0 : Reference impedance of the measurement instrument (50 Ω)

Core Principle

- Object temperature is proportional to its emitted thermal noise power.

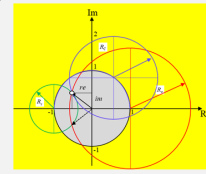
Key Challenge

- LNA input impedance varies with time and temperature, altering the voltage divider ratio.
- Without accurate compensation, the calculated object temperature contains significant error.

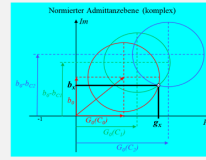
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Setup

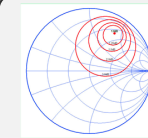
$$T_n = \frac{V_o^2 \cdot \left| \frac{Z_n + Z_{LNA}}{Z_n} \right|^2 - |V_n|^2 - L_n^2 \cdot |Z_n|^2 - 2 \cdot \text{Re}\{Z_{LNA}\} \cdot \text{Re}\{L_n \cdot V_n^*\} - 2 \text{Im}\{Z_n\} \cdot \text{Im}\{L_n \cdot V_n^*\}}{G_{\text{voltage}}^2 \cdot 4k\Delta f \text{Re}\{Z_n\}}$$



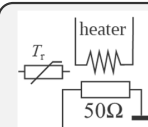
6 sweeps
 $\Rightarrow Z_{LNA}, V_n$



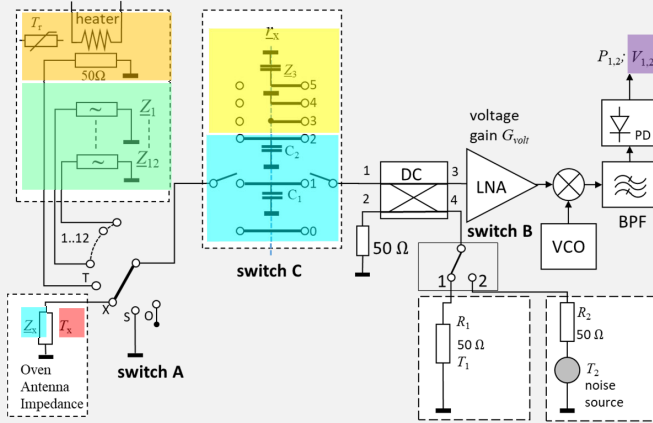
6 sweeps
 $\Rightarrow Z_{ant}$



6n sweeps: (n = no. of noise impedances)
 $\Rightarrow |L_n|^2; \text{Re}(L_n V_n^*); \text{Im}(L_n V_n^*)$

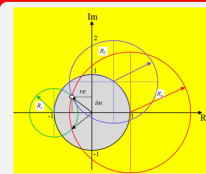


2 sweeps
 @ 2 temperatures
 $\Rightarrow G_{\text{voltage}}, V_n$

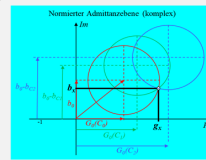


Setup

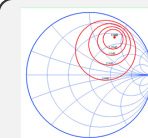
$$T_n = \frac{V_o^2 \cdot \left| \frac{Z_n + Z_{LNA}}{Z_n} \right|^2 - |V_n|^2 - L_n^2 \cdot |Z_n|^2 - 2 \cdot \text{Re}\{Z_{LNA}\} \cdot \text{Re}\{L_n \cdot V_n^*\} - 2 \text{Im}\{Z_n\} \cdot \text{Im}\{L_n \cdot V_n^*\}}{G_{\text{voltage}}^2 \cdot 4k\Delta f \text{Re}\{Z_n\}}$$



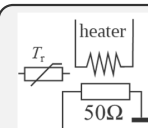
6 sweeps
 $\Rightarrow Z_{LNA}, V_n$



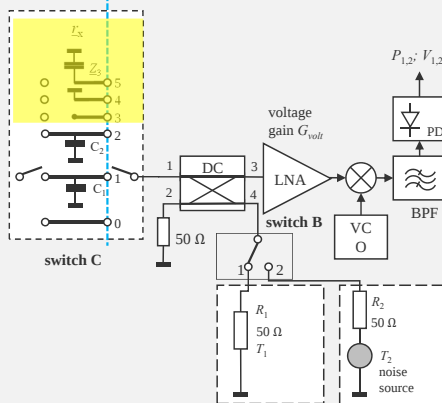
6 sweeps
 $\Rightarrow Z_{ant}$



6n sweeps: (n = no. of noise impedances)
 $\Rightarrow |L_n|^2; \text{Re}(L_n V_n^*); \text{Im}(L_n V_n^*)$



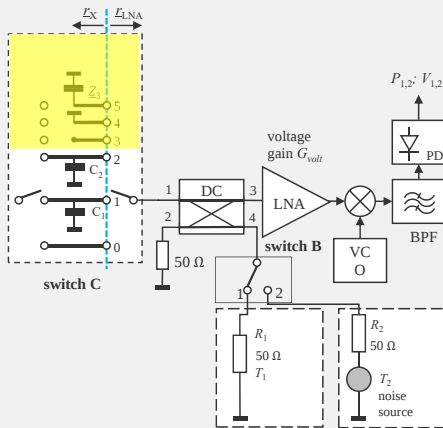
2 sweeps
 @ 2 temperatures
 $\Rightarrow G_{\text{voltage}}, V_n$



Principle of measurement

$$a_{1,2}^0 = \beta \cdot \sqrt{T_{1,2} \cdot k_B \cdot \Delta f} \longrightarrow V_x = G \cdot \sqrt{(a_2^0)^2 - (a_1^0)^2} \longrightarrow V_{out} \approx V_x \cdot \left| \frac{r_x}{1 - r_x r_{LNA}} \right| + V_{int}$$

$$b_{1,2}^{\text{tot}} = a_{1,2}^0 \cdot \left| \sum_{n=1}^{\infty} \underline{r}_{\text{x}}^n \cdot \underline{r}_{\text{LNA}}^{n-1} \right| = a_{1,2}^0 \cdot \left| \frac{\underline{r}_{\text{x}}}{1 - \underline{r}_{\text{x}} \cdot \underline{r}_{\text{LNA}}} \right|$$



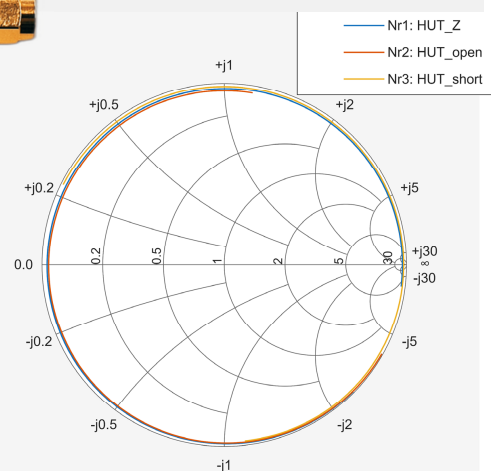
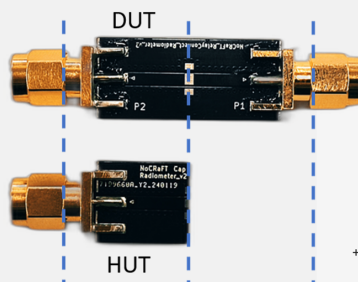
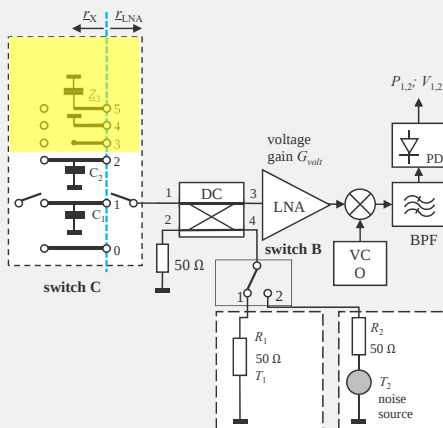
V_{int} : impedance-dependent,
intrinsic noise of the LNA

$$\Delta V_{\text{out}} = \sqrt{V_2^2 - V_1^2} = V_x \cdot \left| \frac{r_x}{1 - r_x r_{\text{LNA}}} \right|$$

- Noise voltage measured with a spectrum analyser during setup
- V_x is an unknown parameter
- Subtracting uncorrelated voltages V_1 and V_2
- Intrinsic noise of the LNA and impedance noise of the HUTs eliminated

Principle of measurement

- Symmetrical PCBs as DUT connections (needed for determination of target impedance)
- PCB with half the length (HUT) as an open circuit, short circuit and with a shunt capacitor
- S-parameters measured in advance with VNA



$$\Delta V_{\text{out}} = \sqrt{V_2^2 - V_1^2} = V_x \cdot \left| \frac{r_x}{1 - r_x r_{\text{LNA}}} \right|$$

Determination of LNA impedance / LNA reflection coefficient

- Sought: Input impedance of the LNA Z_{LNA} or the LNA reflection factor r_{LNA} . The following applies: $r_{LNA} = re + j \cdot im = \frac{Z_{LNA} - 50 \Omega}{Z_{LNA} + 50 \Omega}$
- The HUT measurements (open/short/C) result in three circuit equations in the reflection plane.

Solution in 3 steps:

Step 1:

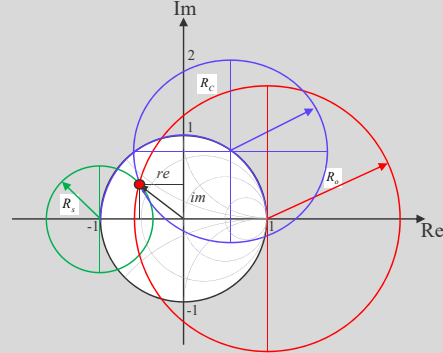
Determination of the circle equations:

Short: $R_s^2 = (1 + re)^2 + im^2 = \frac{V_x^2}{V_s^2}$ $\text{---} \text{---} Y_i = \infty$

Open: $R_o^2 = (1 - re)^2 + im^2 = \frac{V_x^2}{V_o^2}$ $\text{---} \text{---} Y_i = 0$

C: $R_{Z3}^2 = (X_{Z3} - re)^2 + (Y_{Z3} - im)^2 = \frac{V_x^2}{V_{Z3}^2}$ $\text{---} \text{---} Y_i = G_i + j \cdot B_i$

Where the centre points of circles are: $X_{Zi} = \frac{Z_0^{-1} - G_i^2 - B_i^2}{(Z_0^{-1} - G_i)^2 + B_i^2}$; $Y_{Zi} = \frac{2B_i}{(Z_0^{-1} - G_i)^2 + B_i^2}$



- Centres are known, calibration voltage V_X is unknown $\Rightarrow V_X$ is selected, so that an intersection exists
- Ideally, the centres of the circles lie on the unit circle (C), at $[-1;0]$ (short) and at $[1;0]$ (open).
- All three circles must have exactly ONE common intersection.
- This results in the calibration voltage V_X , which is proportional to the radii and will be used later.

Determination of LNA impedance / LNA reflection coefficient

Step 2:

Combining two circle equations to form one $\Rightarrow$ elimination of calibration voltage V_X .

Short: $R_s^2 = (X_s - re)^2 + (Y_s - im)^2 = \frac{V_x^2}{V_s^2}$ $\rightarrow (re - X_{os})^2 + (im - Y_{os})^2 = R_{os}^2$

Open: $R_o^2 = (X_o - re)^2 + (Y_o - im)^2 = \frac{V_x^2}{V_o^2}$ $\rightarrow (re - X_{oc})^2 + (im - Y_{oc})^2 = R_{oc}^2$

C: $R_{Z3}^2 = (X_{Z3} - re)^2 + (Y_{Z3} - im)^2 = \frac{V_x^2}{V_{Z3}^2}$

where:

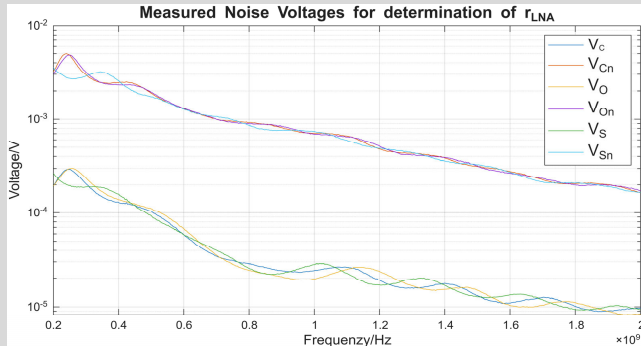
$$X_{os} = \frac{X_s \frac{V_o^2}{V_s^2} - X_s}{\frac{V_o^2}{V_s^2} - 1}; B_{os} = \frac{X_o \frac{V_o^2}{V_s^2} - X_s^2}{\frac{V_o^2}{V_s^2} - 1}; Y_{os} = \frac{Y_o \frac{V_o^2}{V_s^2} - Y_s}{\frac{V_o^2}{V_s^2} - 1}; D_{os} = \frac{Y_o^2 \frac{V_o^2}{V_s^2} - Y_s^2}{\frac{V_o^2}{V_s^2} - 1}; R_{os}^2 = X_{os}^2 - B_{os} + Y_{os}^2 - D_{os}$$

$$X_{oc} = \frac{X_o \frac{V_o^2}{V_{Z3}^2} - X_{Z3}}{\frac{V_o^2}{V_{Z3}^2} - 1}; B_{oc} = \frac{X_o \frac{V_o^2}{V_{Z3}^2} - X_{Z3}^2}{\frac{V_o^2}{V_{Z3}^2} - 1}; Y_{oc} = \frac{Y_o \frac{V_o^2}{V_{Z3}^2} - Y_{Z3}}{\frac{V_o^2}{V_{Z3}^2} - 1}; D_{oc} = \frac{Y_o^2 \frac{V_o^2}{V_{Z3}^2} - Y_{Z3}^2}{\frac{V_o^2}{V_{Z3}^2} - 1}; R_{oc}^2 = X_{oc}^2 - B_{oc} + Y_{oc}^2 - D_{oc}$$

Determination of LNA impedance / LNA reflection coefficient

Raw Data

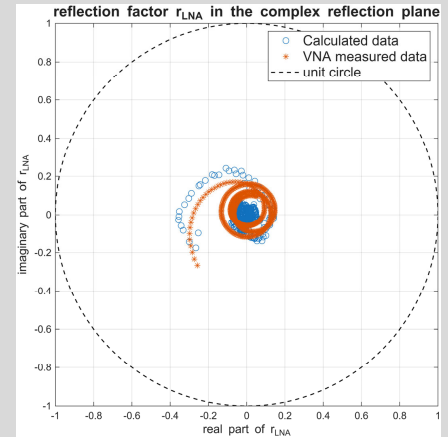
- Noise voltage measured with SA during setup



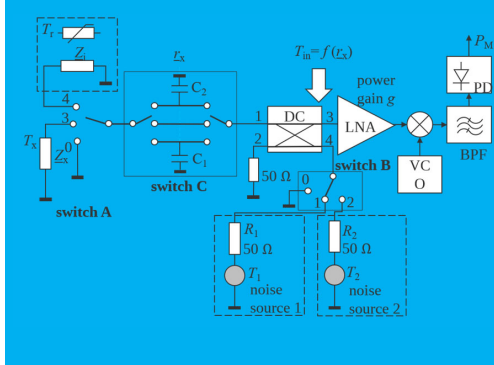
with $r_{HUT} \rightarrow$

Calculated Data

- Reflection factor of LNA input



Conclusion



Problem addressed

- Assuming ideal 50Ω LNA impedance introduces significant error in wideband radiometry.

Method introduced

- Open–Short–Capacitance three-impedance calibration with intersecting-circle resolution in the complex plane.

Outcome

- Object temperature determination enabled even with frequency-dependent, non-ideal LNA input impedance.

Supported by BMBF Program FH-Kooperativ, Project NoCrAFT (13FH034KX1)

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Determination of LNA impedance / LNA reflection coefficient

Step 3:

Calculation of the 2 intersection points and selection

Open/Short: $(re - X_{os})^2 + (im - Y_{os})^2 = R_{os}^2$

Open/C: $(re - X_{oc})^2 + (im - Y_{oc})^2 = R_{oc}^2$

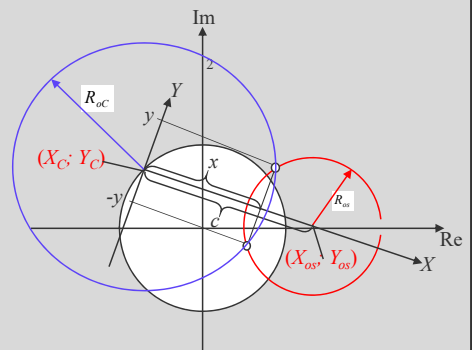
Intersection 1: $re_1 = X_{oc} + x \cdot \frac{X_{os} - X_{oc}}{c} - y \cdot \frac{Y_{os} - Y_{oc}}{c}; im_1 = Y_{oc} + x \cdot \frac{Y_{os} - Y_{oc}}{c} + y \cdot \frac{X_{os} - X_{oc}}{c};$

Intersection 2: $re_2 = X_{oc} + x \cdot \frac{X_{os} - X_{oc}}{c} + y \cdot \frac{Y_{os} - Y_{oc}}{c}; im_2 = Y_{oc} + x \cdot \frac{Y_{os} - Y_{oc}}{c} - y \cdot \frac{X_{os} - X_{oc}}{c};$

where: $c = \sqrt{(X_{os} - X_{oc})^2 + (Y_{os} - Y_{oc})^2}; x = \frac{c^2 - R_{os}^2 + R_{oc}^2}{2c}; y_{1,2} = \sqrt{R_{oc}^2 - x^2}$

Selection:

- The intersection point within the unit circle is selected.
- For ideal components (open/short/C), the other point lies outside the unit circle.



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- Optical Sensors
- Quantum Sensors
- Sensor Networks
- Sensor Data Processing
- Non-invasive measurements
- Radiometry
- RF & Microwave Sensors
- Radar Applications
- Environmental Measurement
- Novel Sensor Principles



The venue is located in the heart of the Ruhrgebiet. It has a rich industrial heritage and cultural significance. Today, it offers a diverse range of attractions such as UNESCO World Heritage Sites, art galleries, museums, theatres, outdoor activities, lively markets, delicious cuisine and vibrant nightlife. The Ruhrtourismus GmbH has built a platform and service to present a large variety of cultural activities. <https://www.ruhr-tourismus.de>

Venue:

**Hochschule Ruhr West
Institut Mess- und Sensor-
technik
Duisburger Str. 100
45479 Mülheim an der Ruhr**

