An Accurate and Efficient Recursive T-Matrix Algorithm without Violating the Addition Theorem

M. Degen, V. Jandieri, A. Rennings, and D. Erni

General and Theoretical Electrical Engineering (ATE), University of Duisburg-Essen, Duisburg, Germany, and CENIDE–Center for Nanointegration Duisburg-Essen, Duisburg, Germany, marvin.degen@uni-due.de

The accurate and efficient analysis of electromagnetic scattering from clusters of randomly arranged particles is a challenging but relevant task in many practical applications. Particularly when the scattering for (many) different illuminations is of interest, the computational effort can be substantial because often many computations must be repeated for each illumination. The latter can be overcome by determining the T-matrix of the cluster which then reduces–once the T-matrix is known– the computational effort to a single matrix-vector multiplication for each illumination. However, the calculation of the cluster T-matrix using the standard superposition T-matrix approach is a non-trivial and computationally costly task, especially for a large number of particles due to the increasing number of unknowns in the system of equations to be solved. To obviate this difficulty, a variety of recursive T-matrix algorithms have been developed where the number of unknowns to be determined in each recurrence step is significantly reduced allowing the use of direct solvers that are independent of the right-hand site. Unfortunately, as is well known, the fast recursive algorithms inherently violate the involved addition theorems (AT) for the

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translations [1] preventing the convergence of the solutions or significantly restricting the geometries which can be analyzed. The recursive centered T-matrix algorithm (RCTMA) [4] is a recursive algorithm that does not violate the AT as it is based on scatterer-centered T-matrices $\mathbf{T}_{j,i}$ relating the scattered fields of particle j to the incident field impinging on particle i. The reason is to obviate the numerical (truncated) evaluation of indirect translations that are the source of the aforementioned violations. This comes on the cost of computational efficiency as the number of matrices to be determined in each recurrence step is the squared number of particles taken into account in the respective step (in each recurrence step, the mutual interactions with one more particle is calculated) resulting in a computational complexity of $\mathcal{O}(N^3)$ for a cluster of N particles which is as slow as the direct application of Gaussian elimination.

The developed recursive aggregated centered T-matrix algorithm (RACTMA) [1] is also based on the scatterer-centered T-matrices and additionally accelerated using an aggregation procedure as in the recursive aggregated T-matrix algorithm (RATMA) [3]. Although the mutual interactions between all particles considered in the respective recurrence step are taken into account, only a subset of them is aggregated into an equivalent single particle. The particles to be aggregated are selected in each recurrence step based on rules to ensure the non-violation of the AT [1]

$$d_{0,j} < d_{0,k} - \rho_k \quad \forall \quad 1 \leq j \leq p \quad \text{and } n < k \leq N, \tag{0.1}$$

$$d_{0,k} > d_{0,j} + \rho_j \quad \forall \quad 1 \leq j \leq p \quad \text{and } n < k \leq N, \tag{0.2}$$

where $d_{j,k}$ is the distance between the centers of particles j and k (substricpt 0 referres to the center of the equivalent aggregate), ρ_j is the radius of particle j, p is the number of particles to be aggregated, and n is the number of the current recurrence step. (0.1) forces that in recurrence step $n \leq N$ (when particles $1 \ldots n$ are taken into account), the smallest circle [sphere in 3D] with its origin at the center of the equivalent aggregate comprising the origins of all aggregated particles [dashed line in Figure 1] does not intersect with or comprise any particle that has not

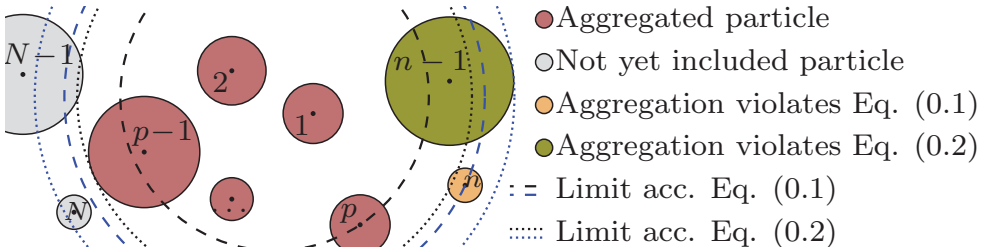

Figure 1: Visualisation of the aggregation rules (0.1) and (0.2).

yet been considered in the recurrence procedure [gray particles in Figure 1]. Accordingly, (0.2) ensures that the circumscribing circle [sphere in 3D] of the equivalent aggregate [dotted line in Figure 1] does not contain any origin of a particle that has not yet been considered in the recurrence procedure [gray particle in Figure 1]. (0.1) and (0.2) are equivalent in case of identical circumscribing circles [spheres in 3D] of all particles.

The cluster T-matrix is obtained by adding all not-yet aggregated particles in the last step. Due to the aggregation scheme, the computational complexity of the RACTMA scales as $\mathcal{O}(N^2)$ [1] allowing the analysis of clusters with a large number of particles. Further savings of computational costs can be achieved by nesting the RACTMA in itself [2]: since the cluster T-matrix allows an accurate representation of its scattering properties for all illuminations, well-separated clusters can be analyzed separately using the RACMTA and then the mutual interactions between the clusters can be taken into account using the RACTMA again. Although this procedure can be nested to any depth the geometry allows a separation, a 2-level procedure reduces both the computational complexity to $\mathcal{O}(N^\xi)\,(5/3 \leq \xi \leq 2)$, and the factor suppressed in the $\mathcal{O}(\bullet)$ notation [2]. The generalization to a) an n-level procedure and b) general and thus not well-separable geometries is under investigation from which we expect a further speedup and advantages in the application to structural optimization and inverse scattering analysis.

The numerical efficiency is demonstrated in Figure 2: Figure 2 (a) shows the CPU time to calculate the cluster T-matrix comprising up to $N = 3159$ circular-cylindrical particles with different material properties under TM-polarized normal inci-

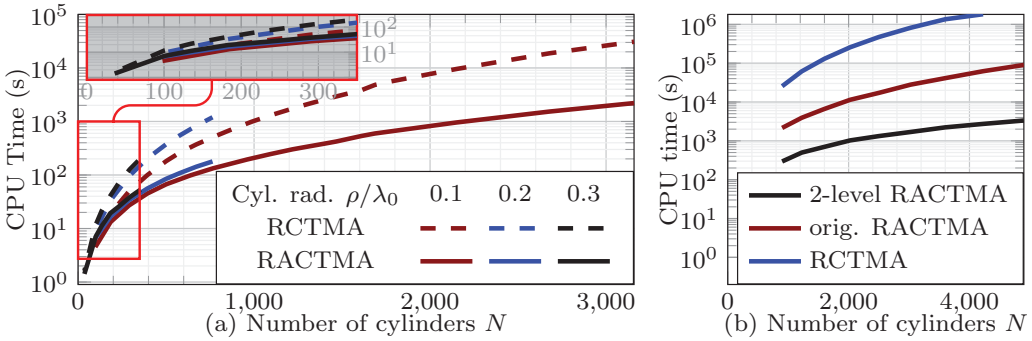

Figure 2: CPU time to calculate the cluster T-matrix of randomly arranged circular cylinders with different material properties using different methods: (a) under normal incidence and (b) for locally concentrated cylinders.

dence and Figure 2 (b) the CPU time of the 2-level implementation of the RACTMA contrasted to the RACTMA and RCTMA of up to $N = 4900$ circular cylinders with different material properties and radius $\rho/\lambda_0 = 0.15$, that are locally concentrated in $\sqrt{N}$ non-identical clusters, when oblique incidence is assumed.

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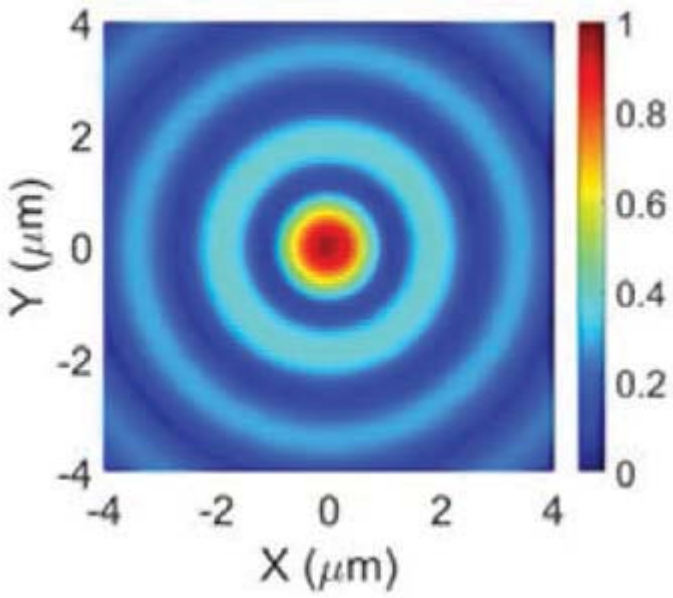

Thomas Wriedt

Jonas Gienger

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