A Full-wave Numerical Approach for Leaky Modes in EBG Structures

Vakhtang Jandieri[1], **Paolo Baccarelli**[2,3], **Guido Valerio**[4], **and Giuseppe Schettini**[2,3]

[1]General and Theoretical Electrical Engineering (ATE), Faculty of Engineering
University of Duisburg-Essen, CENIDE — Center for Nanointegration Duisburg-Essen
Duisburg D-47048, Germany
[2]EMLAB3 Laboratory of Electromagnetic Fields, Department of Engineering, "Roma Tre" University, Italy
[3]CNIT, "Roma Tre" Unit, Via Vito Volterra 62, Rome 00146, Italy
[4]Laboratoire d'Electronique et Electromagnetisme, Sorbonne Université, Paris 75252, France

Abstract— During the last decade, an extensive research effort has been aimed at studying and developing electromagnetic bandgap (EBG) structures [1]. EBGs are artificial periodic dielectric or metallic structures in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. This frequency range is called the bandgap, which is analogous to the energy bandgaps for electrons in semiconductors [2]. A periodic array of infinitely long parallel cylinders is a typical kind of discrete periodic system. EBG structures composed of cylindrical inclusions, enclosed in a finite number of stacked layers, have inspired great interest because of their novel scientific and engineering application as narrow-band filters, guiding devices, and substrates/covers for antennas.

EBG waveguides can be designed by removing one or more rows of the rods thus allowing to guide the electromagnetic waves with relatively low losses along particular directions [1]. When the number of the layers of cylinders is decreased, the EBG structure loses its complete band-gap properties and the modal field of the waveguide leaks out from the guiding structure. The losses along the structure are then described by a complex wavenumber. Knowledge of the real and complex propagation wavenumbers of bound and leaky modes supported by 2D EBG waveguides is essential for understanding of the fundamental parameters governing the design of leaky-wave antennas and of a variety of microwave and optical guiding devices. Therefore, the effective and rigorous full-wave modal analysis of EBG waveguides composed by multilayered arrays of 2D cylindrical inclusions is of particular interest.

In this regard, a rigorous and efficient full-wave numerical approach devoted to the modal analysis of 2D EBG waveguides is presented [3]. The proposed technique allows for the numerical study of bound and leaky modes propagating in various types of periodic and EBG structures. The method adopted here is based on the transition matrix (T-matrix) method, the Lattice Sums (LSs) and the generalized reflection and transmission matrices characterizing the nature of the EBG structure. Recently developed fast and accurate calculation for the LSs, based on higher-order spectral and spatial Ewald representations that are highly convergent also in the case of complex propagation wavenumbers [3, 4], is used. The proposed approach has shown a Gaussian convergence and allows for the correct spectral determination of each spatial harmonic constituting the leaky modal field.

We have numerically tested the accuracy and efficiency of the method for several types of periodic and EBG structures, including conventional dielectric periodic and EBG waveguides as well as most challenging plasmonic chains, where the intrinsic losses of the scatterers are properly considered. An excellent agreement has been observed in all cases. Future developments concern the application of the proposed method to the analysis and design of EBG based leaky-wave antennas, where the radiative features can be explained in terms of suitable leaky modes propagating along the EBG waveguides [5]. Radiation patterns both in infinite and truncated EBG structures are under investigation.

REFERENCES

1. Yasumoto, K., ed., *Electromagnetic Theory and Applications for Photonic Crystals*, CRC Press, Boca Raton, FL, 2005.
2. Yablonovitch, E., "Photonic band-gap structures," *J. Opt. Soc. Am. B*, Vol. 10, No. 2, 283–295, 1993.
3. Jandieri, V., P. Baccarelli, G. Valerio, and G. Schettini, "1-D Periodic lattice sums for complex and leaky waves in 2-D structures using higher-order ewald formulation," *IEEE Transactions on Antennas and Propagation*, (in press).

4. Baccarelli, P., V. Jandieri, G. Valerio, and G. Schettini, "Efficient computation of the lattice sums for leaky waves using the ewald method," *Proceedings of the 11-th European Conference on Antennas and Propagation*, 3233–3234, Paris, France, 2017.
5. Baccarelli, P., S. Ceccuzzi, V. Jandieri, C. Ponti, and G. Schettini, "Recent advances on EBG cavity antennas," *Proceedings of XXII Italian National Meeting on Electromagnetics (RINEM), Session: Antennas II*, 301–304, Cagliari, Italy, 2018.

Rome, Italy, June 17 – 20, 2019

A Full-Wave Numerical Approach for Leaky Modes in EBG Structures

V. Jandieri [1], P. Baccarelli [2], G. Valerio [3] and G. Schettini [2]

UNIVERSITÄT DUISBURG ESSEN Open-Minded — ROMA TRE UNIVERSITÀ DEGLI STUDI — SORBONNE UNIVERSITÉ

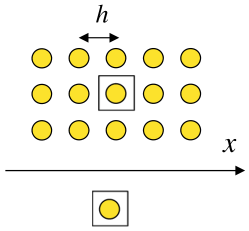
[1] *General and Theoretical Electrical Engineering (ATE), Faculty of Engineering, University of Duisburg-Essen, D-47048, Duisburg, Germany.*

[2] *Roma Tre University, Department of Engineering, Via Vito Volterra 62, 00146 Rome, Italy.*

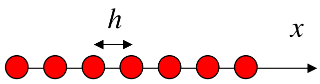
[3] Sorbonne Université, Laboratoire d'Electronique et Electromagnetisme, 75252, Paris, France.

Motivation (1)

2D Electromagnetic-Band-Gap (EBG) Structure

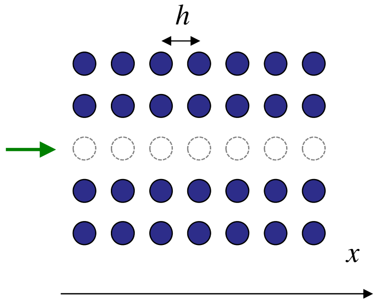


Periodic chain



Modal analysis of 2D periodic dielectric or metallic structures composed by *cylindrical* inclusions in a hosting dielectric medium

Planar 2D waveguide structure



The modal field is represented as a superposition of an *infinite* number of space harmonics

Goal: Derivation of the complex wavenumbers for *bound* modes in their stop-band regimes and *leaky* modes in their physical and non-physical regions

Space-harmonic complex wavenumber

$$k_{xn} = \beta_n + i\alpha, \quad n = 0, \pm 1, \pm 2, ..$$

$$\beta_n = \beta_0 + \frac{2\pi n}{h}$$

Space-harmonic phase constant

Modal attenuation (leakage) constant

Motivation (2)

- The complex wavenumbers are found by applying a rigorous and efficient formulation based on the Lattice Sums (LSs) technique combined with the Transition-matrix (T-matrix) approach and the recursive algorithm for the multilayered structure [1].

- The method is highly efficient, since the LSs are evaluated by using an effective Ewald approach [2] and a recursive relation for the layered structure is based on a simple matrix multiplication.

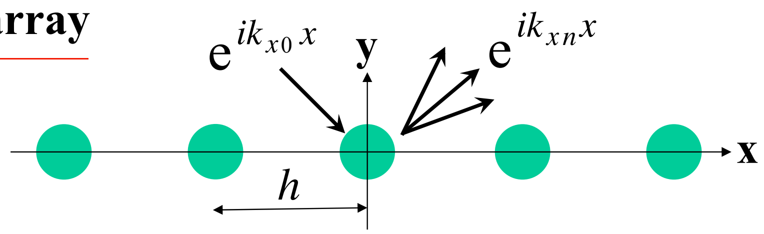
- The method allows for the appropriate choice of the spectral determination for each space harmonic in order to consider both proper and improper modal solutions [2].

- Radiative features of EBG Fabry-Perot cavities excited by simple localized sources (line or Hertzian dipole sources) at microwave and millimeter waves can be explained in terms of the leaky modes supported by the relevant open waveguide [3].

1. K. Yasumoto, H. Toyama and T. Kushta, *IEEE Transaction on Antennas and Propagation*, vol.52, pp.2603-2611, 2004.
2. V. Jandieri, P. Baccarelli, G. Valerio and G. Schettini, "1-D Periodic lattice sums for complex and leaky waves in 2-D structures using higher-order Ewald formulation," *IEEE Transaction on Antennas and Propagation*, vol. 67, no. 4, pp. 2364 - 2378, 2019.
3. S. Ceccuzzi, V. Jandieri, P. Baccarelli, C. Ponti and G. Schettini, *JOSA A*, vol.33, no.4, pp.764-770, 2016.

Formulation of the Problem (1)

Single array



$$e^{ik_{x0}x} \Rightarrow e^{ik_{xn}x}, \quad k_{xn} = k_{x0} + \frac{2n\pi}{h}$$

Reflected fields $(y>0)$

Transition matrix (T-matrix) — Lattice Sums

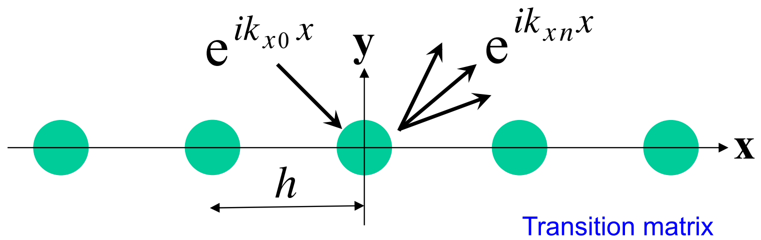
$$\psi_n^r(x,y) = r_n^{(+)} a^i e^{i(k_{xn}x + k_{yn}y)}, \quad r_n^{(+)} = \boldsymbol{u}_n^{(+)T} \cdot (\mathbf{I} - \mathbf{T}\cdot\mathbf{L})^{-1} \cdot \mathbf{T} \cdot \boldsymbol{p}^-$$

$r_n^{(+)}$: **0-th incident wave** ➡ ***n*-th reflected wave**

T-matrix is obtained in a closed form for cylindrical inclusions. It is a diagonal matrix.

Formulation of the Problem (2)

Single array



Transmitted fields $(y<0)$

Transition matrix (T-matrix)

Lattice Sums

$$\psi_n^t(x,y) = f_n^{(-)} a^i \, e^{i(k_{xn}x - k_{yn}y)}, \quad f_n^{(-)} = \delta_{n0} + \boldsymbol{u}_n^{(-)T} \cdot (\mathbf{I} - \mathbf{T}\cdot\mathbf{L})^{-1} \cdot \mathbf{T} \cdot \boldsymbol{p}^-$$

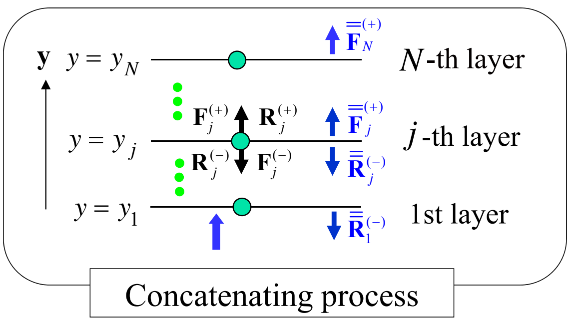
$f_n^{(-)}$: **0-th incident wave** ➡ ***n*-th transmitted wave**

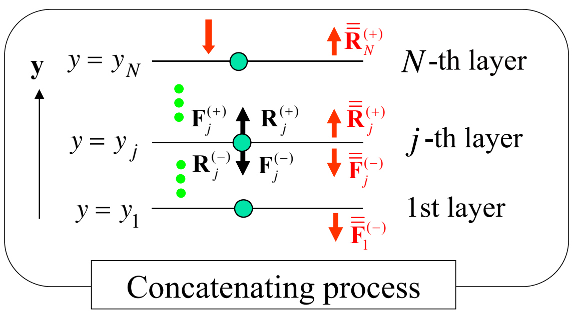
$$\boldsymbol{u}_n^{(\pm)} = [u_{nm}^{(\pm)}] = \left[\frac{2(-i)^m}{k_{yn}h} e^{\pm i m \alpha_n} \right],$$

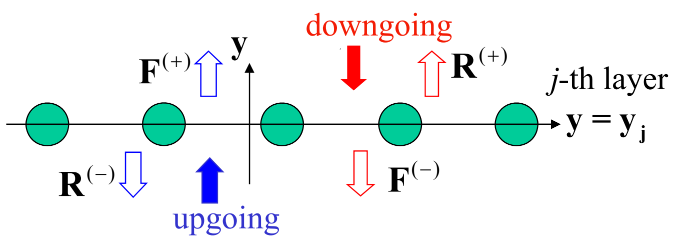
$$\alpha_n = \cos^{-1}\left(\frac{k_{xn}}{k_0}\right) \qquad k_{yn} = \sqrt{k^2 - k_{xn}^2}$$

The ***proper*** or ***improper*** features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber k_{yn}.

Formulation of the Problem (3)







$\mathbf{R}^{(\pm)}$: Reflection matrix for downgoing and upgoing space harmonics

$\mathbf{F}^{(\mp)}$: Transmission matrix for downgoing and upgoing space harmonics

Lattice-Sum (LS)

$$L_m(kh, k_{x0}h) = \sum_{n=1}^{\infty} \mathrm{H}_m^{(1)}(nkh)[\mathrm{e}^{ik_{x0}hn} + (-1)^m \mathrm{e}^{-ik_{x0}hn}]$$

L_m depends only on k, h (i.e., the period), and k_{x0} (i.e., the fundamental space-harmonic wavenumber); It is independent of the polarization of the incident field. It uniquely characterizes a periodic array of sources.

$$k_{x0} = \beta_0$$

Real wavenumber [1, 2]:
Slow converging series

$$k_{x0} = \beta_0 + i\alpha$$

Complex wavenumber [3, 4]:
Exponentially diverging series

[1] K. Yasumoto and K. Yoshitomo, *IEEE TAP*, vol.47, pp.1050-1055, 1999.

[2] C. M. Linton, "Lattice sums for the Helmholtz equation," SIAM Review, vol. 52, no. 4, pp. 630-674, 2010.

[3] P. Baccarelli, V. Jandieri, G. Valerio and G. Schettini, "Efficient computation of the lattice sums for leaky waves using the Ewald method," *Proceedings of the 11-th European Conference on Antennas and Propagation*, Paris, France, pp. 3233-3234, March, 2017.

[4] V. Jandieri, P. Baccarelli, G. Valerio and G. Schettini, "1-D Periodic lattice sums for complex and leaky waves in 2-D structures using higher-order Ewald formulation," *IEEE Transaction on Antennas and Propagation*, vol. 67, no. 4, 2019.

LSs for the complex wavenumber can be accurately calculated using *Ewald method*.
We calculate separately spectral and spatial series [3, 4]:

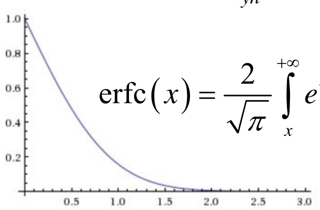
$$L_m = L_m^{E_{spectral}}(kh, k_{x0}h) + L_m^{E_{spatial}}(kh, k_{x0}h)$$

Lattice-Sum Ewald Spectral Series

After several mathematical manipulations, for the **Spectral** series we finally obtain:

$$L_m^{E_{spectral}}(kh, k_{x0}h) = \frac{2i^m}{h} \sum_{n=-\infty}^{\infty} \left(\frac{k_{xn}}{k}\right)^m \sum_{q=0}^{[m/2]} (-1)^q \binom{m}{2q} \left(\frac{k_{yn}}{k_{xn}}\right)^{2q} C_{q,n}, \quad m \geq 0$$

$$C_{q,n} = \frac{1}{k_{yn}} \mathrm{erfc}\left(-i\frac{hk_{yn}}{2E_{spl}}\right) - \frac{e^{\left(\frac{hk_{yn}}{2E_{spl}}\right)^2}}{k_{yn}} \sum_{s=1}^{q} \frac{\left(-i\frac{hk_{yn}}{2E_{spl}}\right)^{1-2s}}{\Gamma\left(\frac{3}{2} - s\right)},$$



$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2} \mathrm{d}t$$

$$k_{yn} = \sqrt{k^2 - k_{xn}^2}$$

The ***spectral*** higher-order Ewald series presents a ***very fast Gaussian convergence*** also for **complex** k_{x0}

The ***proper*** or ***improper*** features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber k_{yn}.

Lattice-Sum Ewald Spatial Series (1)

For the **Spatial** series, we obtain:

$$L_m^{E_{spatial}}(kh, k_{x0}h) = \delta_{m,0}\left[-1 - \frac{i}{\pi}\mathrm{Ei}\left(\frac{k^2h^2}{4E_{spl}^2}\right)\right]$$

$$+\frac{2^{m+1}}{i\pi}\sum_{n=1}^{\infty}[e^{ink_{x0}h} + (-1)^m e^{-ink_{x0}h}]\left(\frac{n}{kh}\right)^m \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2+\frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

How to compute this integral? A numerical integration could be slow and not robust…

$$I_m = \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2+\frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

Lattice-Sum Ewald Spatial Series (2)

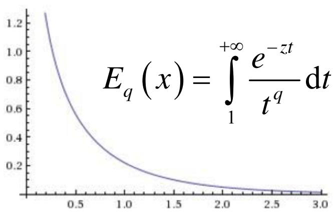
$$I_m = \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2+\frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

A recurrence relation in m have been obtained to **significantly speed up** the evaluation of the integrals in the spatial Ewald series

$$I_{m+1} = \frac{1}{2n^2}\left(2mI_m - \frac{k^2h^2}{2}I_{m-1} + E_{spl}^{2m} e^{-n^2E_{spl}^2} e^{k^2h^2/4E_{spl}^2}\right)$$

I_0 and I_1 can be easily obtained as

$$I_0 = \frac{1}{2}\sum_{p=0}^{\infty}\left(\frac{kh}{2E_{spl}}\right)^{2p}\frac{1}{p!}E_{p+1}\left(n^2E_{spl}^2\right) \qquad I_1 = \frac{E_{spl}^2}{2}\left[\frac{1}{n^2E_{spl}^2}e^{-n^2E_{spl}^2} + \sum_{p=1}^{\infty}\left(\frac{kh}{2}\right)^{2p}\frac{1}{p!}\frac{1}{E_{spl}^{2p}}E_p\left(n^2E_{spl}^2\right)\right]$$



$$E_q(x) = \int_1^{+\infty}\frac{e^{-zt}}{t^q}\mathrm{d}t$$

The spatial higher-order Ewald series also presents a ***very fast Gaussian convergence*** also for **complex** k_{x0}.

Spectral Properties of the Modal Solution

Behavior of each space harmonic in the air region, e.g.: $y > 0,\ e^{ik_{yn}y}$

$$k_{yn} = \sqrt{k^2 - k_{xn}^2} = \sqrt{k^2 - (\beta_n + i\alpha)^2} = \beta_{yn} + i\alpha_{yn}$$

Exponentially decaying

Proper determination: $\mathrm{Im}\{k_{yn}\} > 0$

Exponentially growing

Improper determination: $\mathrm{Im}\{k_{yn}\} < 0$

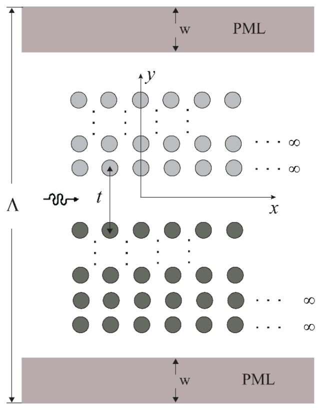
Physical choice for each space harmonic of the spectral series

	Surface Wave *(proper)*	**Forward Wave** *(improper leaky wave)*	**Backward Wave** (proper leaky wave)
Slow harmonics:	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$ $\alpha = 0$	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$
Fast harmonics:		$\lvert\beta_n\rvert < k : \mathrm{Im}\{k_{yn}\} < 0$ $\beta_n\alpha > 0$	$\lvert\beta_n\rvert < k : \mathrm{Im}\{k_{yn}\} > 0$ $\beta_n\alpha < 0$

Full-Wave Modal Analysis. Numerical Results

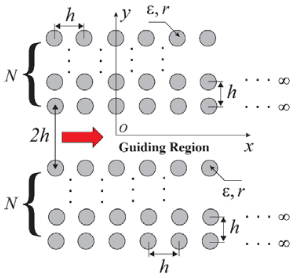
Fourier Series Expansion Method (FSEM)
Combined with Perfectly Matched Layers (PMLs)

- For validation purposes, a Fourier Series Expansion method (FSEM) with perfectly matched layers (PMLs) has been implemented to analyze 2-D EBG waveguides composed by cylindrical inclusions, whose section can have an arbitrary geometry.
- The electric and magnetic fields are approximated by truncated Fourier series.
- The FSEM uses the staircase approximation of the circular section by applying several multilayered thin rectangular strips.
- A substantial number of numerical tests are required to properly choose the PML parameters in order to distinguish the leakage loss from the material loss caused by the assumed conductivity in the PMLs.



D. Zhang and H. Jia, "Numerical analysis of leaky modes in two-dimensional photonic crystal waveguides using Fourier series expansion method with perfectly matched layer," *IEICE Transactions on Electronics*, vol. E90-C, pp. 613-622, 2007.

W1 Type EBG Waveguide: Improper Leaky Mode (1)

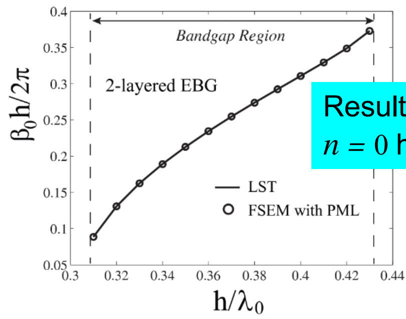


$N = 2$

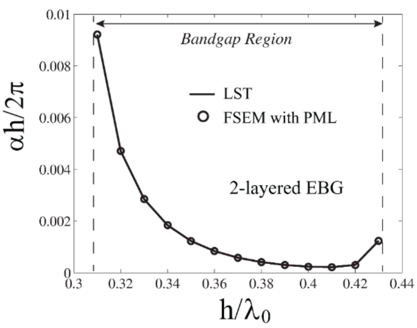
$\varepsilon = 11.9\ \varepsilon_0$
$r = 0.2\ h$

The periodic array of dielectric cylindrical rods has a bandgap region in the normalized frequency range $0.303 < h/\lambda_0 < 0.432$

Lowest order TE leaky mode (E_z, H_x, H_y)



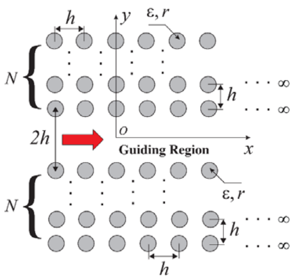
Results for
$n = 0$ harmonic



The $n = 0$ space harmonic is fast and has an improper determination in the LST, whereas all other harmonics are proper

The results obtained with the LST and FSEM with PML are in very close agreement with an accuracy of at least four digits

W1 Type EBG Waveguide: Improper Leaky Mode (2)



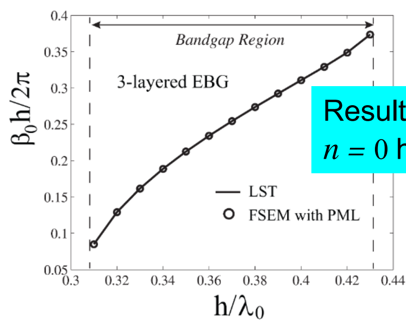
$N = 3$

$\varepsilon = 11.9\ \varepsilon_0$
$r = 0.2\ h$

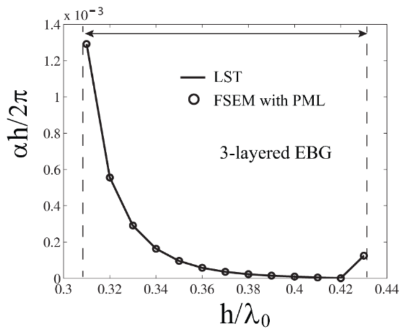
When the number of the EBG layers is increasing the attenuation constant substantially decreases.

$n = 0$ space harmonic is fast and has an improper determination in the LST

Lowest order TE leaky mode (E_z, H_x, H_y)

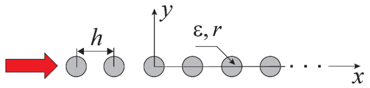


Results for
$n = 0$ harmonic



The LST is more efficient than the FSEM with PML: 0.02 s against 20 s per one frequency point with the same 3.6 GHz Intel Core i7 with 8 GB RAM

Periodic Chain of *Dielectric* Circular Rods



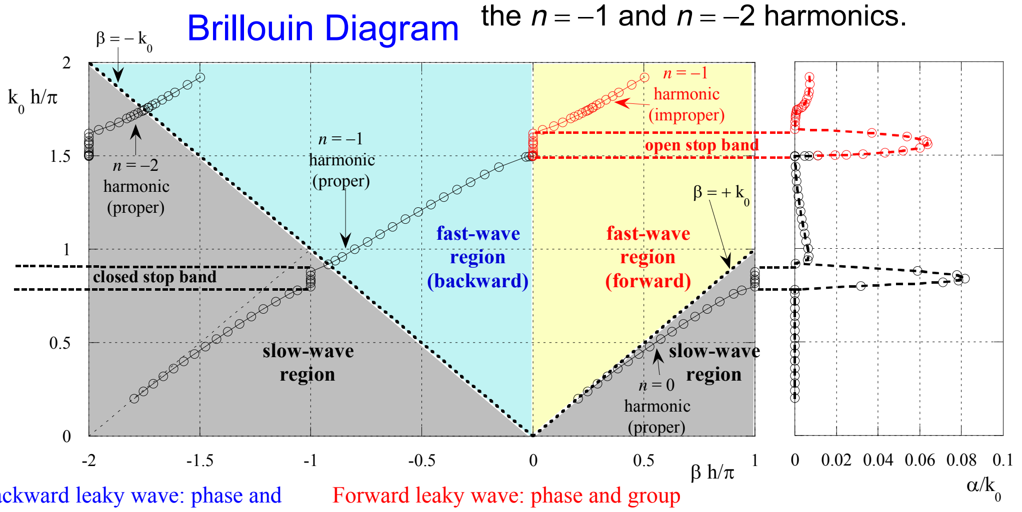
$\varepsilon = 2.25\varepsilon_0$

$r = 0.4167\ h$

Lowest order
TE leaky mode
(E_z, H_x, H_y)

Complete Brillouin diagram with the details of the backward and forward fast-wave regions for the $n = -1$ harmonic (proper/improper determinations), the closed and open stop-band regions, and the grating lobe due to the simultaneous radiation from the $n = -1$ and $n = -2$ harmonics.

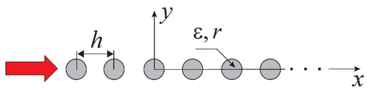
Brillouin Diagram



Backward leaky wave: phase and group velocities of opposite signs

Forward leaky wave: phase and group velocities are in the same direction

Periodic Chain of *Dielectric* Circular Rods



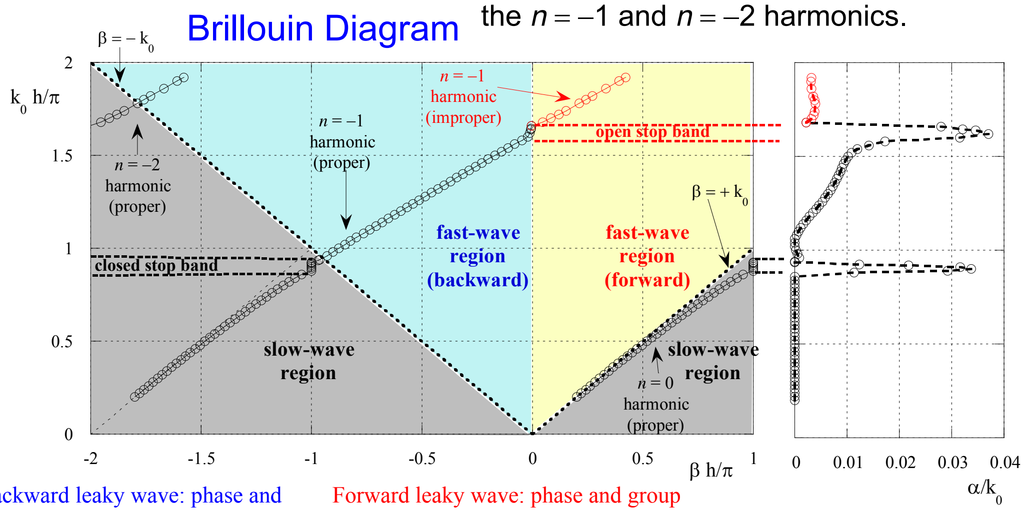
$\varepsilon = 2.25\varepsilon_0$

$r = 0.4167\ h$

Lowest order
TM leaky mode
(H_z, E_x, E_y)

Complete Brillouin diagram with the details of the backward and forward fast-wave regions for the $n = -1$ harmonic (proper/improper determinations), the closed and open stop-band regions, and the grating lobe due to the simultaneous radiation from the $n = -1$ and $n = -2$ harmonics.

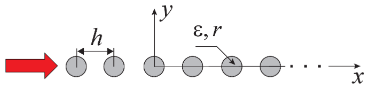
Brillouin Diagram



Backward leaky wave: phase and group velocities of opposite signs

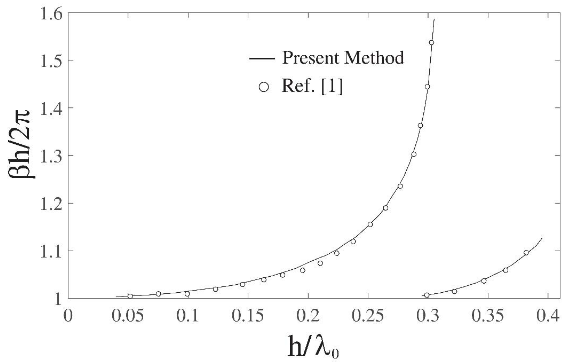
Forward leaky wave: phase and group velocities are in the same direction

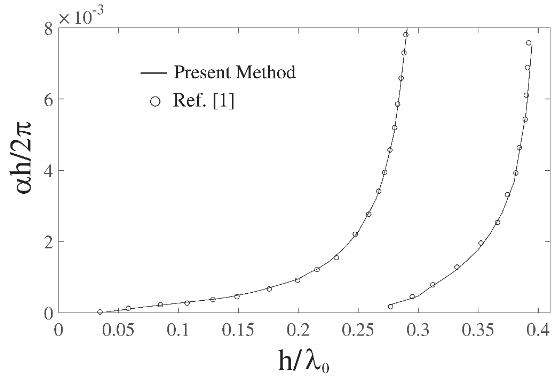
Periodic Chain of *Plasmonic* Circular Rods



$$\varepsilon = 1 - i\frac{\nu\omega_p^2}{\omega(1+i\nu\omega)}$$
$$\nu = 1.45\cdot10^{-14}\,s$$
$$\omega_p = 1.32\cdot10^{16}\,rad/s$$
$$r = 0.4167\,h$$

TM mode (H_z, E_x, E_y)

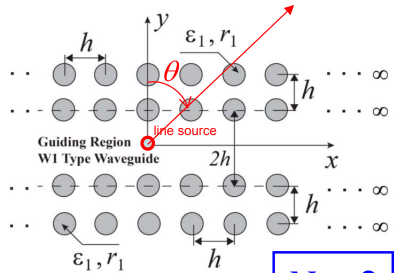




Propagation constant βh/2π and attenuation constant αh/2π versus the normalized frequency h/λ_0 for the fundamental and higher-order mode of *H*-wave for the 2-D periodic chain composed of the silver circular rods having radius *r* = 0.4167*h*. Solid line represent the results obtained based on the present method and the circles represent the results shown in Ref. [1].

[1] A. Hochman and Y. Leviatan, "Rigorous modal analysis of metallic nanowire chains," Optics Express, **17**, 13561-13575 (2009).

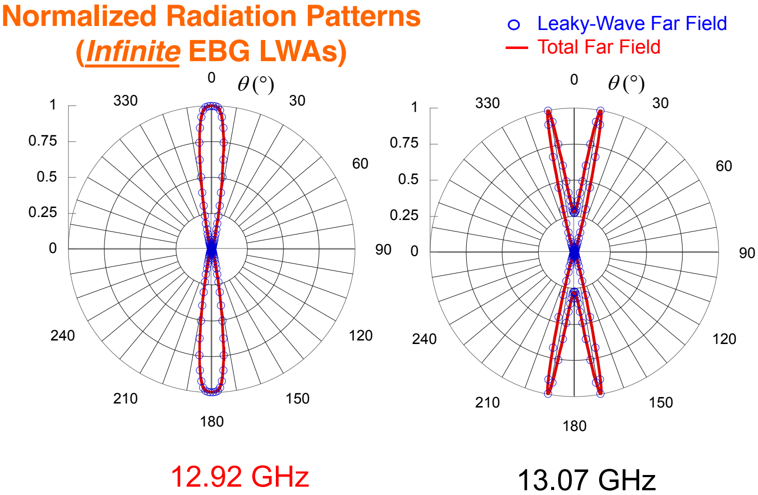
LW Radiation from infinite EBG structures



$N = 2$

$$h = 7\ mm$$
$$\varepsilon_1 = 11.9\varepsilon_0$$
$$r_1 = 0.2\,h$$

Normalized Radiation Patterns
(*Infinite* EBG LWAs)



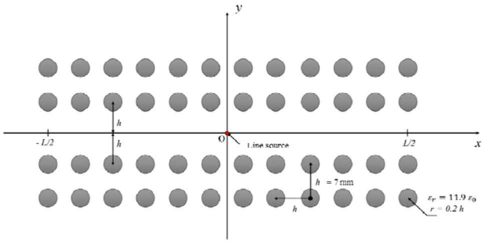
Broadside frequency →

f (GHz)	β/k_0	α/k_0	θ_{LW} (°)
12.92	0.1065	0.1066	0
13.07	0.1923	0.0531	10.5

Perfect agreement between normalized radiation patterns:
Total Field excited by an electric line source vs. Leaky-Wave field

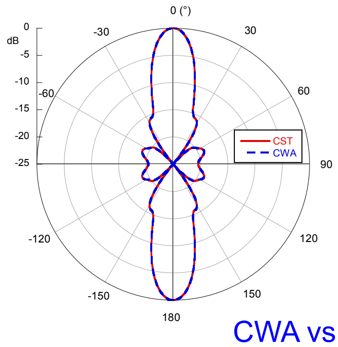
LWAs based on Truncated EBG Structures: Radiation at Broadside

Far-field features of ***Truncated*** EBG LWAs



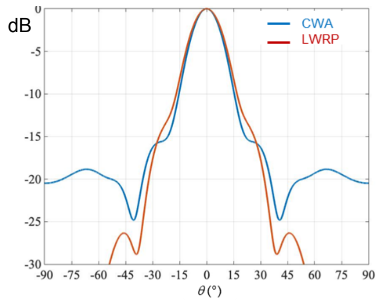
The length L/2 (=38.5 mm*) of the LWA has been fixed to radiate 90% of the injected power at the broadside frequency* (**12.92 GHz**)

- Theoretical LW Radiation Pattern (LWRP) based on the physical-optics approximation (i.e., perfect absorbers at the antenna truncations).
- Full-wave calculation of the field radiated by an elementary source in a truncated structures through an ad-hoc implementation of the Cylindrical Wave Approach (CWA).
- CST microwave studio: The excitation of a 2-D structure has ben properly emulated.



CWA vs CST

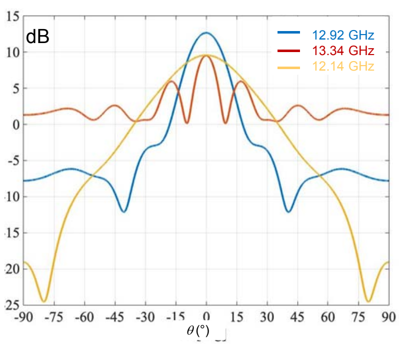
Excellent agreement between normalized radiation patterns obtained with the LW theory, CWA, and CST



LWRP vs CWA

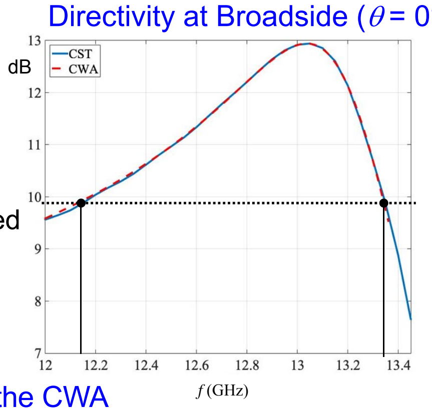
LWAs based on Truncated EBG Structures: 3dB Bandwidth at Broadside

Directivity Patterns



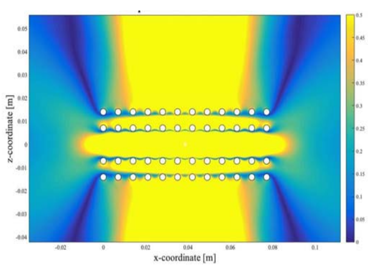
Directivity patterns obtained with the CWA and validated with CST show a fractional 3dB bandwidth at broadside a.e. to 10% for a truncated EBG LWA with a 90% radiation efficiency at broadside frequency

Directivity at Broadside ($\theta = 0°$)

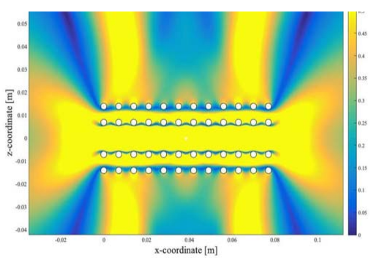


Near-field patterns obtained with the CWA

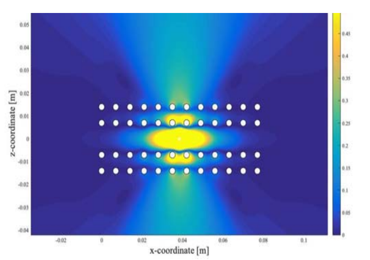
12.92 GHz
(Broadside frequency)



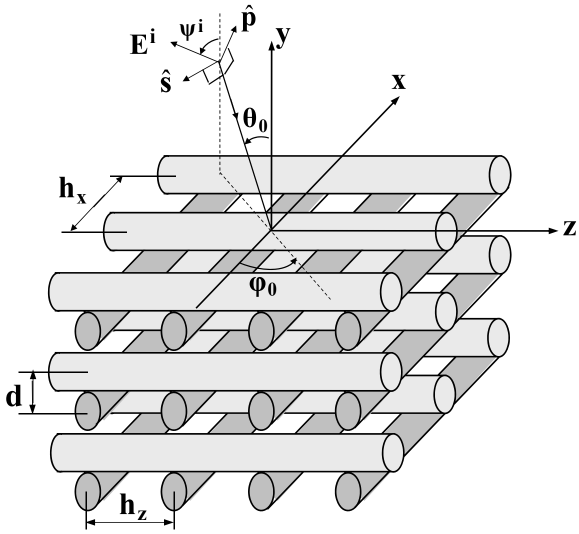
13.34 GHz



12.14 GHz

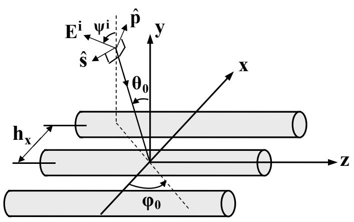


Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (1)

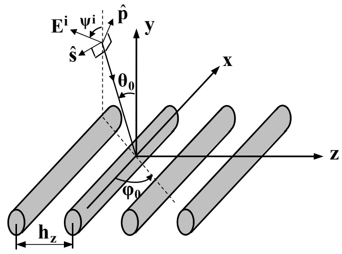


Layered crossed-arrays of cylindrical objects

Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (2)



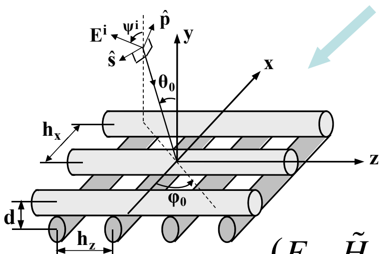
Z-array: $\mathbf{R}_{\parallel}$, $\mathbf{F}_{\parallel}$ $(E_z, \tilde{H}_z)$



X-array: $\mathbf{R}_x$, $\mathbf{F}_x$ $(E_x, \tilde{H}_x)$

Coordinate transformation

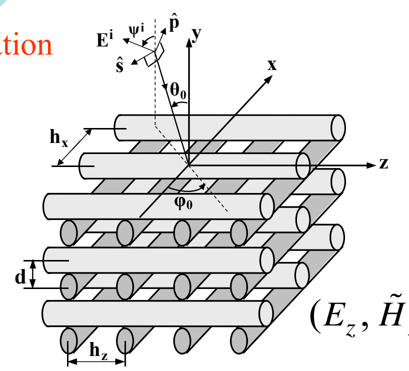
$\mathbf{R}_{\perp}$, $\mathbf{F}_{\perp}$ $(E_z, \tilde{H}_z)$



$(E_z, \tilde{H}_z)$

Crossed-array: $\mathbf{R}_{\times}^{(+)}$, $\mathbf{F}_{\times}^{(-)}$, $\mathbf{R}_{\times}^{(-)}$, $\mathbf{F}_{\times}^{(+)}$

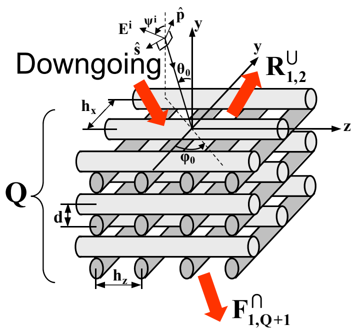
Stacked

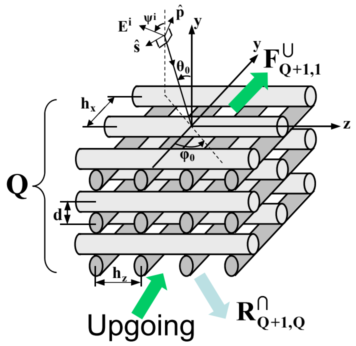


$(E_z, \tilde{H}_z)$

Layered crossed-arrays

Woodpile LWAs: Extension of the modal and radiative analyses to 3-D EBG structures (3)





$$\mathbf{R}^{\cup}_{\nu,\nu+1} = \mathbf{R}^{(+)}_{\times} + \mathbf{F}^{(+)}_{\times}\mathbf{W}\mathbf{R}^{\cup}_{\nu+1,\nu+2}\mathbf{\Lambda}^{(-)}_{\nu+1}\mathbf{W}\mathbf{F}^{(-)}_{\times}$$

$$\mathbf{F}^{\cap}_{1,Q+1} = \mathbf{F}^{(-)}_{\times}\mathbf{\Lambda}^{(-)}_{Q}\mathbf{\Lambda}^{(-)}_{Q-1}\cdots\mathbf{\Lambda}^{(-)}_{3}\mathbf{\Lambda}^{(-)}_{2}\mathbf{W}\mathbf{F}^{(-)}_{\times}$$

$$\mathbf{R}^{\cap}_{\nu+1,\nu} = \mathbf{R}^{(-)}_{\times} + \mathbf{F}^{(-)}_{\times}\mathbf{W}\mathbf{R}^{\cap}_{\nu,\nu-1}\mathbf{\Lambda}^{(+)}_{\nu}\mathbf{W}\mathbf{F}^{(+)}_{\times}$$

$$\mathbf{F}^{\cup}_{Q+1,1} = \mathbf{F}^{(+)}_{\times}\mathbf{\Lambda}^{(+)}_{2}\mathbf{\Lambda}^{(+)}_{3}\cdots\mathbf{\Lambda}^{(+)}_{Q-1}\mathbf{\Lambda}^{(+)}_{Q}\mathbf{W}\mathbf{F}^{(+)}_{\times}$$

$$\mathbf{\Lambda}^{(-)}_{\nu+1} = \left[\mathbf{I} - \mathbf{W}\mathbf{R}^{(-)}_{\times}\mathbf{W}\mathbf{R}^{\cup}_{\nu+1,\nu+2}\right]^{-1}$$

$$\mathbf{\Lambda}^{(+)}_{\nu} = \left[\mathbf{I} - \mathbf{W}\mathbf{R}^{(+)}_{\times}\mathbf{W}\mathbf{R}^{\cap}_{\nu,\nu-1}\right]^{-1}$$

$$\mathbf{W} = \left[\mathrm{e}^{i\gamma_{\ell}(m)d}\ \delta_{\ell\ell'}\right]$$

$$\mathbf{R}^{\cup}_{Q+1,Q+2} = \mathbf{0} \Rightarrow \mathbf{R}^{\cup}_{1,2},\quad \mathbf{F}^{\cap}_{1,Q+1}$$

$$\mathbf{R}^{\cap}_{1,0} = \mathbf{0} \Rightarrow \mathbf{R}^{\cap}_{Q+1,Q},\quad \mathbf{F}^{\cup}_{Q+1,1}$$

Conclusions

- A full-wave numerical approach for the analysis of modes with complex propagation wavenumber in periodic and bandgap structures composed of 2D cylindrical inclusions has been proposed.
- The method is based on the lattice sums (LSs) technique and has been suitably adapted to the analysis of modes with complex propagation wavenumbers, by applying higher-order Ewald representation, in terms of spectral and spatial series having Gaussian convergence.
- All the possible bound and leaky modes propagating along periodic and bandgap structures composed of 2D cylindrical inclusions can be considered.
- An exhaustive analysis of two reference 2D EBG waveguides has allowed us to characterize the relevant Pass-Band and Band-Gap Zones and the Radiative Regions.

Future works:

- Analysis of leakage and radiative phenomena in 2D EBG structures
- Design of filters and periodic Leaky Wave Antennas based on EBG structures

Thank You!