A concatenated Reed–Solomon–SPC coding scheme with soft decision *

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The concatenation of Reed–Solomon (RS) codes and Single Parity Check (SPC) codes is considered for two different combining concepts. In the first concept each Reed–Solomon code symbol is extended with a parity bit (symbolwise SPC encoding). In the second concept for each block of $(N-1)$ Reed–Solomon codewords a parity word is generated, which is also a Reed–Solomon codeword (blockwise SPC encoding). Soft decision SPC decoding is applied in both concepts. Simulation results for the decoding error rate of the different concatenated coding systems are given and compared with the results of standard Reed–Solomon coded systems. The digital simulations are based on the assumptions of a binary transmission with antipodal signaling and coherent detection over an Additive White Gaussian Noise Channel (AWGNC) as well as a Rayleigh fading channel.

Keywords: soft-decision, Reed-Solomon, concatenation, coding

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1. Introduction

The class of Reed–Solomon $((n, k)-\text{RS})$ codes has found widespread applications in satellite communications. RS codes, being maximum distance separable, can be advantageously used in channels characterized by burst errors as in fading channels. The high decoding speed of algebraic RS decoders is a second benefit which makes RS codes attractive particularly for high data rate applications. Reed–Solomon decoders with decoding speed up to 40 Mbit/s are commercially available. However, with algebraic RS decoders only hard decision decoding can be realized. Hard decision decoding leads to a loss in signal-to-noise ratio of more than 2 dB as compared with full soft decision decoding.

For soft decision decoding of block codes various algorithms have been proposed [1]–[3]. The application of those decoding algorithms in general increases the decoding complexity considerably. This tends to be impractical for RS codes. On the other hand, it is known, that a "soft

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decision gain" of about 2 dB can be obtained using a $(N, N-1)$ Single Parity Check (SPC) code [4]. The SPC decoding is characterized by its extremely low complexity since, in practice, the decoder has simply to invert the least reliable bit (corresponding to the smallest absolute demodulator output) whenever a parity error occurs in the binary quantized received word.

We consider in this paper two different concatenated RS–SPC coding schemes. Both systems differ in the combined code structure. The code structures and the decoding procedures are subject of section 2. In section 3 the measured performance of the different coding schemes is given for a binary transmission with antipodal signaling over a channel with additive white Gaussian noise (AWGNC) as well as a Rayleigh fading channel, which is of central interest for satellite mobile communications.

2. Decoding procedures

First we describe a soft decision decoding method for RS codes in accordance to the decoding algorithm proposed by Chase [2].

Following the so-called Chase algorithm "2" and considering a RS codeword in its binary representation, the following decoding steps have to be performed:

(1) Make a hard decision on each received sequence (of analogue demodulator outputs).

(2) For a code with minimum distance d generate test vectors, each representing one of the $2^{\lfloor d/2 \rfloor}$ possible combinations of values in the $\lfloor d/2 \rfloor$ least reliable *positions* of a received word, add it to the hard decision word and decode each created codeword.

(3) Compute the distance from each of the codewords to the received sequence and select the codeword which is closest.

As a reliability measure we calculate the a-posteriori probability L_i of a hard quantized digit c_i that follows from a received analogue channel output r_i, i.e.

$$L_i = \Pr(c_i \mid r_i) = \frac{\Pr(r_i \mid c_i)}{\Pr(r_i)} \cdot \Pr(c_i). \tag{1}$$

Obviously the Chase algorithm "2" requires a high decoding complexity. An alternative with lower complexity follows Chase algorithm "3" by declaring the l least reliable *symbols* as erasures (for d odd $l = 0, 2, \ldots, d-1$, while for d even $l = 0, 1, 3, \ldots, d-1$). For a AWGNC the a-posteriori probability L_i is maximized for hard quantized symbols $\boldsymbol{c}_i$ and given by

$$\begin{aligned} L_i &= \frac{\Pr(\boldsymbol{r}_i \mid \boldsymbol{c}_i)}{\Pr(\boldsymbol{r}_i)} \cdot \Pr(\boldsymbol{c}_i) \\ &= \prod_{j=1}^{m} \frac{1}{1 + \exp\left(-4\sqrt{E_s \mid N_0} \mid r_{i,j} \mid\right)}, \end{aligned} \tag{2}$$

where E_s is the transmitted energy per digit and m the number of bits per RS code symbol.

A distance d code is capable of correcting e errors and s erasures if $2e + s \le d - 1$. Hence, it is highly important to be able to declare erasure positions precisely. The maximum coding gain that can be expected from erasure decoding is 3 dB (distance doubling).

We now describe a concatenated RS–SPC code in two encoding concepts.

2.1. Symbolwise SPC coding

Each symbol of the RS code is extended with a parity bit. The decoder first applies soft decision decoding to each symbol. Then, the "corrected" symbols are decoded by the RS decoder. Using the symbol error rate that results from SPC decoding the performance of the concatenated scheme can be easily predicted.

Extension to erasure RS decoding is again possible, but requires calculation of eq. (2) for SPC codewords. If we let $|r_{i,\min}|$ denote the smallest and $|r_{i,\min'}|$ the smallest but one of the absolute values of the received digits within a symbol, then it suffices to take as an approximation for L_i

$$L_i = \begin{cases} \left| |r_{i,\min}| - |r_{i,\min'}| \right| \\ \quad \text{if a SPC error occurs} \\ |r_{i,\min}| + |r_{i,\min'}| \\ \quad \text{if no SPC error occurs.} \end{cases} \tag{3}$$

This measure indicates that for small values of L_i a decoding error is more likely. The values of L_i

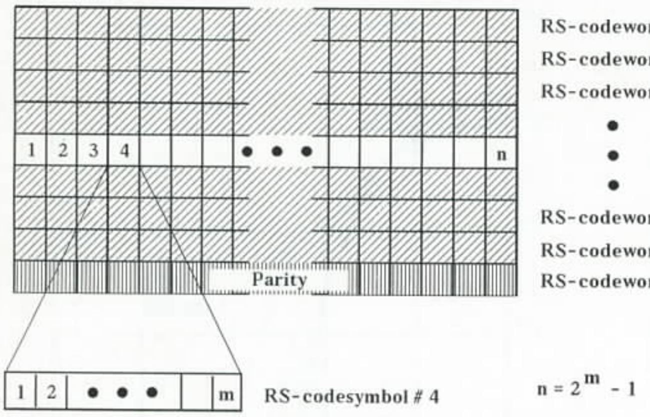

Fig. 1. Structure of concatenated RS–SPC code.

are then used in the same way as in the previously described alternative Chase decoding method.

2.2. Blockwise SPC coding

The second concept assumes transmission of a set of $(N-1)$ RS codewords followed by a word for which the jth $(1 \leq j \leq nm)$ digit is the modulo-2 sum of the jth digits of the $(N-1)$ RS codewords. By linearity, this last word is also a RS codeword, see Fig. 1.

For each received block of RS codewords, the decoder performs the following three major steps:

(1) Apply soft decision decoding for each individual column (each column is a SPC codeword);

(2) Apply RS decoding for each row.

If for a specific row an invalid syndrome occurs (detected, but uncorrectable error pattern), mark this row with a flag. In addition define a set

$$F_E := \{k \,|\, 1 \leq k \leq N; \text{ codeword } RS_k \text{ has erasure flag}\}, \qquad (4)$$

and let

$$\boldsymbol{E} := \sum_{k \notin F_E} RS_k, \qquad (5)$$

where Σ denotes a bitwise modulo-2 addition. Initially, $\boldsymbol{E} := \boldsymbol{0}$.

(3) After decoding the whole block, four different situations occur:

(a) $F_E = \theta$; $\boldsymbol{E} = \boldsymbol{0}$. No erasures and no detected errors. Continue with the next block.

(b) The number of elements in the set F_E is $|F_E| = 1$. Hence, the block contains *one* erasure flag. Assuming correct decoding for all $k \notin F_E$, the erased RS codeword can be reconstructed according to eq. (5).

(c) $|F_E| > 1$. More than one erasure occurs. The value of $\boldsymbol{E}$ is calculated, and this value is now being considered as the new overall parity check sequence for the block of erased RS codewords. These erased RS codewords are subsequently considered as a new block with less rows. With this block, the sequence $\boldsymbol{E}$, and the received demodulator outputs decoding is restarted at step 1. To avoid looping, we demand that $|F_E|$ in the new block has to be smaller than $|F_E|$ in the old one. If not, decoding is stopped.

(d) $F_E = \theta$; $\boldsymbol{E} \neq \boldsymbol{0}$. In this case we assume that there is only *one* RS codeword in error.

The decoder now calculates

$$\max_{k=1,N} \Delta_k := \log \frac{\Pr(\boldsymbol{r}_k \,|\, \mathrm{RS}_k \oplus \boldsymbol{E})}{\Pr(\boldsymbol{r}_k \,|\, \mathrm{RS}_k)} \qquad (6)$$

and gives as output $\mathrm{RS}_{\max} \oplus \boldsymbol{E}$ instead of $\mathrm{RS}_{\max}$ for the RS codeword that maximizes eq. (6). Note that in eq. (6) we only have to consider the locations in RS_k where $\boldsymbol{E}$ has nonzero components.

3. Performance

3.1. AWGNC

Considering the RS–SPC coding scheme of structure A (symbolwise SPC coding) in an

AWGNC the error rate of the RS codesymbols ${}_{A}P_S$ can be approximated for high signal-to-noise ratios by

$$ {}_{A}P_S = \binom{m+1}{2} Q\left(\sqrt{\frac{4RE_b}{N_0}}\right), \tag{7} $$

where $R = R_{rs}m/(m+1)$ denotes the overall code rate, R_{rs} is the code rate of the (n, k) RS code in $GF(2^m)$ and E_b the transmitted energy per information bit. For the concatenated RS–SPC coding system of structure B the symbol error rate ${}_{B}P_S$ for high signal-to-noise ratios is approximately

$$ {}_{B}P_S = m(N-1)Q\left(\sqrt{\frac{4RE_b}{N_0}}\right) \tag{8} $$

with $R = R_{rs}(N-1)$. Since the symbol error rate P_S for hard decision RS decoding can be approximated by

$$ P_S = mQ\left(\sqrt{\frac{2R_{rs}E_b}{N_0}}\right) \tag{9} $$

an asymptotic gain due to soft decision decoding of 10 log $(2m/(m+1))$ dB (structure A) and 10 log $(2(N-1)/N)$ dB (structure B) is expected; for $n = 31$ $(m = 5)$ that is about 2.2 dB.

Since the minimum distance of the concatenated code (structure B) is twice the minimum distance of the (n, k) RS code, an additional gain corresponding to the distance doubling can be expected. For exploiting this potential gain erasure RS decoding as given B is employed in addition to the soft decision SPC inner decoding.

The simulation results for the different coding schemes based on the use of antipodal signaling with coherent detection in a channel with additive white Gaussian noise are given in Fig. 2. The Reed–Solomon code employed is the short (31,21) code. For block structure B, $N = 9$ is chosen in the simulations; in addition the maximum number of iterations (decoding step 3) in limited to 3. As can be seen, symbolwise soft decision SPC (inner) decoding (curve 3) results in a gain over the hard decision RS decoding of to about 1.5 dB at a bit error rate (BER) of 10^{-5}; this corresponds to an overall coding gain of about 4 dB. Additional erasure decoding (curve 4) lead to an improvement of 0.3 dB. For code structure B the best performance (curve 5) for BER $< 10^{-3}$ is obtained.

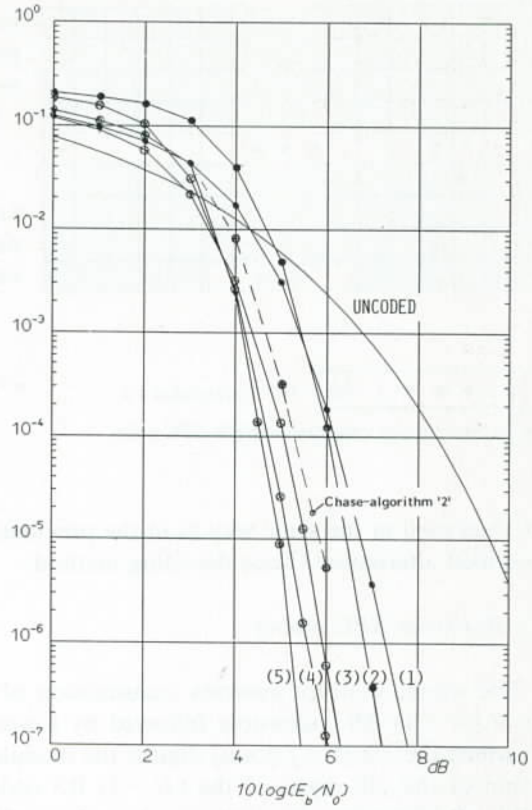

Fig. 2. Bit error rate *vs.* signal-to-noise ratio for antipodal signaling in an AWGNC: (1) hard decision (31,21)-RS decoding, $R \cong 0.68$; (2) hard decision error and erasure (31,21)–RS decoding, Chase algorithm "3"; (3) concatenated (6,5)-SPC–(31,21)-RS decoding, $R \cong 0.56$, structure A; (4) concatenated (6,5)-SPC–(31,21)-RS decoding, $R \cong 0.56$, structure A, erasure decoding; (5) concatenated (9.8)-SPC–(31,21)-RS decoding, $R \cong 0.60$, structure B.

Compared with the symbolwise soft decision SPC decoding (curve 3) the gain is about 0.5 dB at BER $= 10^{-5}$.

3.2. Rayleigh fading channel

A Rayleigh fading channel is considered, with signal fading *slowly* varying compared with the data rate. Different types of data interleaving for the different coding schemes are assumed:

(1) Symbolwise full data interleaving for Reed–Solomon coding, the fading amplitude is assumed to be constant over the code symbol

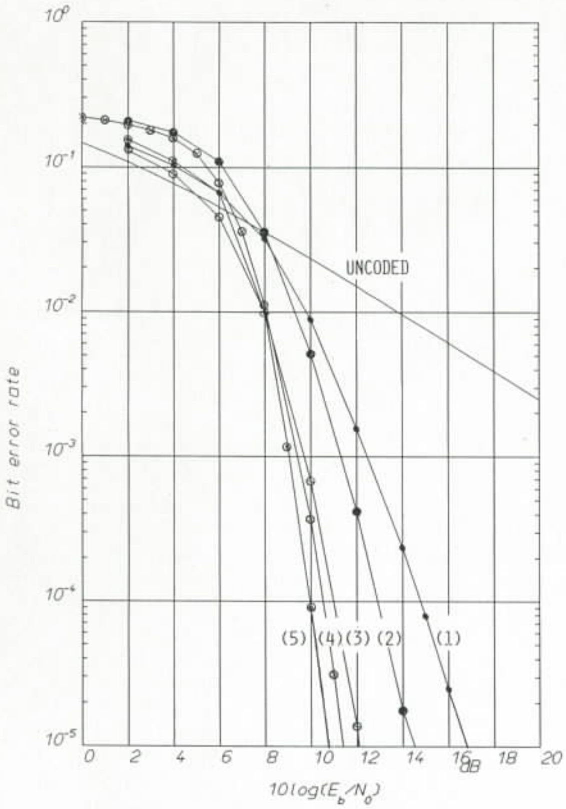

Fig. 3. Bit error rate *vs.* signal-to-noise ratio for antipodal signaling in a Rayleigh fading channel: (1) hard decision (31,21)-RS decoding, $R \cong 0.68$; (2) hard decision error and erasure (31,21)-RS decoding, Chase algorithm "3"; (3) concatenated (6.5)-SPC–(31,21)-RS decoding, $R \cong 0.56$, structure A; (4) concatenated (6,5)-SPC–(31,21)-RS decoding, $R \cong 0.56$, structure A, erasure decoding; (5) concatenated (9.8)-SPC–(31,21)-RS decoding, $R \cong 0.60$, structure B.

duration and symbolwise statistically independent.

(2) Symbolwise full data interleaving for the concatenated scheme, structure B.

(3) Bitwise full data interleaving for concatenated scheme, structure A.

Figure 3 shows the performance of the different coding schemes in the considered Rayleigh fading channel. It can be seen that the relation between the different schemes is qualitatively the same as for the AWGNC. For the concatenated coding scheme with symbolwise soft decision SPC decoding (curve 3) a gain of about 5 dB, for structure B (curve 5) a gain of about 6 dB over hard decision RS decoding is obtained at BER $= 10^{-5}$.

4. Conclusions

The concatenated Reed–Solomon–Single Parity Check coding systems are considered The measured performance of both schemes is given and compared to the results for Reed–Solomon codes. It is shown that a concatenated coding system characterized by symbolwise SPC encoding, followed by hard decision RS decoding, lead in comparison to hard decision RS decoding to a gain of about 1.5 dB in an AWGNC and to a gain of about 5 dB in a Rayleigh fading channel at BER $= 10^{-5}$. The additional decoding complexity due to the soft decision SPC decoding is almost negligible and the decoding speed is determined by the RS decoder only. The method B of course can be applied to other classes of block codes as well as to convolutional codes [5].

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