URSI GASS 2023, Sapporo, Japan, 19 – 26 August 2023

A Study on Eigenmode Analysis of Electromagnetic Wave Propagation in Post-Wall Waveguide

Koki Watanabe(1) and Vakhtang Jandieri(2)
(1) Fukuoka Institute of Technology, Fukuoka, Japan
(2) General and Theoretical Electrical Engineering (ATE), Faculty of Engineering, University of Duisburg-Essen and CENIDE - Center for Nanointegration Duisburg-Essen, D-47048 Duisburg, Germany

Post-wall waveguides (PWWs, also known as substrate integrated waveguides) are widely used in millimeter and sub-millimeter regions. The PWWs are formed between two parallel metal planes and the fields propagates along the cylindrical posts arranged between the metal plates. Usually, the distance between two metal planes is much smaller than the wavelength and the metal plates are regarded as perfect conductors. The electric field is therefore directed perpendicular to both plates and the fields between the plates are uniform in this direction. This implies that the fields propagating in the PWW are treated as a two-dimensional problem. Various PWW devices are realized by cascading post-wall elements or adding some cylindrical posts. Many of these devices can be thought as multilayered structures, in which each layer contains several cylindrical posts, and their properties are often characterized by numerical approaches for the multilayer structure scatterings.

This study deals with a straight PWW and consider a numerical method to obtain a practical set of the eigenmode fields in it. If the PWW devices have the input/output ports in the form of straight PWW, the eigenmode expansion expression of the fields in the straight PWW is required to give the input/output conditions. The structure under consideration is periodic in the propagation direction and the Floquet theorem asserts that the eigenmode fields have a pseudo-periodic property. The eigenmode fields propagating in the straight PWW are usually calculated by the following method. The eigenmode fields are pseudo-periodic, and the fields in homogeneous region can be therefore expressed in the Floquet-Fourier expansion. The post-walls are periodic array of circular cylinders and the scattering-matrices for the post-walls can be derived by the recursive transition-matrix algorithm with lattice sum technique. Then, the dispersion equation for the eigenmode fields are derived from the existence condition of the fields. The dispersion equation is numerically solved by using such as Müller’s method and the eigenmode fields are obtained. This method provides very accurate results for each eigenmode, but it is not easy to obtain a practical set of eigenmodes to express arbitrary fields in the PWW.

The formulation under consideration introduces the periodic boundary conditions and the fields are expressed in the Fourier series expansions [1]. Then, a set of eigenmode fields are simultaneously obtained by an eigenvalue/eigenvector analysis of the transfer matrix for the periodicity cell in the propagation direction. We also use the cylindrical-wave expansion expressions for the fields to treat the boundary conditions on the cylinder surface in an adequate manner and accelerate the convergence [2]. Main difficulty is to evaluate the leakage from the waveguiding structure. We use the perfectly matched layer [3] for this purpose.

References

[1] T. Hosono, T. Hinata, and A. Inoue, “Numerical Analysis of the Discontinuities in Slab Dielectric Waveguides,” *Radio Science*, **17**, 1, 1982, pp. 75-83.
[2] W. C. Chew, *Waves and Fields in Inhomogeneous Media*, Van Nostrand Reinhold, New York, 1990.
[3] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, 3rd ed., Artech House Publishers, 2005.

A Study on Eigenmode Analysis of Electromagnetic Wave Propagation in Post-Wall Waveguide

Koki Watanabe[1] and Vakhtang Jandieri[2]

[1] Fukuoka Institute of Technology, Fukuoka, Japan

[2] General and Theoretical Electrical Engineering (ATE), Faculty of Engineering, University of Duisburg-Essen and CENIDE - Center for Nanointegration Duisburg-Essen, D-47048 Duisburg, Germany

25 August 2023

URSI GASS 2023 (Sapporo, Japan)

Introduction (1/2)

- Periodic Waveguides
  - waveguiding structures with periodicity in the main propagation direction
    - photonic crystal waveguide (PBG/EBG waveguide), post-wall waveguide (substrate integrated waveguide), fiber grating, etc.
    - various ability to control and manipulate the propagation of EM waves.

  Waveguiding structure formed by periodic array of circular cylinders will be considered in this presentation.

- Fields in Periodic Structures (period in the x-direction: d)
  - Fields are supposed to be with the time-dependence of $\exp(-i\,\omega\,t)$.
  - Fields in a periodic waveguide are expressed as a superposition of Floquet-modes (eigenmodes in periodic waveguides) with pseudo-periodic property.
    - Pseudo-periodicity of each Floquet-mode ($(\boldsymbol{e}_m(x,y), \boldsymbol{h}_m(x,y))$: mth-modal fields):

$$\begin{pmatrix} \boldsymbol{e}_m(x+d,y) \\ \boldsymbol{h}_m(x+d,y) \end{pmatrix} = \eta_m \begin{pmatrix} \boldsymbol{e}_m(x,y) \\ \boldsymbol{h}_m(x,y) \end{pmatrix}$$

η_m: growing factor for periodicity cell

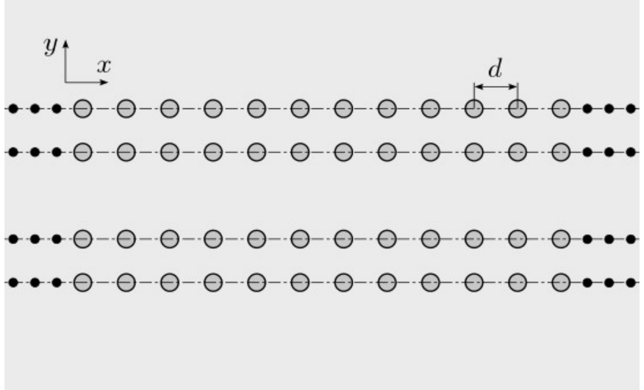
      - x-directional propagation constant of the mth-Floquet-mode

$$\xi_m = -\frac{i}{d}\ln(\eta_m)$$

        - complex logarithm is periodic in the imaginary direction.
        - ξ_m is here defined in the first Brillouin zone ($-\pi/d < \Re(\xi_m) \le \pi/d$).

Introduction (2/2)

- Numerical Methods for Floquet-mode Analysis



  - Multilayer Scattering Approach
    1. Scattering by each periodic circular cylinder array is described by the scattering-matrix among the plane-wave amplitudes.
    2. Reflection-matrices for cladding layers are derived by multilayer scattering technique.
    3. Dispersion equation is derived by the reflection-matrices for the cladding layers and the propagation-matrix for the core region.
    4. Dispersion equation is numerically solved.

    By solving the dispersion equation numerically, it is not easy to obtain sufficient number of Floquet-modes to approximate the fields in the periodic waveguide.
  - Supercell Approach
    1. The analysis region is limited in the perpendicular direction by introducing an artificial boundaries.
    2. The fields in the plane perpendicular to the propagation direction are expressed in the expansion form, and changing of the expansion coefficients for a periodicity cell is expressed in the transfer-matrix form.
    3. Eigenvalues/eigenvectors of the transfer-matrix provide the Floquet-modes.

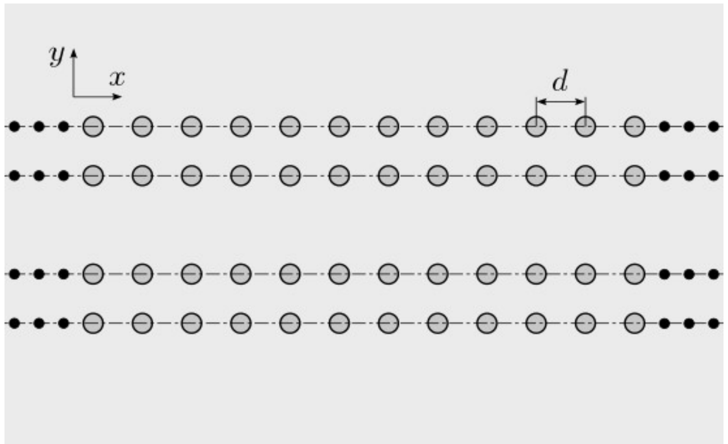
    A set of the Floquet-mode fields is simultaneously obtained.

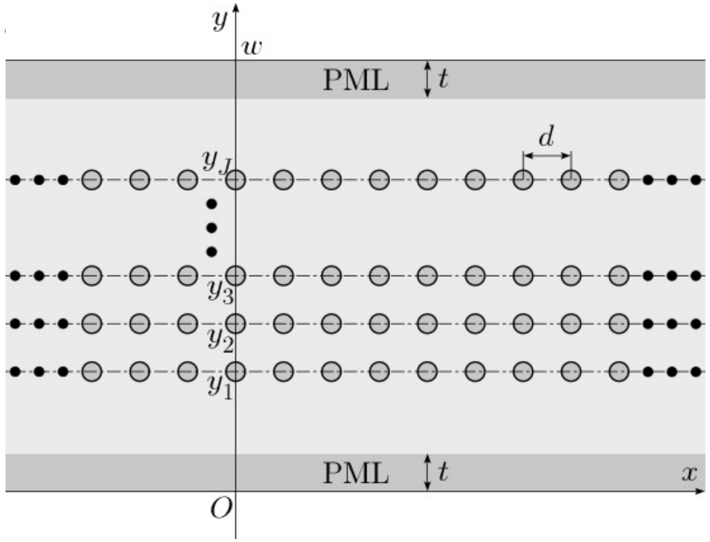
Settings of the Problem

Open-type periodic waveguides formed by periodic arrays of circular cylinders

- Waveguide Structure
  - surrounding medium: ε_s, μ_s, $k_s = \omega\sqrt{\varepsilon_s\,\mu_s}$
  - J cylinders are located at $x = l\,d$ (l: any integer)
    - jth-cylinder ($j = 1, \ldots, J$): ε_j, μ_j, radius a_j, location $(x, y) = (l\,d, y_j)$
- Analysis Region
  - Artificial boundaries at $y = 0,\ w$ (analysis region: $0 < y < w$)
    - Fields inside the analysis region are expressed in the Fourier series.
    - PMLs with thickness t are introduced near the artificial boundaries.

The parameters are chosen so that the cylinders





Eigenmodes in Surrounding Medium (1/2)

- Relations among the field components (use of the stretched-coordinate PML)

$$\frac{\partial H_y(x,y)}{\partial x} - \xi(y)\,\frac{\partial H_x(x,y)}{\partial y} = -i\,\omega\,\varepsilon_s\,E_z(x,y)$$
$$\frac{\partial E_z(x,y)}{\partial x} = -i\,\omega\,\mu_s\,H_y(x,y)$$
$$\xi(y)\,\frac{\partial E_z(x,y)}{\partial y} = i\,\omega\,\mu_s\,H_x(x,y)$$
$$\frac{\partial E_y(x,y)}{\partial x} - \xi(y)\,\frac{\partial E_x(x,y)}{\partial y} = i\,\omega\,\mu_s\,H_z(x,y)$$
$$\frac{\partial H_z(x,y)}{\partial x} = i\,\omega\,\varepsilon_s\,E_y(x,y)$$
$$\xi(y)\,\frac{\partial H_z(x,y)}{\partial y} = -i\,\omega\,\varepsilon_s\,E_x(x,y)$$

$$\Rightarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\,\boldsymbol{h}_y(x) - i\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{h}_x(x) = -i\,\omega\,\varepsilon_s\,\boldsymbol{e}_z(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\,\boldsymbol{e}_z(x) = -i\,\omega\,\mu_s\,\boldsymbol{h}_y(x)$$
$$i\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{e}_z(x) = i\,\omega\,\mu_s\,\boldsymbol{h}_x(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\,\boldsymbol{e}_y(x) - i\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{e}_x(x) = i\,\omega\,\mu_s\,\boldsymbol{h}_z(x)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\,\boldsymbol{h}_z(x) = i\,\omega\,\varepsilon_s\,\boldsymbol{e}_y(x)$$
$$i\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{h}_z(x) = -i\,\omega\,\varepsilon_s\,\boldsymbol{e}_x(x)$$

  - $\xi(y) = 1/(1 + i\,\sigma(y))$ (conductivity of PML: $\sigma(y)$)

- Fourier series expansion ($\Psi = \boldsymbol{E}, \boldsymbol{H}$ and $p = x,\ y,\ z$)

$$\Psi_p(x,y) = \sum_{n=-N}^{N} \Psi_{p,n}(x)\,e^{i\,n\,k_w\,y}, \qquad k_w = \frac{2\,\pi}{w}$$

$$\big(\boldsymbol{\psi}_p(x)\big)_n = \Psi_{p,n}(x)$$
$$(\boldsymbol{Y})_{n,m} = \delta_{n,m}\,n\,k_w$$
$$([\![\xi]\!])_{n,m} = \frac{1}{w}\int_0^w \xi(y)\,e^{-i(n-m)k_w\,y}\,\mathrm{d}y$$

Eigenmodes in Surrounding Medium (2/2)

- Differential-equations for the Fourier coefficient of z-components

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\,\boldsymbol{\psi}_z(x) = -\boldsymbol{M}\,\boldsymbol{\psi}_z(x), \qquad \boldsymbol{M} = {k_s}^2\,\boldsymbol{I} - ([\![\xi]\!]\,\boldsymbol{Y})^2$$

  - General solution
    - Eigenvalues of M: $\{\gamma_m(x)^2\}_{m\in[1,2N+1]}$ and associated eigenvectors: $\{p_m\}_{m\in[1,2N+1]}$

$$\boldsymbol{\psi}_z(x) = \boldsymbol{P}\left(\boldsymbol{a}^{(\psi,+)}(x) + \boldsymbol{a}^{(\psi,-)}(x)\right), \qquad \boldsymbol{P} = \begin{pmatrix}\boldsymbol{p}_1 & \cdots & \boldsymbol{p}_{2N+1}\end{pmatrix}$$

$$\boldsymbol{a}^{(\psi,\pm)}(x) = \boldsymbol{U}(\pm(x-x'))\,\boldsymbol{a}^{(\psi,\pm)}(x'), \qquad (\boldsymbol{U}(x))_{m,n} = \delta_{m,n}\,e^{i\,\gamma_m\,x}$$

    - elements of $a^{(\psi,\pm)}(x)$ give the eigenmode amplitudes propagating in the $\pm\,z$-direction.
  - Fourier coefficients of the x, y-components

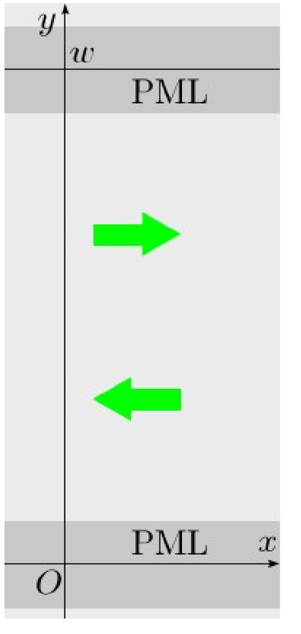
$$\boldsymbol{h}_x(x) = \frac{1}{\omega\,\mu_s}\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{P}\left(\boldsymbol{a}^{(e,+)}(x) + \boldsymbol{a}^{(e,-)}(x)\right)$$

$$\boldsymbol{h}_y(x) = -\frac{1}{\omega\,\mu_s}\,\boldsymbol{P}\,\boldsymbol{\Gamma}\left(\boldsymbol{a}^{(e,+)}(x) - \boldsymbol{a}^{(e,-)}(x)\right)$$

$$\boldsymbol{e}_x(x) = -\frac{1}{\omega\,\varepsilon_s}\,[\![\xi]\!]\,\boldsymbol{Y}\,\boldsymbol{P}\left(\boldsymbol{a}^{(h,+)}(x) + \boldsymbol{a}^{(h,-)}(x)\right)$$

$$\boldsymbol{e}_y(x) = \frac{1}{\omega\,\varepsilon_s}\,\boldsymbol{P}\,\boldsymbol{\Gamma}\left(\boldsymbol{a}^{(h,+)}(x) - \boldsymbol{a}^{(h,-)}(x)\right)$$

$$(\boldsymbol{\Gamma})_{m,n} = \delta_{m,n}\,\gamma_m$$



Scattering by Cylinder Array Located at $x = 0$ (1/2)

- Incident Fields for Cylinder Array ($\varPsi = \boldsymbol{E}, \boldsymbol{H}$)

$$\varPsi_z^{(i)}(x,y) = \sum_{n=-N}^{N} \left(\boldsymbol{P} \left(\boldsymbol{U}(x)\, \boldsymbol{a}^{(\psi,+)}(-0) + \boldsymbol{U}(-x)\, \boldsymbol{a}^{(\psi,-)}(+0) \right) \right)_n e^{i\,n\,k_w\,y}$$

  - cylindrical-wave expansion representation near the jth-cylinder

$$\varPsi_z^{(i)}(x,y) = \boldsymbol{g}^{(J)}(x, y - y_j)^t\, \boldsymbol{c}_j^{(\psi,i)}$$

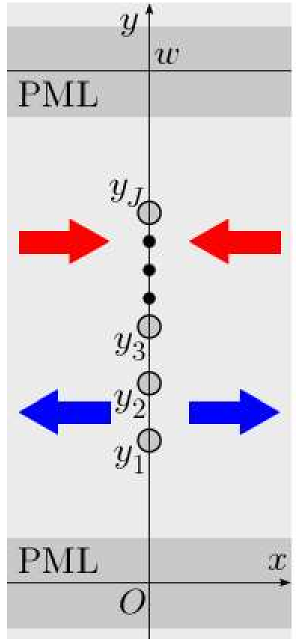
    - cylindrical-wave expansion bases ($Z = J, H^{(1)}$)

$$\left(\boldsymbol{g}^{(Z)}(x,y) \right)_n = Z_n(k_s\, \rho(x,y))\, e^{i\,n\,\phi(x,y)}$$

$$\rho(x,y) = \sqrt{x^2 + y^2}, \qquad \phi(x,y) = \arg(x + i\,y)$$

    - expansion coefficients (concrete representation of $\boldsymbol{A}_j^{(\pm)}$ is omitted here)

$$\boldsymbol{c}_j^{(\psi,i)} = \boldsymbol{A}_j^{(+)}\, \boldsymbol{a}^{(\psi,+)}(-0) + \boldsymbol{A}_j^{(-)}\, \boldsymbol{a}^{(\psi,-)}(+0)$$



- Scattering Fields from Cylinder Array

$$\varPsi_z^{(s)}(x,y) = \sum_{j=1}^{J} \boldsymbol{g}^{(H^{(1)})}(x, y - y_j)^t\, \boldsymbol{c}_j^{(\psi,s)}$$

  - expansion coefficients (concrete representation of $\widetilde{\boldsymbol{C}}^{(\psi)}$ is omitted here)

$$\begin{pmatrix} \boldsymbol{c}_1^{(\psi,s)} \\ \vdots \\ \boldsymbol{c}_J^{(\psi,s)} \end{pmatrix} = \widetilde{\boldsymbol{C}}^{(\psi)} \begin{pmatrix} \boldsymbol{c}_1^{(\psi,i)} \\ \vdots \\ \boldsymbol{c}_J^{(\psi,i)} \end{pmatrix}$$

Scattering by Cylinder Array Located at $x=0$ (2/2)

- Fields Propagating in the $+x$-direction for $y > a_{\max}$ ($a_{\max} = \max\{a_j\}$)

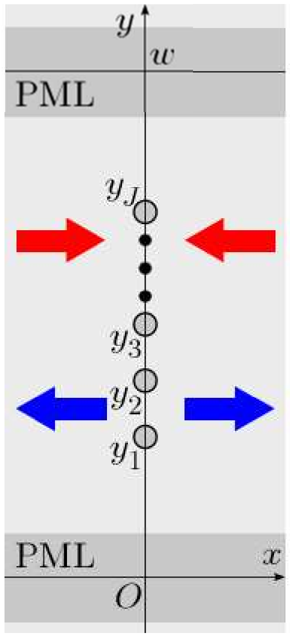
$$\sum_{n=-N}^{N} \left(\boldsymbol{P}\,\boldsymbol{U}(x)\,\boldsymbol{a}^{(\psi,+)}(+0)\right)_n e^{i\,n\,k_w\,y} = \sum_{n=-N}^{N} \left(\boldsymbol{P}\,\boldsymbol{U}(x)\,\boldsymbol{a}^{(\psi,+)}(-0)\right)_n e^{i\,n\,k_w\,y} + \sum_{j=1}^{J} \boldsymbol{g}^{(H^{(1)})}(x, y-y_j)^t\, \boldsymbol{c}_j^{(\psi,s)}$$

- Fields Propagating in the $-x$-direction for $y < -a_{\max}$

$$\sum_{n=-N}^{N} \left(\boldsymbol{P}\,\boldsymbol{U}(x)\,\boldsymbol{a}^{(\psi,-)}(-0)\right)_n e^{i\,n\,k_w\,y} = \sum_{n=-N}^{N} \left(\boldsymbol{P}\,\boldsymbol{U}(x)\,\boldsymbol{a}^{(\psi,-)}(+0)\right)_n e^{i\,n\,k_w\,y} + \sum_{j=1}^{J} \boldsymbol{g}^{(H^{(1)})}(x, y-y_j)^t\, \boldsymbol{c}_j^{(\psi,s)}$$

  - eigenmodal amplitudes (concrete representation of $\boldsymbol{B}_j^{(\pm)}$ is omitted here)

$$\boldsymbol{a}^{(\psi,\pm)}(\pm 0) - \boldsymbol{a}^{(\psi,\pm)}(\mp 0) = \sum_{j=1}^{J} \boldsymbol{B}_j^{(\pm)}\, \boldsymbol{c}_j^{(\psi,s)}$$



- Input/Output Relation for Scattering by Cylinder Array

$$\begin{pmatrix} \boldsymbol{a}^{(\psi,-)}(-0) \\ \boldsymbol{a}^{(\psi,+)}(+0) \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{S}}_{11}^{(\psi)} & \widetilde{\boldsymbol{S}}_{12}^{(\psi)} \\ \widetilde{\boldsymbol{S}}_{21}^{(\psi)} & \widetilde{\boldsymbol{S}}_{22}^{(\psi)} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}^{(\psi,+)}(-0) \\ \boldsymbol{a}^{(\psi,-)}(+0) \end{pmatrix}$$

  - scattering-matrices

$$\boldsymbol{S}_{11}^{(\psi)} = \widetilde{\boldsymbol{B}}^{(-)}\, \widetilde{\boldsymbol{C}}^{(\psi)}\, \widetilde{\boldsymbol{A}}^{(+)}$$
$$\boldsymbol{S}_{12}^{(\psi)} = \widetilde{\boldsymbol{B}}^{(-)}\, \widetilde{\boldsymbol{C}}^{(\psi)}\, \widetilde{\boldsymbol{A}}^{(-)} + \boldsymbol{I}$$
$$\boldsymbol{S}_{21}^{(\psi)} = \widetilde{\boldsymbol{B}}^{(+)}\, \widetilde{\boldsymbol{C}}^{(\psi)}\, \widetilde{\boldsymbol{A}}^{(+)} + \boldsymbol{I}$$
$$\boldsymbol{S}_{22}^{(\psi)} = \widetilde{\boldsymbol{B}}^{(+)}\, \widetilde{\boldsymbol{C}}^{(\psi)}\, \widetilde{\boldsymbol{A}}^{(-)}$$

$$\widetilde{\boldsymbol{A}}^{(\pm)} = \begin{pmatrix} \boldsymbol{A}_1^{(\pm)} \\ \vdots \\ \boldsymbol{A}_J^{(\pm)} \end{pmatrix}$$

$$\widetilde{\boldsymbol{B}}^{(\pm)} = \begin{pmatrix} \boldsymbol{B}_1^{(\pm)} & \cdots & \boldsymbol{B}_J^{(\pm)} \end{pmatrix}$$

Floquet-Modes in the Periodic Waveguide

- Input/Output Relation for Periodicity Cell ($U(x)$: propagation matrix)

$$\begin{pmatrix} \boldsymbol{a}^{(\psi,-)}(-0) \\ \boldsymbol{a}^{(\psi,+)}(d-0) \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{S}}_{11}^{(\psi)} & \widetilde{\boldsymbol{S}}_{12}^{(\psi)}\,\boldsymbol{U}(d) \\ \boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{21}^{(\psi)} & \boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{22}^{(\psi)}\,\boldsymbol{U}(d) \end{pmatrix} \begin{pmatrix} \boldsymbol{a}^{(\psi,+)}(-0) \\ \boldsymbol{a}^{(\psi,-)}(d-0) \end{pmatrix}$$

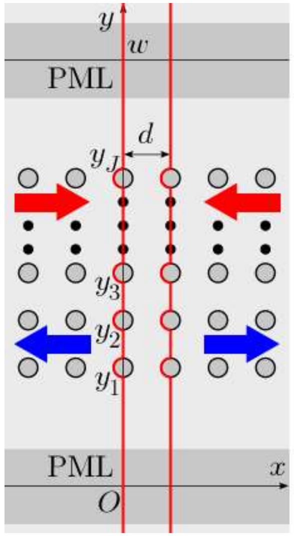
- Transfer Relation

$$\begin{pmatrix} \boldsymbol{0} & \widetilde{\boldsymbol{S}}_{12}^{(\psi)}\,\boldsymbol{U}(d) \\ \boldsymbol{I} & -\boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{22}^{(\psi)}\,\boldsymbol{U}(d) \end{pmatrix} \begin{pmatrix} \boldsymbol{a}^{(\psi,+)}(d-0) \\ \boldsymbol{a}^{(\psi,-)}(d-0) \end{pmatrix} = \begin{pmatrix} -\widetilde{\boldsymbol{S}}_{11}^{(\psi)} & \boldsymbol{I} \\ \boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{21}^{(\psi)} & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{a}^{(\psi,+)}(-0) \\ \boldsymbol{a}^{(\psi,-)}(-0) \end{pmatrix}$$

  - Corresponding generalized eigenvalue equation

$$\eta_m^{(\psi)} \begin{pmatrix} \boldsymbol{0} & \widetilde{\boldsymbol{S}}_{12}^{(\psi)}\,\boldsymbol{U}(d) \\ \boldsymbol{I} & -\boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{22}^{(\psi)}\,\boldsymbol{U}(d) \end{pmatrix} \boldsymbol{r}_m^{(\psi)} = \begin{pmatrix} -\widetilde{\boldsymbol{S}}_{11}^{(\psi)} & \boldsymbol{I} \\ \boldsymbol{U}(d)\,\widetilde{\boldsymbol{S}}_{21}^{(\psi)} & \boldsymbol{0} \end{pmatrix} \boldsymbol{r}_m^{(\psi)}$$

    - Eigenvalues $\{\eta_m^{(\psi)}\}_{m=1}^{2N+1}$ give the growing factors of the Floquet-modes.
      - Propagation constant of the mth-Floquet-mode: $\xi_m^{(\psi)} = -\dfrac{i}{d}\ln(\eta_m^{(\psi)})$
    - Associated eigenvectors $\{\boldsymbol{r}_m^{(\psi)}\}_{m=1}^{2N+1}$ give the modal fields.



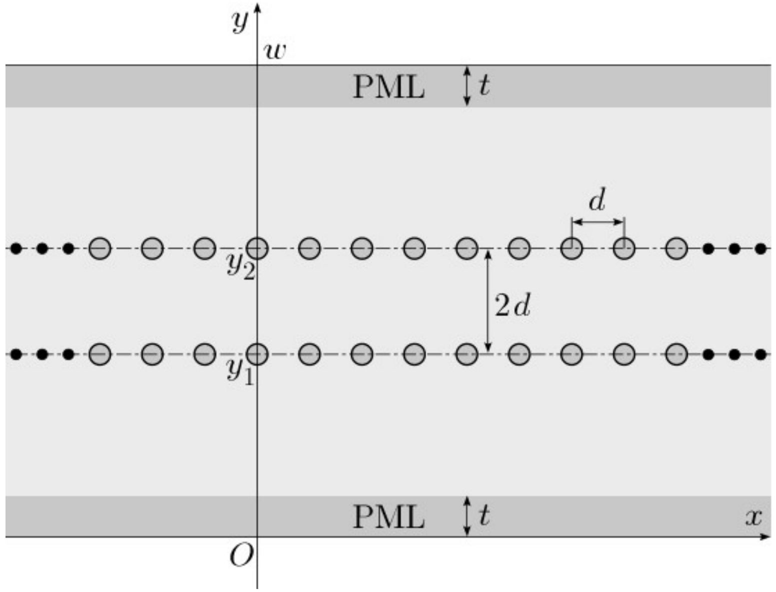
Numerical Examples (1/2)

- Used Prameters
  - surrounding medium: ε_0, μ_0
  - cylinders: $11.56\,\varepsilon_0,\ \mu_0,\ a = 0.353\,\mathrm{mm},\ d = 1.96\,\mathrm{mm}$
  - width of analysis region: $w = 14\,\mathrm{mm}$
  - truncation order for
    - Fourie series expansions: N
    - cylindrical-wave expansions: K
  - PML ($t = 2\,\mathrm{mm},\ \sigma_{\max} = 8$)
    - type of conductivity distribution
      - power function distribution

$$\sigma(y) = \begin{cases} \sigma_{\max}\left(\frac{t-y}{t}\right)^q & : 0 \le y < t \\ 0 & : t \le y < w-t \\ \sigma_{\max}\left(\frac{y-w+t}{t}\right)^q & : w-t \le y < w \end{cases}$$

      - step distribution

$$\sigma(y) = \begin{cases} \sigma_{\max} & : 0 \le y < t \\ 0 & : t \le y < w-t \\ \sigma_{\max} & : w-t \le y < w \end{cases}$$

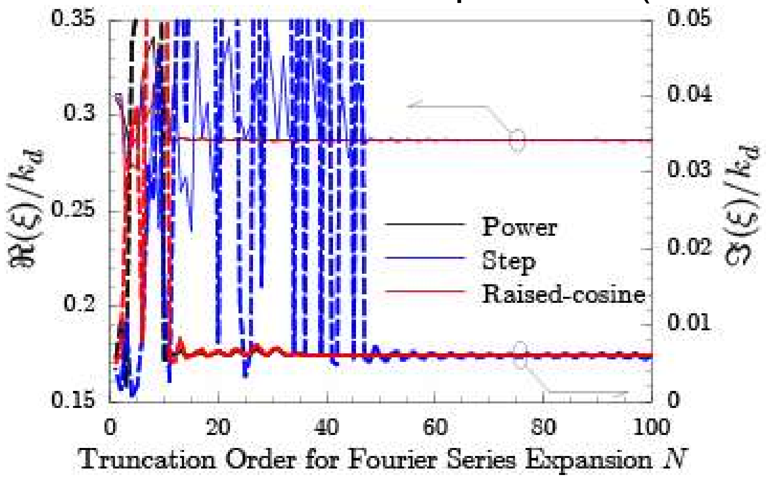


      - raised cosine distribution

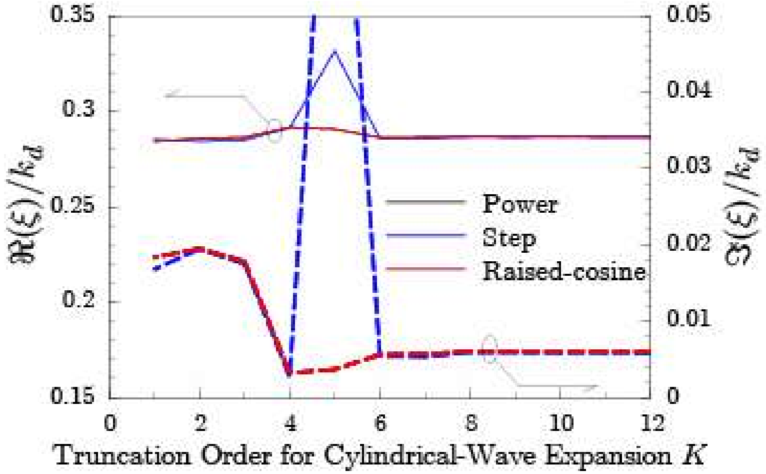
$$\sigma(y) = \begin{cases} \frac{\sigma_{\max}}{2}\left(1 + \cos(\frac{\pi}{t}\,y)\right) & : 0 \le y < t \\ 0 & : t \le y < w-t \\ \frac{\sigma_{\max}}{2}\left[1 + \cos(\frac{\pi}{t}\,(y-w))\right] & : w-t \le y < w \end{cases}$$

Numerical Examples (2/2)

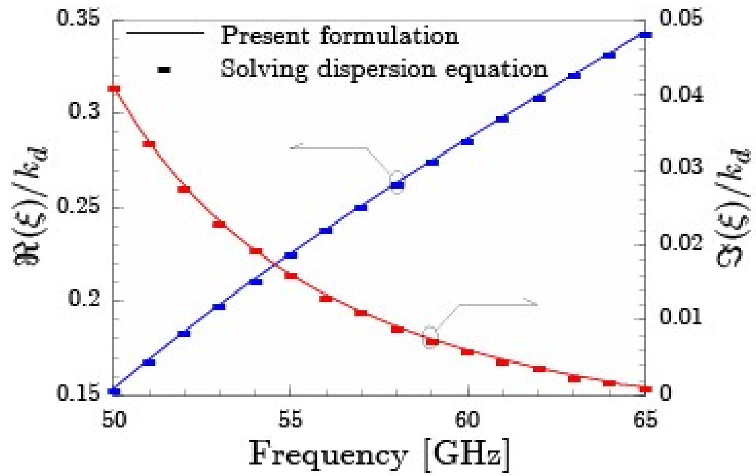
- Convergence Test
  - for Fourier series expansions ($K = 10$)



  - for cylindrical-wave expansions ($N = 50$)



- Example of dispersion curve
  - truncation orders: $N = 50$, $K = 10$
  - conductivity distribution: raised-cosine



Concluding Remarks

- This presentation has shown a techniques to compute a set of the Floquet-modes propagating in open-type periodic waveguides formed by periodic arrays of circular cylinders.
  - The periodic boundaries are introduced to limit the analysis region.
    - The leaky waves are absorbed by the PMLs.
  - Set of the Floquet-modes are obtained by solving an eigenvalue/eigenvector problem for the wave propagation for a periodicity cell.
    - The transfer relation for a periodicity cell is derived by using the recursive transition-matrix algorithm.
- Numerical experiments are performed for a waveguide formed by 2 layers of periodic dielectric cylinder arrays.
  - Convergence for the truncation order is very fast, and accurate results are obtained in very short computation time.