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An algebraic approach to generate super-set of perfect complementary codes for interference-free CDMA

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Summary

This paper introduces an algebraic approach to generate the super-set *of* perfect complementary (PC) codes suitable for new generation CDMA applications, characterized by isotropic multiple access interference (MAI) free and multipath interference (MI) free properties. The code design methodology proposed in this paper takes into account major impairing factors existing in real applications, such as MAI, MI, asynchronous transmissions, and random signs in consecutive symbols, such that a CDMA system using the generated codes can insure a truly interference-free operation. Two important facts will be revealed by the analysis given in this paper. First, implementation of an interference-free CDMA will never be possible unless using complementary code sets, such as the PC code sets generated in this paper. In other words, all traditional spreading codes working on an one-code-per-user basis are not useful for implementation of an MAI-free and MI-free CDMA system. Second, to enable the interference-free CDMA operation, the flock size of the PC codes should be made equal to the set size of the codes, implying that a PC code set can support as many users as the flock size of the code set. A systematic search has been carried out to generate the super-set of various PC codes with the help of carefully selected seed codes belonging to distinct sub-sets. This paper will also propose an implementation scheme based on multi-carrier CDMA architecture and its performance is compared by simulations with the ones using traditional spreading codes. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: Spreading code; CDMA; multiple access interference; multipath interference

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1. Introduction

It is well known that the performance of a CDMA system is always interference-limited due to the presence of multiple access interference (MAI) and multipath interference (MI), the former stemmed from non-ideal cross-correlation functions amid all spreading codes and the latter caused by non-trivial auto-correlation side lobes of any individual code used in the system. A direct consequence from such interference-limited

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property is that the capacity of all currently available CDMA-based 2–3G wireless systems [1–5] can offer a capacity equal to merely about one-third to a half of its processing gain, even using so called *orthogonal codes* (such as Walsh–Hadamard sequences [12], OVSF codes [3–5], etc.). As the core technologies of 2G and 3G CDMA systems are very similar, we would like to refer them as to *the first generation* CDMA (1G-CDMA) technologies, all of which make use of traditional *unitary* spreading codes for channelization and need a great deal of auxiliary sub-systems, such as the power control and RAKE, etc., to mitigate the problems associated with the spreading codes, such as near–far effect and MI, etc.

Being the most important part of the CDMA technologies, the spreading codes or signature codes play a critical role in overall performance of a CDMA system. It has to be admitted that many problems existing in the current 2–3G CDMA systems, such as low capacity, slow transmission rate, and complex system implementation and so on, are because of unsatisfactory properties of the spreading codes adopted by the systems. For instance, the choice of Walsh–Hadamard sequences and OVSF codes in IS-95 [1,2] and WCDMA [3,4] standards, respectively, has made it impossible to insure a symmetric data throughput in their up-link and down-link channels at the very beginning of the system design, because of their completely different correlation characteristics in asynchronous and synchronous transmission modes.

The study on spreading codes for CDMA applications is a traditional research topic and many candidate codes have been found in the literature, such as Gold codes [6], GMW codes [7], No codes [8], Bent sequences [9], Kasami codes [11], m-sequence [10], Walsh–Hadamard sequence [12], and OVSF codes [3–5], to just name a few as examples, all of which are *unitary codes*, meaning that only one code should be assigned to each user. Some of them have already been integrated into 2–3G mobile cellular standards as an indispensable part of the systems. The problems associated with those systems based on the traditional spreading codes were mainly attributable to the fact that their design approaches simply ignored real application scenarios in a wireless system, such as up-link asynchronous transmissions, varying signs in bit stream and MI. In fact, they were generated and applied to the systems based only on the knowledge of seemly acceptable properties in their periodic auto-correlation and cross-correlation functions, which are relevant only to the down-link synchronous operation in a real system without MI. However, a wireless system has to cope with many other impairing factors, the most serious one of which is MI. Without considering them, any attempt to design a spreading code set will never meet our requirements.

Very recently, the CDMA technologies have faced a serious challenge from other traditional multiple access technologies. Having been fed up with the annoying MAI and MI problems with the 1G-CDMA technologies, someone have turned to work on some other new possible candidates to replace CDMA for the next generation wireless applications, which require much higher data rate than current 2–3G systems. One of such possible candidates is so called high data rate (HDR) TDMA technology or channel-driven TDMA. Therefore, it is likely that 4G wireless will probably run on a non-CDMA based platform, leaving CDMA technologies lag behind other competing multiple access technologies. In order to prevent such thing from happening, the evolution of CDMA technologies should be speeded up and many innovative techniques, including promising spreading codes, are badly needed to bring CDMA back on track.

Motivated by the call for innovation of CDMA technologies, in this paper we intend to propose a new methodology to generate ideal spreading codes as an effort to improve the performance of the current CDMA systems. In particular, we will make use of an algebraical approach for spreading code design and generation by taking into account major operational scenarios such that resultant spreading codes should effectively address those adverse operational conditions in their code structure. To make the study as general as possible, we should not limit our attention to the candidates in some particular code form. Instead, we would rather start from a mathematical problem that covers all possible codes, either real or complex and either unitary or complementary codes. Therefore, the design problem will be formulated initially from complementary codes, as any non-complementary codes are only the special cases with the flock size being one.

Nevertheless, as the results obtained in this paper are related closely to complementary codes, its history has to be briefed in the sequel. The complementary codes were first introduced by Golay in 1961 [13]. Later in 1963, Turyn studied the ambiguity functions of the complementary codes for their possible applications in radar systems [14]. Both Golay and Turyn studied pairs of binary complementary codes whose autocorrelation function is zero for all even shifts except the zero shift. Suehiro [15] extended the concept to generation of so called complete complementary (CC) codes whose autocorrelation function is zero for all even and odd shifts except the zero shift and whose cross-correlation function for any pair is zero for all possible shifts. The work carried out in [15] had paved the way for possible applications of the CC codes in CDMA systems, a possible architecture of which has been proposed and studied in [16]. However, the major problem with the CC codes is its very small set size, given a certain PG value. Specifically, it can only support $L^{1/3}$ users with its PG being equal to L, making it hardly practical for any real system, as shown in Table I. For instance, if we want to support 64 users in a cell, we have to use a CC code set with it PG being equal to $64^3 = 262,144$, which is too complicated to implement from the view point of currently available hardware. On the other hand, the resultant codes generated from the method proposed in this paper form a super-set of perfect complementary (PC) codes to distinguish them from the previously reported complete complementary codes. The set size of some PC codes can be equal to their PG value, and thus they can support a lot more users than the CC codes with a fixed PG value.

The rest of the paper is outlined as follows. In the next section, we will introduce the method to formulate a design problem under the frame of a non-linear equation set. Section 3 will explain necessary conditions to solve for the PC codes from a set of homogenous linear equations based on some seed codes from different sub-sets. It will also reveal that an interference-free CDMA can never be made possible if considering only unitary codes, thus making the complementary codes a very important part of the future CDMA applications. The relation derived in Section 4 specifies that set size K and flock size M of a complementary code set must satisfy the inequality $M \geq K$ to formulate an interference-free CDMA, and an example will be given to show step by step how to generate the PC codes. A possible implementation scheme is to be proposed in Section 5, together with performance comparison of the systems based on different codes, followed by the conclusion. Finally, a PC code set concerned in the simulation study is given in Appendix.

2. Formulation of the Non-Linear Problem

Let us start with a generic spreading code set, whose element code length, flock size, and set size are N, M, and K, respectively. If $M = 1$, the generic code set reduces to a conventional unitary spreading code set, which works on one-code-per-user basis. On the other hand, if $M > 1$, the generic code set represents a complementary code set, in which a flock of codes should be assigned to a user and ought to be sent to a receiver individually via different channels.

Let us consider any two flocks in a code set, $x = \{x_1, x_2, \ldots, x_M\}$ and $y = \{y_1, y_2, \ldots, y_M\}$, where $x_i = \{x_{i1}, x_{i2}, \ldots, x_{iN}\}$ and $y_i = \{y_{i1}, y_{i2}, \ldots, y_{iN}\}$ for $1 \leq i \leq M$. All chips, x_{ij} or y_{ij} for $1 \leq i,\ j \leq M$, could take any value, either binary or multiple-leveled and either real or complex.

Assume that the signal transmission from each user forms a continuous bit stream and the signs of two consecutive bits can be arbitrary, either same or different. We are concerning an asynchronous channel (thus a synchronous one is only a special case), where both MAI and MI are present. Thus, the spreading code designing problem can be expressed as: to design a perfect complementary code set such that its periodic and aperiodic out-of-phase auto-correlation functions as well as its periodic and aperiodic cross-correlation functions should be zero, to insure MAI-free and MI-free operation. To formulate a mathematical expression for such a design problem, let us examine in particular two different cases: even and odd lengths of the element codes. As all element codes have the same length N, we could focus on any one of them, say the ith one or x_i ($1 \leq i \leq M$), without losing generality. For representation simplicity, let us assume $N = 6$ for the even case and $N = 5$ for the odd case, as shown in Figures 1 and 2, where the ith element code of a complementary code x and its j chips shifted versions ($0 \leq j \leq N - 1$) are shown.

It is seen from Figures 1 and 2 that, if two relatively shifted versions of an element code with their offsets being a and b and $a + b = N$ (as shown by the pairs indicated by arrowed lines in the figures), they will generate the same periodic and aperiodic auto-correlation functions with the zero-shifted version. Thus, we can establish an equation set for the element code x with in total MN unknown variables as follows:

For either an even or odd N, there are $\lfloor \frac{N}{2} \rfloor + \lfloor \frac{N-1}{2} \rfloor + 1$ independent non-linear equations as:

$$\begin{cases} \sum_{i=1}^{M} \left[x_{i1}^2 + x_{i2}^2 + \cdots + x_{iN}^2\right] = NM \\ \sum_{i=1}^{M} \left[x_{iN}x_{i1} + x_{i1}x_{i2} + \cdots + x_{i(N-1)}x_{iN}\right] = 0 \\ \sum_{i=1}^{M} \left[x_{i(N-1)}x_{i1} + x_{iN}x_{i2} + \cdots + x_{i(N-2)}x_{iN}\right] = 0 \\ \sum_{i=1}^{M} \left[x_{i(N-2)}x_{i1} + x_{i(N-1)}x_{i2} + \cdots + x_{i(N-3)}x_{iN}\right] = 0 \\ \cdots\cdots \\ \sum_{i=1}^{M} \left[-x_{iN}x_{i1} + x_{i1}x_{i2} + \cdots + x_{i(N-1)}x_{iN}\right] = 0 \\ \sum_{i=1}^{M} \left[-x_{i(N-1)}x_{i1} - x_{iN}x_{i2} + \cdots + x_{i(N-2)}x_{iN}\right] = 0 \\ \cdots\cdots \end{cases} \tag{1}$$

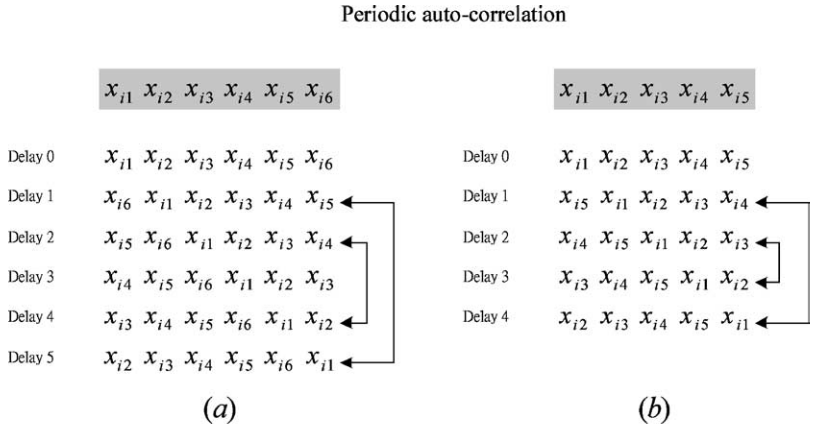

Fig. 1. Illustration of periodic auto-correlation functions of an even ($N = 6$ in (a)) or odd ($N = 5$ in (b)) length element code x_i ($1 \leq i \leq M$) with two consecutive bits having the same sign in an asynchronous channel, where $N - 1 = 5$ (in (a)) or $N - 1 = 4$ (in (b)) delayed multipath returns or periodical cyclic-shifted versions are present. The shaded sequences represent the local element code of x_i ($1 \leq i \leq M$) generated at a correlator and the arrowed lines indicate the two relatively cyclic-shifted versions of an element code that will generate the same out-of-phase auto-correlation functions with the local element code.

where the first $\lfloor \frac{N}{2} \rfloor + 1$ equations are from ideal periodic auto-correlation functions and the last $\lfloor \frac{N-1}{2} \rfloor$ ones from ideal aperiodic auto-correlation functions. Here $\lfloor x \rfloor$ stands for the largest integer less than x.

Now, let us introduce the second code y into the set, which consists of x and y only, such that they should satisfy all conditions as a pair of perfect complementary codes. Similarly, we can establish the equation sets based on the ideal periodic and aperiodic cross-correlation functions between the codes y and x as follows.

For either an even or odd N, there are $2N - 1$ nonlinear homogenous equations as

$$\begin{cases} \sum_{i=1}^{M} \left[y_{i1}x_{i1} + y_{i2}x_{i2} + \cdots + y_{iN}x_{iN} \right] = 0 \\ \sum_{i=1}^{M} \left[y_{iN}x_{i1} + y_{i1}x_{i2} + \cdots + y_{i(N-1)}x_{iN} \right] = 0 \\ \sum_{i=1}^{M} \left[y_{i(N-1)}x_{i1} + y_{iN}x_{i2} + \cdots + y_{i(N-2)}x_{iN} \right] = 0 \\ \sum_{i=1}^{M} \left[y_{i(N-2)}x_{i1} + y_{i(N-1)}x_{i2} + \cdots + y_{i(N-3)}x_{iN} \right] = 0 \\ \cdots\cdots \\ \sum_{i=1}^{M} \left[-y_{iN}x_{i1} + y_{i1}x_{i2} + \cdots + y_{i(N-1)}x_{iN} \right] = 0 \\ \sum_{i=1}^{M} \left[-y_{i(N-1)}x_{i1} - y_{iN}x_{i2} + \cdots + y_{i(N-2)}x_{iN} \right] = 0 \\ \cdots\cdots \end{cases} \quad (2)$$

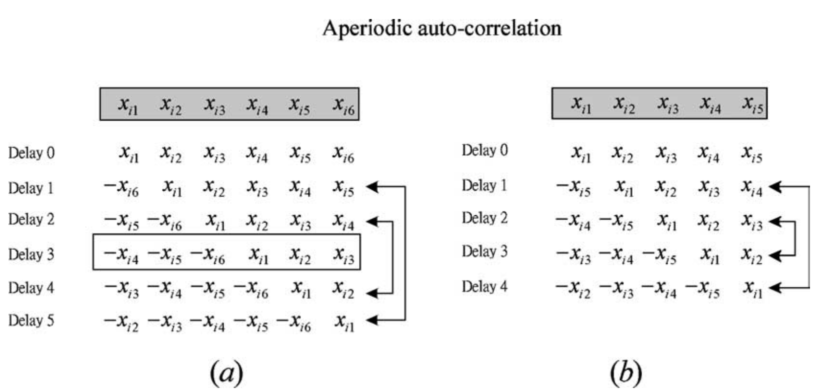

Fig. 2. Illustration of aperiodic auto-correlation functions of an even ($N = 6$ in (a)) or odd ($N = 5$ in (b)) length element code x_i ($1 \leq i \leq M$) with two consecutive bits having different signs in an asynchronous channel, where $N - 1 = 5$ (in (a)) or $N - 1 = 4$ (in (b)) delayed multipath returns or aperiodically cyclic-shifted versions are present. The shaded sequences represent the local element code of x_i ($1 \leq i \leq M$) generated at a correlator. The arrowed lines indicate the two relatively cyclic-shifted versions of an element code that will generate the same out-of-phase auto-correlation functions with the local element code and the code inside the solid-line frame yields zero out-of-phase auto-correlation function.

where the upper N equations are due to ideal periodic cross-correlation functions and the lower $N-1$ ones from the ideal aperiodic cross-correlation functions. In addition, y itself should also satisfy the conditions for ideal auto-correlation functions as

$$\begin{cases} \sum_{i=1}^{M}\left[y_{i1}^2 + y_{i2}^2 + \cdots + y_{iN}^2\right] = NM \\ \sum_{i=1}^{M}\left[y_{iN}y_{i1} + y_{i1}y_{i2} + \cdots + y_{i(N-1)}y_{iN}\right] = 0 \\ \sum_{i=1}^{M}\left[y_{i(N-1)}y_{i1} + y_{iN}y_{i2} + \cdots + y_{i(N-2)}y_{iN}\right] = 0 \\ \sum_{i=1}^{M}\left[y_{i(N-2)}y_{i1} + y_{i(N-1)}y_{i2} + \cdots + y_{i(N-3)}y_{iN}\right] = 0 \\ \cdots\cdots \\ \sum_{i=1}^{M}\left[-y_{iN}y_{i1} + y_{i1}y_{i2} + \cdots + y_{i(N-1)}y_{iN}\right] = 0 \\ \sum_{i=1}^{M}\left[-y_{i(N-1)}y_{i1} - y_{iN}y_{i2} + \cdots + y_{i(N-2)}y_{iN}\right] = 0 \\ \cdots\cdots \end{cases} \quad (3)$$

where the first $\lfloor \frac{N}{2} \rfloor + 1$ equations reflect ideal periodic auto-correlation functions and the last $\lfloor \frac{N-1}{2} \rfloor$ equations specify the ideal aperiodic auto-correlation functions, respectively.

In general, to determine a complementary code set with its element code length, flock size, and set size being N, M, and K respectively, where $K > 1$, we can obtain the following non-linear equations, regardless of even or odd N.

(1) $(\lfloor \frac{N}{2} \rfloor + 1)K$ non-linear equations from perfect periodic auto-correlation conditions;
(2) $\frac{NK(K-1)}{2}$ non-linear equations from zero periodic cross-correlation conditions;
(3) $(\lfloor \frac{N-1}{2} \rfloor + 1)K$ non-linear equations from perfect aperiodic auto-correlation conditions; and
(4) $\frac{(N-1)K(K-1)}{2}$ non-linear equations from zero aperiodic cross-correlation conditions, where $K > 1$.

Thus, we will have altogether $K(\lfloor \frac{N}{2} \rfloor + \lfloor \frac{N-1}{2} \rfloor + 1) + (2N-1)\frac{K(K-1)}{2}$ non-linear equations, which contain MNK unknown variables or K unknown complementary codes. It is possible that those non-linear equations might be solvable under a necessary condition that the following inequality must be satisfied

$$K\left(\left\lfloor \frac{N}{2} \right\rfloor + \left\lfloor \frac{N-1}{2} \right\rfloor + 1\right) + (2N-1)\frac{K(K-1)}{2} \geq MNK \quad (4)$$

However, the solutions to a non-linear equation set are not guaranteed even if Equation (4) is satisfied. To have a clearer interpretation for the problem, we would like to turn to the following methodology based on the linear equation sets. Before doing so, let us make several observations worthy mentioning as follows.

It is noticed that Equation (2) in fact can be transformed into a homogenous linear equation set if x is already known. We will take this known first code x as a seed code, which itself should satisfy all conditions for MI-free operation or both periodic and aperiodic out-of-phase auto-correlation functions of the codes should be zero. If so, we readily have an linear equation set from Equation (2) for us to solve the second code y jointly with Equation (3), which is not a linear equation set but can be used effectively to determine all unknown variables in Equation (2). The solution to those equation sets or code y must have satisfied all MAI-free and MI-free conditions, or equations from Equations (1–3). The same procedure can be repeated until all codes in the set are determined. This observation is significant to facilitate our analysis followed.

It should be again emphasized that the above mathematical design problem did not impose any restrictions on the form of codes, neither the values each chip might take, which can be either binary or polyphase and either real or complex. We have chosen the complementary codes as our starting point simply because they represent the most general case, including all spreading codes currently available or $M \geq 1$. If $M = 1$, the obtained results is reduced to traditional unitary spreading codes; otherwise if $M > 1$, the complementary codes will yield. In other words, the solutions from Equations (1–3) should include all possible codes satisfying the MAI-free and MI-free conditions. If the above equations are not solvable, we then can conclude that there will not exist such perfect codes offering an MAI-free and MI-free operation. Therefore, we would like to know in particular

(1) whether or not such perfect codes exist?
(2) if yes, what form the codes will take?
(3) how many such perfect code sets will exist?

which are the questions of interest in the next section.

3. Necessary Conditions to Generate Perfect Codes

The following theorems in linear algebra will be useful in our code design problem based on the linear equation sets.

Consider an $m \times n$ coefficient matrix $\mathbf{A}$, a variable vector $\boldsymbol{x}$ and an $m \times 1$ constant vector $\boldsymbol{b}$, which are related by the equation $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$, where the elements of $\mathbf{A}$, $\boldsymbol{x}$ and $\boldsymbol{b}$ in general can be complex. It is noted that m is the number of linear equations and n gives the number of unknown variables in $\boldsymbol{x}$.

(1) **Theorem 1.** *For a given $m \times n$ matrix $\mathbf{A}$, it is always true that* $\text{rank}(\mathbf{A}) \le \min(m, n)$.

(2) **Theorem 2.** *The solution to the linear equation set $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has only the following three possible outcomes*
   (a) if $rank(\mathbf{A}|\boldsymbol{b}) > rank(\mathbf{A})$, $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has no solution;
   (b) if $rank(\mathbf{A}|\boldsymbol{b}) = rank(\mathbf{A})$ and $rank(\mathbf{A}) = n$, $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has only one solution;
   (c) if $rank(\mathbf{A}|\boldsymbol{b}) = rank(\mathbf{A})$ and $rank(\mathbf{A}) < n$, $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has more than one solutions;
   where $(\mathbf{A}|\boldsymbol{b})$ stands for an extended matrix for the linear equation set $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$. For a homogenous linear equation set, we always have $rank(\mathbf{A}|\boldsymbol{b}) = rank(\mathbf{A})$ as $\boldsymbol{b} = \mathbf{0}$. Therefore, it will have at least one solution or all-zero solution as long as $rank(\mathbf{A}) \le n$.

(3) **Theorem 3.** *If $rank(\mathbf{A}) \le min(m, n)$, then matrix $\mathbf{A}$ must have exactly $rank(\mathbf{A})$ independent row or column vectors, which can be uniquely determined by a series of linear transformation operations against either rows or columns of $\mathbf{A}$.*

(4) **Theorem 4.** *If two vectors, $\boldsymbol{x}$ and $\boldsymbol{y}$, are independent, then their inner product, $\boldsymbol{x}\boldsymbol{y}$, must be zero.*

Assume that there exists a flock of codes, $\boldsymbol{x} = \{x_{11}, x_{12}, \ldots, x_{2N}; x_{21}, x_{22}, \ldots, x_{2N}; \ldots; x_{M1}, x_{M2}, \ldots, x_{MN}\}$, which satisfies the conditions for an ideal auto-correlation function, as defined by Equation (1), where x_{ij} represents the jth chip in the ith element code in the flock. To determine the second flock of codes, $\boldsymbol{y} = \{y_{11}, y_{12}, \ldots, y_{2N}; y_{21}, y_{22}, \ldots, y_{2N}; \ldots; y_{M1}, y_{M2}, \ldots, y_{MN}\}$, we can obtain a $(2N-1) \times (NM)$ coefficient matrix of a homogenous linear equation set from Equation (2), which specifies ideal periodic and aperiodic cross-correlation functions between x and y and can be written into $\mathbf{A}_y y = \mathbf{0}$, as

$$\mathbf{A}_y = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1N}, & x_{21} & x_{22} & \ldots & x_{2N}, & \ldots, & x_{M1} & x_{M2} & \ldots & x_{MN} \\ x_{12} & x_{13} & \ldots & x_{11}, & x_{22} & x_{23} & \ldots & x_{21}, & \ldots, & x_{M2} & x_{M3} & \ldots & x_{M1} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{1N} & x_{11} & \ldots & x_{1(N-1)}, & x_{2N} & x_{21} & \ldots & x_{2(N-1)}, & \ldots, & x_{MN} & x_{M1} & \ldots & x_{M(N-1)} \\ x_{12} & x_{13} & \ldots & -x_{11}, & x_{22} & x_{23} & \ldots & -x_{21}, & \ldots, & x_{M2} & x_{M3} & \ldots & -x_{M1} \\ x_{13} & x_{14} & \ldots & -x_{12}, & x_{23} & x_{24} & \ldots & -x_{22}, & \ldots, & x_{M3} & x_{M4} & \ldots & -x_{M2} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{11} & -x_{12} & \ldots & -x_{1N}, & x_{21} & -x_{22} & \ldots & -x_{2N}, & \ldots, & x_{M1} & -x_{M2} & \ldots & -x_{MN} \end{bmatrix} \quad (5)$$

where the upper half of $\mathbf{A}_y$ is the result of the periodic cross-correlation functions between x and y and the lower half is from the aperiodic cross-correlation functions between the two codes.

Because x must satisfy the all conditions for ideal auto-correlation functions specified by Equation (1), all row vectors in the upper half of Equation (5) have to be mutually independent. To illustrate it more clearly, let us examine a matrix formed by all possible relatively cyclic-shifted versions of the ith element code x_i $(1 \le i \le M)$ in a flock of codes x, where $N = 4$ is assumed for illustration simplicity.

$$\begin{pmatrix} x_{i1} & x_{i2} & x_{i3} & x_{i4} \\ x_{i2} & x_{i3} & x_{i4} & x_{i1} \\ x_{i3} & x_{i4} & x_{i1} & x_{i2} \\ x_{i4} & x_{i1} & x_{i2} & x_{i3} \end{pmatrix} \quad (6)$$

which in fact is just one of the M sub-matrices in the upper half of Equation (5). Since x has a perfect periodic auto-correlation function, the following equation set must be true

$$x_{i1}x_{i2} + x_{i2}x_{i3} + x_{i3}x_{i4} + x_{i4}x_{i1} = 0 \quad (7)$$

$$x_{i1}x_{i3} + x_{i2}x_{i4} + x_{i3}x_{i1} + x_{i4}x_{i2} = 0 \quad (8)$$

$$x_{i1}x_{i4} + x_{i2}x_{i1} + x_{i3}x_{i2} + x_{i4}x_{i3} = 0 \quad (9)$$

which merely states the fact that three pairs of the row vectors in Equation (6) are mutually independent. Now let us establish the inner products of the other three pairs of relatively cyclic-shifted versions of the element code $x = \{x_{i1}, x_{i2}, x_{i3}, x_{i4}\}$, which are

$$x_{i2}x_{i3} + x_{i3}x_{i4} + x_{i4}x_{i1} + x_{i1}x_{i2} \quad (10)$$

$$x_{i3}x_{i4} + x_{i4}x_{i1} + x_{i1}x_{i2} + x_{i2}x_{i3} \quad (11)$$

$$x_{i2}x_{i4} + x_{i3}x_{i1} + x_{i4}x_{i2} + x_{i1}x_{i3} \quad (12)$$

where Equation (10) is the inner product between the second and third rows of Equations (6) and (11) is that between the third and fourth rows and Equation (12) is that between the second and fourth rows, respectively. By comparing Equations (10) and (7), (11) and (9), (12) and (8), we can find that they are all equal. Therefore, we have

$$x_{i2}x_{i3} + x_{i3}x_{i4} + x_{i4}x_{i1} + x_{i1}x_{i2} = 0 \quad (13)$$

$$x_{i3}x_{i4} + x_{i4}x_{i1} + x_{i1}x_{i2} + x_{i2}x_{i3} = 0 \quad (14)$$

$$x_{i2}x_{i4} + x_{i3}x_{i1} + x_{i4}x_{i2} + x_{i1}x_{i3} = 0 \quad (15)$$

or all rows in matrix Equation (6) are mutually independent with each other, implying that the matrix has a rank equal to $N = 4$, or the length of the element code x. Although the above conclusion is obtained from the observation of the single element code with a particular length $N = 4$, it can be easily shown that the conclusion is equally applicable to the cases for any M and N.

In general, it can be shown that the rank of a matrix formed by all possible cyclic-shifted versions of an element code $x = \{x_1, x_2, \ldots, x_M\}$, where $x_i = \{x_{i1}, x_{i2}, \ldots, x_{iN}\}$ and $1 \leq i \leq M$, must be equal to the length of the element code or N, as long as the element code itself has a perfect auto-correlation function specified by Equation (1).

Observed from the upper half of the $(2N - 1) \times (NM)$ matrix $\mathbf{A}_y$ shown in Equation (5), formed by all possible cyclic-shifted versions of x, it is concluded that the rank of this coefficient matrix of the homogenous linear equation set $\mathbf{A}_y\ y = \mathbf{0}$ is also equal to N. If we let $M = 1$, we will get a homogenous linear equation set with its rank equal to the number of variables or $NM = N$, resulting in an all-zero solution or $y = \mathbf{0}$ from Theorem 2(b). It means that, when $M = 1$, we can not find the second code y such that it will ensure a perfect periodic and aperiodic cross-correlation functions with the existing code x. However if we let $M > 1$, the solution of $\mathbf{A}_y\ y = \mathbf{0}$ could possibly exist according to Theorem 2(c). Therefore, we have the first important conclusion obtained in this paper as follows.

Corollary 1. *To insure perfect periodic and aperiodic auto- and cross-correlation functions for a generic spreading code set, the flock size M of the code set must be greater than one* (*or* $M > 1$)*. In other words, only a complementary code set can possibly achieve the perfect periodic and aperiodic auto-correlation and cross-correlation functions. All traditional unitary codes, such as Gold, Kasami, Walsh–Hadamard, OVSF codes and so on, can never yield such perfect periodic and aperiodic auto-correlation and cross-correlation functions.*

It should be noted that the above conclusion was obtained without imposing any limits on the values the complementary codes might take. Therefore, the conclusion will be valid for any types of complementary codes, either binary, or multiple leveled and either real or complex.

4. Relation Between Flock Size and Set Size

If $M > 1$, we then know from Theorem 2(c) that it is impossible for us to readily determine a unique solution to the homogenous linear equation set $\mathbf{A}_y\ y = \mathbf{0}$, which specifies zero periodic and aperiodic cross-correlation functions between the codes x and y. Fortunately, the relations specifying the ideal periodic and aperiodic auto-correlation functions of the code y itself have never been used, and thus they can help us to find the final solutions of the second code y. The number of suitable solutions may not be necessarily one.

With the two codes in our hands, we can proceed to find the third one or $z = \{z_1, z_2, \ldots, z_M\}$, where $z_i = \{z_{i1}, z_{i2}, \ldots, z_{iN}\}$ and $1 \leq i \leq M$. Similarly, we will have a $2(2N - 1) \times (NM)$ homogenous linear equation set to specify the zero periodic and aperiodic cross-correlation functions between the new code z and the existing ones, or x and y, as follows.

$$\mathbf{A}_z = \left[\begin{array}{ccccccccccccc}
x_{11} & x_{12} & \cdots & x_{1N}, & x_{21} & x_{22} & \cdots & x_{2N}, & \ldots, & x_{M1} & x_{M2} & \cdots & x_{MN} \\
x_{12} & x_{13} & \cdots & x_{11}, & x_{23} & x_{23} & \cdots & x_{21}, & \ldots, & x_{M2} & x_{M3} & \cdots & x_{M1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
x_{1N} & x_{11} & \cdots & x_{1(N-1)}, & x_{2N} & x_{21} & \cdots & x_{2(N-1)}, & \ldots, & x_{MN} & x_{M1} & \cdots & x_{M(N-1)} \\
x_{12} & x_{13} & \cdots & -x_{11}, & x_{22} & x_{23} & \cdots & -x_{21}, & \ldots, & x_{M2} & x_{M3} & \cdots & -x_{M1} \\
x_{13} & x_{14} & \cdots & -x_{12}, & x_{23} & x_{24} & \cdots & -x_{22}, & \ldots, & x_{M3} & x_{M4} & \cdots & -x_{M2} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
x_{11} & -x_{12} & \cdots & -x_{1N}, & x_{21} & -x_{22} & \cdots & -x_{2N}, & \ldots, & x_{M1} & -x_{M2} & \cdots & -x_{MN} \\
y_{11} & y_{12} & \cdots & y_{1N}, & y_{21} & y_{22} & \cdots & y_{2N}, & \ldots, & y_{M1} & y_{M2} & \cdots & y_{MN} \\
y_{12} & y_{13} & \cdots & y_{11}, & y_{23} & y_{23} & \cdots & y_{21}, & \ldots, & y_{M2} & y_{M3} & \cdots & y_{M1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
y_{1N} & y_{11} & \cdots & y_{1(N-1)}, & y_{2N} & y_{21} & \cdots & y_{2(N-1)}, & \ldots, & y_{MN} & y_{M1} & \cdots & y_{M(N-1)} \\
y_{12} & y_{13} & \cdots & -y_{11}, & y_{22} & y_{23} & \cdots & -y_{21}, & \ldots, & y_{M2} & y_{M3} & \cdots & -y_{M1} \\
y_{13} & y_{14} & \cdots & -y_{12}, & y_{23} & y_{24} & \cdots & -y_{22}, & \ldots, & y_{M3} & y_{M4} & \cdots & -y_{M2} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
y_{11} & -y_{12} & \cdots & -y_{1N}, & y_{21} & -y_{22} & \cdots & -y_{2N}, & \ldots, & y_{M1} & -y_{M2} & \cdots & -y_{MN}
\end{array}\right] \tag{16}$$

As shown earlier, the all row vectors in Equation (16) generated from all possible cyclic shifted versions of element codes x and y must be mutually independent. Therefore, the rank of the matrix $\mathbf{A}_z$ should be at least $2N$, where N is the length of the element codes. Now if the flock size M becomes two, we face again the situation that the rank of the homogenous linear equation set $\mathbf{A}_z\, z = \mathbf{0}$ or $rank(\mathbf{A}_z)$ is greater than the number of its unknown variables or $MN = 2N$, yielding either all-zero solution $z = \mathbf{0}$ or no solution according to Theorem 2(a) and (b), which is not the result we want. Therefore, we have to make $M > 2$, say $M = 3$ (note we have $K = 3$ here), to insure possible non-zero solutions for the homogenous linear equation set $\mathbf{A}_z\, z = \mathbf{0}$. Similar results can also be obtained for other values of M, N, and K. Thus, we obtain another interesting conclusion from this paper as follows.

Corollary 2. *To generate a perfect complementary code set with its set size being K, in which all its member codes should have perfect periodic and aperiodic auto-correlation and cross-correlation functions, the flock size M of the all member codes must not be less than the set size K, or $M \geq K$. Usually, $M = K$ suffices.*

The corollaries 1 and 2 should be used jointly to generate the PC code sets. The major steps for generation of the PC codes are summarized in the sequel

(1) First, the three parameters for the PC code set should be given, that is, the length of element codes N, the flock size M, and the set size K, where the conditions for $M > 1$ and $M = K$ must be satisfied.
(2) Second, then we should generate a seed code x with the help of Point (1), which gives all necessary conditions for the seed code to have ideal periodic and aperiodic auto-correlation functions.
(3) Third, with the first code x already in the set, we can proceed to search for the second code y by using the homogenous linear equation set given in Point (2), which specifies the perfect periodic and aperiodic cross-correlation functions between the codes x and y. As long as the conditions of $M > 1$ and $M = K$ are satisfied, the homogenous linear equation set should have some suitable solutions, which usually take the forms as expressions of some undetermined variables.
(4) Fourth, the valid solutions of the problem can usually be obtained by taking into account Point (3), which specifies the ideal periodic and aperiodic auto-correlation functions for y.
(5) By repeating the same procedure over and over again to generate all K codes in the set.

Clearly, the choice of the seed code is of ultimate importance in the code generation procedure given above.

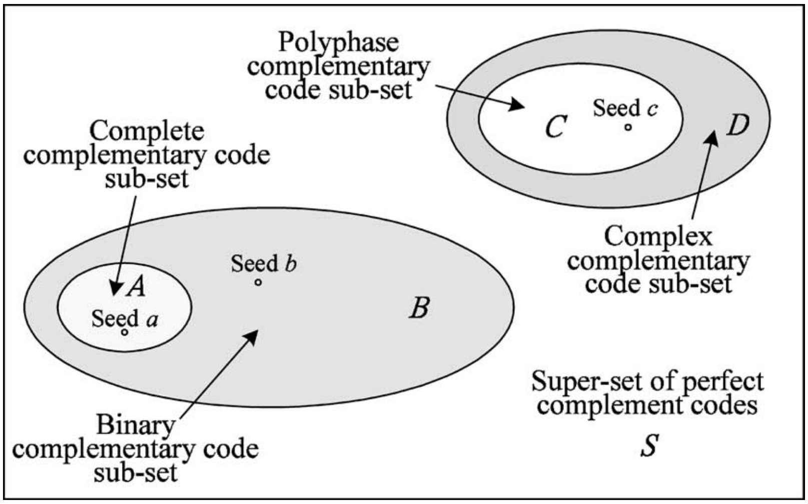

Fig. 3. Illustration of the super-set (**S**) of perfect complementary codes in relation with different sub-sets, which are generated by using different seed codes, such as seed a, seed b, and seed c, yielding complete complementary code sub-set A, binary complementary code sub-set B and polyphase complementary code sub-set C, respectively, in the code generation procedure, where $A \subset B \subset S$ and $C \subset D \subset S$.

Figure 3 shows the possible sub-sets, in which the generation procedure may result due to the use of different seed codes, since all non-linear and linear equations specifying the MAI-free and MI-free operation conditions were introduced on a very general basis, which include all possible candidate codes that satisfy the MAI-free and MI-free operation conditions, either binary or quadraphased and either real or complex sequences. In Figure 3, it is shown that three different seed codes (*a*–*c*) effectively lead to three sub-sets of the PC codes, which are sub-sets *A*–*C*, representing the complete complementary codes [15,16], binary complementary codes and polyphase complementary codes respectively, where $A \subset B \subset S$ and $C \subset S$. Therefore in principle, by using a series of carefully selected seed codes, we can generate the whole super-set **S** of the perfect complementary codes, each of which should insure MAI-free and MI-free operation of a CDMA system.

Now let us give a simple example to show step by step how the generation procedure works. Assume that we already have a seed code $x = \{x_{11}, x_{12}; x_{21}, x_{22}\} = \{+1, +1; +1, -1\}$, where the semi-column ";" separates the two element codes, each of which is 2-chip long. Therefore, the two characteristics parameters, or flock size and element code length, for this code are $M = 2$ and $N = 2$, respectively. Based on the conclusions obtained previously we must have in this case $M = 2 > 1$ (from Corollary 1) and $M = K = 2$ (from Corollary 2), meaning that there must be another code y to complete the whole code set. Let the second code be $y = \{y_{11}, y_{12}; y_{21}, y_{22}\}$. We can obtain a homogenous linear equation set according to Equation (2) as

$$\begin{cases} y_{11} + y_{12} + y_{21} - y_{22} = 0 \\ y_{11} + y_{12} - y_{21} + y_{22} = 0 \\ -y_{11} + y_{12} + y_{21} + y_{22} = 0 \end{cases} \tag{16}$$

the solution to which is

$$\begin{cases} y_{11} = -y_{12} \\ y_{12} = y_{12} \\ y_{21} = -y_{12} \\ y_{22} = -y_{12} \end{cases} \tag{17}$$

Then, using the conditions for ideal periodic and aperiodic auto-correlation functions, we obtain

$$\begin{cases} y_{11}^2 + y_{12}^2 + y_{21}^2 + y_{22}^2 = 4 \\ y_{12}y_{11} + y_{11}y_{12} + y_{22}y_{21} + y_{21}y_{22} = 0 \\ -y_{12}y_{11} + y_{11}y_{12} - y_{22}y_{21} + y_{21}y_{22} = 0 \end{cases} \tag{18}$$

to yield

$$\begin{cases} 4y_{12}^2 = 4 \\ 0 = 0 \\ 0 = 0 \end{cases} \tag{19}$$

Let $y_{12} = 1$ to find one solution as $\{-1, +1; -1, -1\}$. We can obtain another solution by letting $y_{12} = -1$ as $\{+1, -1; +1, +1\}$.

Thus, we find two sets of the PC codes in this case as $\{x, y\} = \{\begin{bmatrix}+1,+1\\+1,-1\end{bmatrix}, \begin{bmatrix}+1,-1\\+1,+1\end{bmatrix}\}$ and $\{x', y'\} = \{\begin{bmatrix}+1,+1\\+1,-1\end{bmatrix}, \begin{bmatrix}-1,+1\\-1,-1\end{bmatrix}\}$, respectively.

From the relation $M = K = 2$, we have already known that it is impossible for us to introduce the third code to either of the two code sets, $\{x, y\}$ or $\{x', y'\}$, if the conditions for MAI-free and MI-free operation are to be satisfied. Nevertheless, we would like to show the validity of Corollary 2 by testing the suitability of the third code $z = \{z_{11}, z_{12}; z_{21}, z_{22}\}$ as follows. If the third code exists, it has to satisfy the following MAI-free condition with the codes x and y as

$$\begin{cases} z_{11} + z_{12} + z_{21} - z_{22} = 0 \\ z_{11} - z_{12} + z_{21} + z_{22} = 0 \\ z_{11} + z_{12} - z_{21} + z_{22} = 0 \\ -z_{11} + z_{12} + z_{21} + z_{22} = 0 \\ -z_{11} + z_{12} - z_{21} - z_{22} = 0 \\ -z_{11} - z_{12} - z_{21} + z_{22} = 0 \end{cases} \tag{20}$$

from which it can be found that the last two equations are not independent and thus can be deleted to yield the following reduced homogenous linear equation set

$$\begin{cases} z_{11} + z_{12} + z_{21} - z_{22} = 0 \\ z_{11} - z_{12} + z_{21} + z_{22} = 0 \\ z_{11} + z_{12} - z_{21} + z_{22} = 0 \\ -z_{11} + z_{12} + z_{21} + z_{22} = 0 \end{cases} \tag{21}$$

which has only all-zero solution, or $z_{11} = z_{12} = z_{21} = z_{22} = 0$. Therefore, we can not find the third code for either of the code sets $\{x, y\}$ or $\{x', y'\}$.

Table I compares characteristic parameters, that is, element code length N, flock size M, and set size K, of the PC codes, complete complementary codes, Gold codes, and OVSF codes with their PG fixed as L. It is seen from the table that the PC codes have the similar set size as that of Gold or OVSF codes at a given PG value; while the complete complementary codes [15,16] offer much smaller set size.

Table I. Characteristic parameters of perfect complementary (PC) codes, complete complementary (CC) codes, gold codes and OVSF codes with their processing gains (PG's) being fixed as L.

Parameter	PC codes	CC codes	Gold codes	OVSF codes
Element code length N	$1 \sim 2^{r}$*	$L^{\frac{2}{3}}$	L	L
Flock size M	$L \sim \frac{L}{2^r}$	$L^{\frac{1}{3}}$	1	1
Set size K	$L \sim \frac{L}{2^r}$	$L^{\frac{1}{3}}$	$L+2$	L

*r can be any integer.

5. Implementation Issues and Performance Study

The CDMA system using the PC codes generated from the super-set can be implemented with a multi-carrier CDMA architecture, where each carrier is responsible to send a particular element code. In other words, all element codes in a PC code should be sent to a receiver via different frequency channels. Obviously, the channel division among the element codes can be done through either time-division multiplexing (TDM) or frequency-division multiplexing (FDM), although only the latter is concerned in this paper. Figures 4 and 5 show an implementation example of the transmitter and the receiver, respectively, based on the FDM scheme, where M different carriers send M different element codes of each user into the channel and a receiver ought to collect all element codes to compute their correlation functions individually, followed by a summation to yield a decision variable.

The computer simulation has been used to study the performance of the CDMA scheme based on the PC codes, whose architecture is shown in Figures 4 and 5. In particular, we would like to make a comparison amid three 8-user CDMA systems based on different spreading codes, that is, the PC codes, Gold codes [6] and OVSF codes [3–5]. The Gold code can be considered as a good example of traditional quasi-orthogonal codes with its well-controlled 3-level cross-correlation functions. On the other hand, the OVSF code is a typical orthogonal code, although in fact it will not be orthogonal at all in asynchronous channels. The both are very popular for their applications in the current 2–3G mobile cellular standards [1–5].

In the simulation, we have taken a particular PC code set with its characteristic parameters being $M = 8$, $N = 8$, and $K = 8$, whose detail information has been given in Appendix.

Figure 6 is to compare the MAI sensitivity to an asynchronous CDMA system based on the three different codes, all operating with eight users. The set size for the PC code is just eight. However, the eight codes of concern are randomly selected from an either Gold code set or OVSF code set, whose sizes are 65 and 64 respectively. The inter-user delay in the asynchronous CDMA system is assumed to be four chips. All schemes use single correlator as their receivers. We did not consider

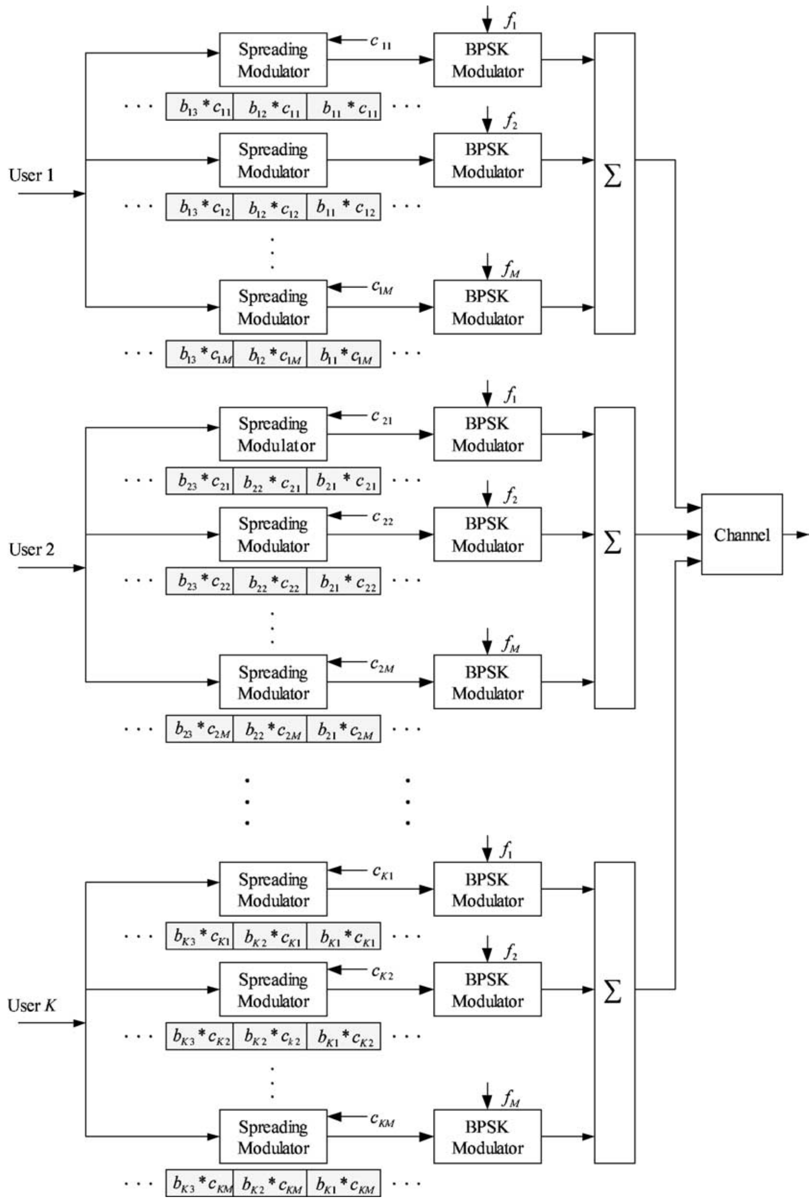

Fig. 4. Block diagram of a K-user CDMA system using the PC codes (a flock of the element codes $\{c_{k1}, c_{k2}, \ldots, c_{kM}\}$ is assigned to the kth user for $1 \leq k \leq K$), where each user employs M element codes, which are sent into channel via M different carriers, and BPSK spreading modulation and carrier modulations are used. The channel model concerned can be either AWGN or multipath channel.

multipath effect in this case. It can be seen from the figure that the system based on the PC code set can offer a well-controlled bit-error-rate (BER) regardless of changing user population of the system, indicating the desirable MAI-free operation property.

Figure 7 examines the system in terms of the impact of operation modes on the BER. Again, we concern an eight-user system, which operates in either synchronous (i.e., down-link channels) or asynchronous (i.e., up-link channels) mode. A great impairment can be seen for a system based on both Gold and OVSF codes; while the performance for the system using the PC codes is not affected at all with almost constant BER for whatever operation modes of the system, verifying another major benefit for the PC codes or the isotropic MAI-free operation. It is noted that the so called orthogonal OVSF codes perform extremely bad in asynchronous channels.

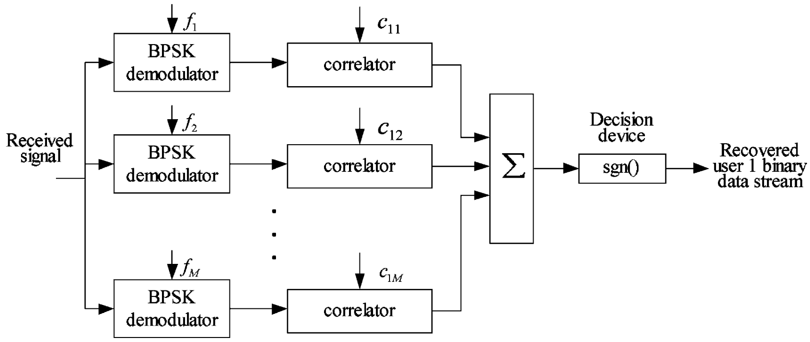

Fig. 5. Illustration of a receiver implementation for the CDMA system based on the PC codes, where the signal of interest to the receiver is the first user transmission and only a simple correlator (matched filter) is used to detect the useful signal.

Figures 8 and 9 consider the multipath effect to the system performance. The three systems will be compared under two different multipath scenarios with their delay profiles being [1, 0.8564, 0.107] and [0.7785, 0.6667, 0.0833], respectively. The inter-path delay of two consecutive multipath returns is assumed to be two chips. The inter-user delay is again four chips, as we concern only an asynchronous system here. Obviously, the first multipath scenario has a stronger first path than that of the second one. All three schemes will use a correlator as their signal detection device for fair comparison. It is observed from Figure 8 that the multipath effect does impose a very serious thread to the signal detection of the systems using either Gold or OVSF code, but no harm to that based on the PC codes. Similar observation can also be made from Figure 9, with only a bit increase BER for the scheme with the PC codes due to the application of single correlator, which captures only the first multipath return in [0.7785, 0.6667, 0.0833] that is lower than that in [1, 0.8564, 0.107], as shown in Figure 8. Nevertheless, Figures 8 and 9 clearly show the superior MI-free operation of the CDMA system based on the PC codes.

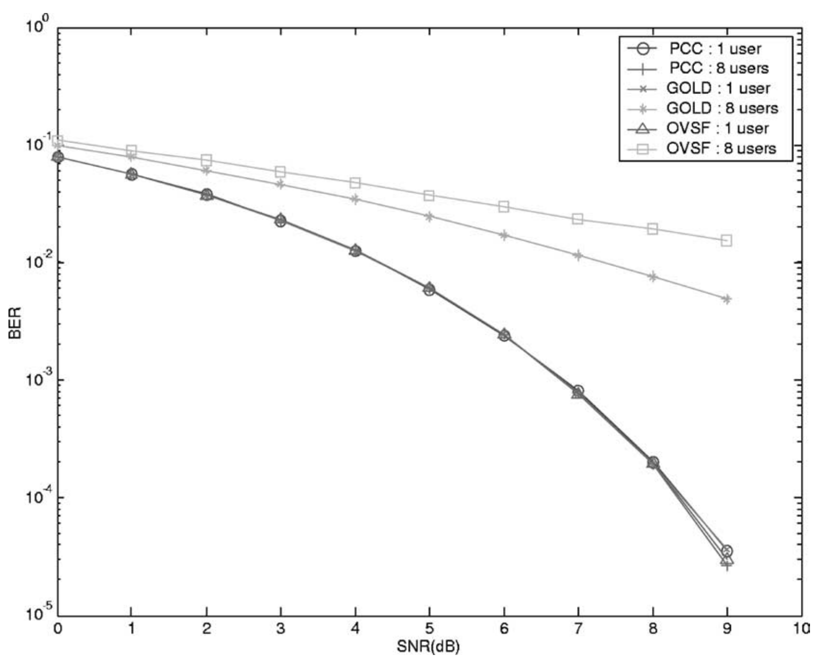

Fig. 6. MAI sensitivity comparison for CDMA systems based on three different types of spreading codes, which are Gold code, OVSF codes and PC codes, versus the change in number of users in AWGN channel. PG's for the PC codes, Gold codes and OVSF codes are 8(flock size)×8(element code length)=64, 63, and 64, respectively. The system operates in an asynchronous mode and the inter-user delay is four chips. The conventional correlator is used for the all three systems.

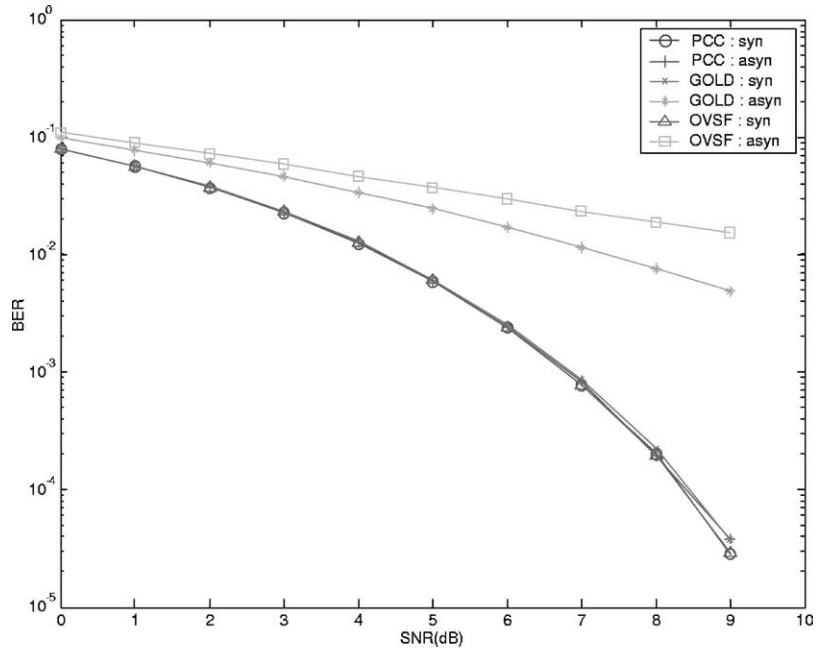

Fig. 7. Comparison for the impact of the operation modes (synchronous or asynchronous) on BER's of the eight-user CDMA systems based on three different types of spreading codes, including Gold code, OVSF codes, and PC codes. PG's for the PC codes, Gold codes, and OVSF codes are 8(flock size)×8(element code length)=64, 63, and 64, respectively. The system operates in a AWGN channel and the inter-user delay (in the asynchronous channel) is four chips. The conventional correlator is used for the receiver in all three systems.

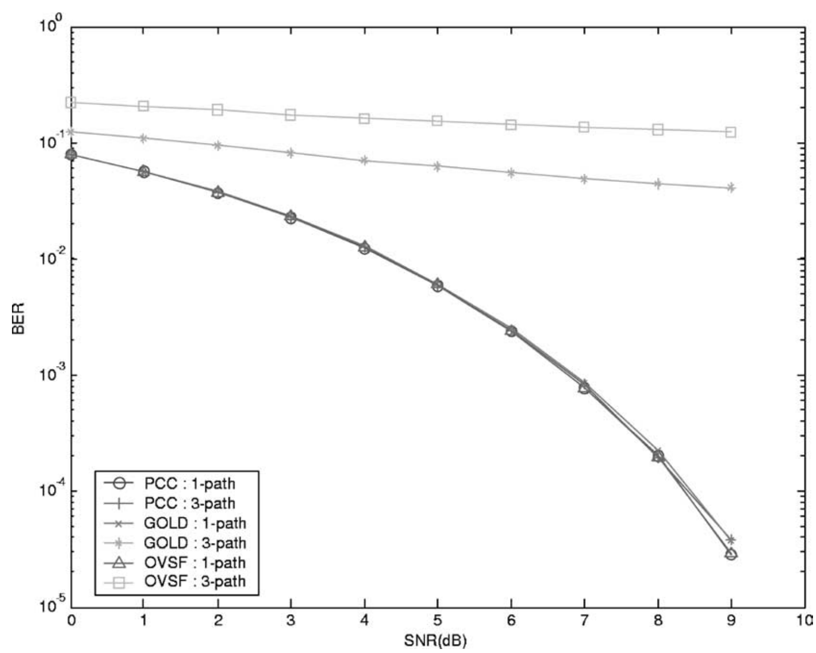

Fig. 8. The impact of multipath effect on BER's of the eight-user CDMA systems based on three different types of spreading codes, including Gold code, OVSF codes, and PC codes. PG's for the PC codes, Gold codes, and OVSF codes are 8(flock size)×8(element code length)=64, 63, and 64, respectively. The 3-path channel is characterized by its delay profile [1, 0.8564, 0.107] with its inter-path delay being two chips. The systems operate in asynchronous mode with its inter-user delay being four chips. The conventional correlator is used for the receiver in all three systems.

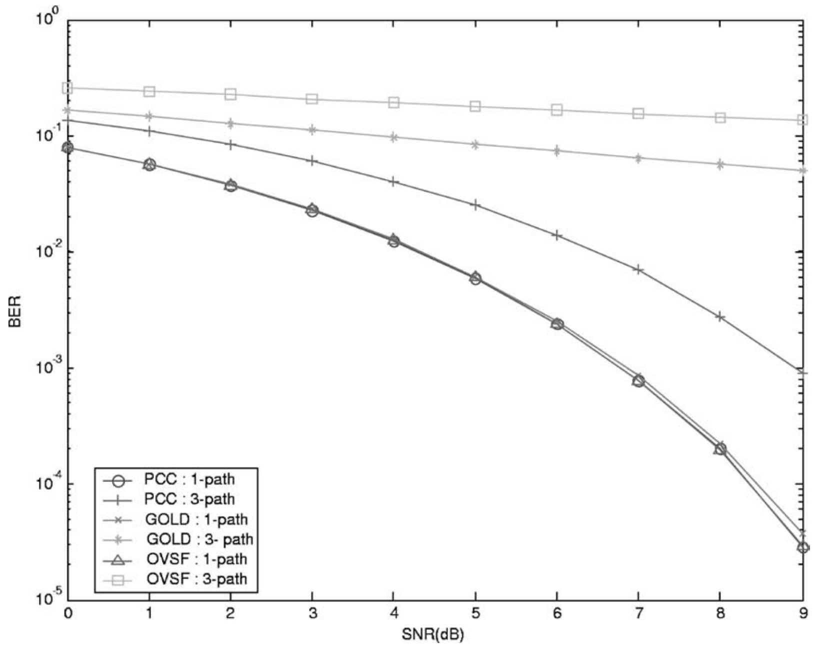

Fig. 9. The impact of multipath effect on BER's of the eight-user CDMA systems based on three different types of spreading codes, including Gold code, OVSF codes, and PC codes. PG's for the PC codes, Gold codes and OVSF codes are 8(flock size)$\times$8(element code length)=64, 63, and 64, respectively. The three-path channel is characterized by its delay profile [0.7785, 0.6667, 0.0833] with its inter-path delay being two chips. The systems operate in asynchronous mode with its inter-user delay being four chips. The conventional correlator is used for the receiver in all three systems.

It should be noted that the decision variable formed at the output of the summation unit in Figure 5 must be very robust due to the use of the PC codes, which have been optimized for the MAI-free and MI-free operation in both up-link and down-link channels. Therefore, basically a simple correlator (instead of a RAKE) is enough to yield a satisfactory detection efficiency, even if multipath effect is present. This feature can be considered as frequence selective fading resistance of the scheme due to its inherent immunity against the MI. In addition, whenever necessary a RAKE can still be applied to further improve the detection efficiency. If so, a genuine multipath-diversity can be achieved with either equal-gain-combining (EGC) or maximal-ratio-combining (MRC) RAKE, as the output from each finger consists of only the useful signals from different multipath returns and nothing else. The use of an EGC-RAKE instead of an MRC-RAKE can effectively reduce the complexity of a channel estimation unit at a receiver, since the EGC-RAKE can already offer a very good performance in the proposed CDMA system owing to the MAI-free and MI-free operation. On the other hand, the output signal-to-interference ratio of a RAKE in a traditional CDMA system will be impaired by the nontrivial auto-correlation side lobes of different paths from self-transmission as well as cross-correlation levels due to the other transmissions.

The scheme proposed in Figures 4 and 5 is basically a multi-carrier CDMA system. However, the multiple carriers used here will not provide any frequency diversity gain, as each carrier will convey different information (or different element codes). Therefore, it may give rise to a concern that the scheme will not be as robust as a traditional multi-carrier CDMA against frequency-selective fading in the channel. This concern is not necessary because the scheme creates an MI-free environment due to the use of the PC codes, designed to work under zero periodic and aperiodic auto-correlation functions. Therefore, the signaling in such a CDMA system has a very good immunity against the MI or frequency-selective fading.

Another advantage for the PC codes based CDMA is its unique signal decorrelating property, which makes a multi-user detection unnecessary. As all codes in a PC code set are designed with zero periodic and aperiodic cross-correlation functions, or simply MAI-free operation. In other words, the signaling structure of the proposed CDMA scheme has already been implanted some sort of de-correlating mechanism. Therefore, all transmissions in the channel have

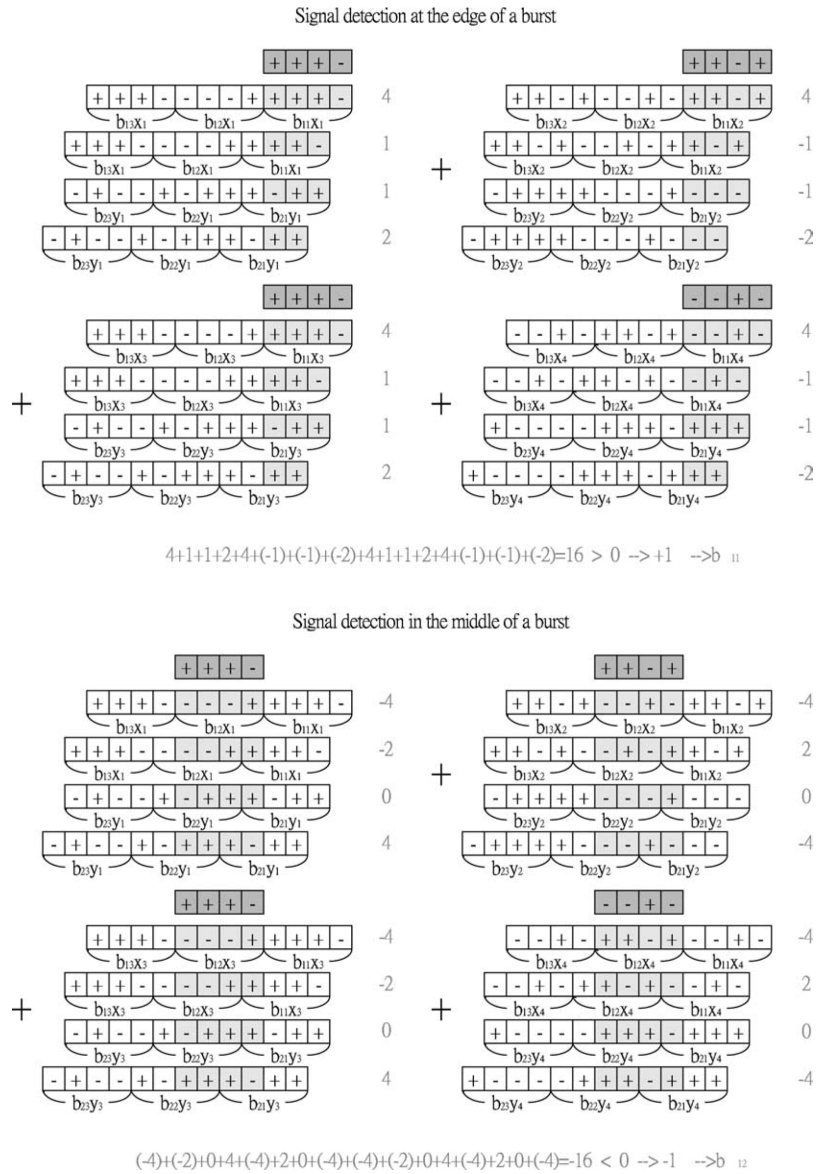

Fig. 10. The illustration of identical detection efficiency at the edge and in the middle of a data burst in the PC code based CDMA, where a particular PC code set of concern takes the following form: user 1: $x_1 = \{+++-\}$, $x_2 = \{++-+\}$, $x_3 = \{+++-\}$, $x_4 = \{--+-\}$; user 2: $y_1 = \{+-++\}$, $y_2 = \{+---\}$, $y_3 = \{+-++\}$, $y_4 = \{-+++\}$; user 3: $z_1 = \{+++-\}$, $z_2 = \{++-+\}$, $z_3 = \{---+\}$, $z_4 = \{++-+\}$; user 4: $w_1 = \{+-++\}$, $w_2 = \{+---\}$, $w_3 = \{-+--\}$, $w_4 = \{+---\}$. Only two users $\{x, y\}$ are present and an asynchronous two-path multipath channel is concerned. The inter-path delay is one chip for illustration simplicity. The three bits in the bursts from the two users are $\{+1-1+1\}$ and $\{+1+1-1\}$.

virtually been decorrelated with one another in either up-link or down-link transmission mode without MAI.

For the same reason, the scheme does not require a precision power control system including open loop and closed loop algorithms, which has been a must to all traditional CDMA systems to mitigate the near-far effect. Due to the perfect periodic and aperiodic cross-correlation functions, the power control pertaining to the proposed CDMA system needs only to maintain minimum emission power for battery life conservation and nothing more than that.

Obviously, the performance of the proposed CDMA scheme has nothing to do with the co-channel interference such that beam-forming algorithms applied to a smart antenna system will no longer be relevant to the improvement of the signal-to-interference-ratio at a receiver. Therefore, we do not need such a complex antenna array system as a means to suppress the co-channel interference in a cellular system.

A CDMA system using the PC codes guarantees a symmetry for transmissions in both up-link and down-link channels due to its isotropic MAI-free and MI-free operation, which can effectively increase the overall throughput in the air-link sector of a wireless system.

The PC codes based CDMA signaling is in particular well suited for burst data traffic, whose message length is usually limited to a few packets, and thus message edges detection will have a much greater impact to the overall signal reception quality than that in continuous transmission mode. However, due to irregularity of partial auto- and cross-correlation functions in most traditional spreading codes, the detection efficiency at message edges is always a problem, which has made a lot of people doubt about the suitability for CDMA to be a good candidate for high-speed burst channel. This is a very serious concern that has to be addressed before CDMA can be considered for applications in B3G wireless. In this sense, the PC codes can offer a very good solution to this problem. It can easily be shown that the PC codes have ideal partial auto- and cross-correlation functions for either synchronous or asynchronous transmission mode, and thus the same MAI-free and MI-free operation can be ensured regardless of detection section in a message, either in edges or middle parts, as illustrated in Figure 10 where the signal detection process for both the edges and middle parts of a message is shown.

With the above advantages combined, the CDMA system implemented by the perfect complementary codes allows us to substantially reduce the hardware complexity as a whole (including the base station and mobiles on a cell basis), while at the same time offering a superior performance with a greater capacity and higher transmission speed than possible by currently available schemes [1–5]. We can also use OFDM technology to implement the multi-carrier CDMA architecture as proposed in Figures 4 and 5 to further reduce the system complexity.

6. Conclusion

This paper has revealed that an interference-free CDMA system can be formulated by using the perfect complementary codes, which can be generated with the help of an algebraic approach taking into account many real operational conditions in a wireless system, such as MI, asynchronous transmission, different signs of consecutive bits in data stream and so on. It has been concluded in this paper that such an ideal CDMA system will never be made possible if using only traditional single-code based channelization techniques, as those applied to the existing 2–3G systems. The possible implementation of such an interference-free CDMA system will make the perfect complementary codes extremely important, whose super-set can be generated by carefully selected seed codes located in different sub-sets. A multi-carrier CDMA architecture based on the PC codes has been proposed in this paper to achieve a noise-limited performance at a very affordable complexity, giving us a great hope for the future CDMA technological revolution.

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APPENDIX A PC Code Set Concerned in Simulation

The perfect complementary code set concerned in the computer simulation study has an identical value for its set size K, flock size M and element code length N, or $K = M = N = 8$, whose detail information is listed as follows (Note: the semi-column ";" is used to separate different element codes.):

(1) The 1st flock:
(+ + + - + + - +; + - + + + - - -; + + + - - - + -; + - + + - + + +; + + + - + + - +; + - + + + - - -; - - - + + + - +; - + - - + - - -)
(2) The 2nd flock:
(+ + + - + + - +; - + - - - + + +; + + + - - - + -; - + - - + - - -; + + + - + + - +; - + - - - + + +; - - - + + + - +; + - + + - + + +)
(3) The 3rd flock:
(+ - + + + - - -; + + + - + + - +; + - + + - + + +; + + + - - - + -; + - + + + - - -; + + + - + + - +; - + - - + - - -; - - - + + + - +)
(4) The 4th flock:
(+ - + + + - - -; - - - + - - + -; + - + + - + + +; - - - + + + - +; + - + + + - - -; - - - + - - + -; - + - - + - - -; + + + - - - + -)
(5) The 5th flock:
(+ + + - + + - +; + - + + + - - -; + + + - - - + -; + -

+ + - + + +; - - - + - - + -; - + - - - + + +; + + + - - - + -; + - + + - + + +)

(6) The 6th flock:
(+ + + - + + - +; - + - - - + + +; + + + - - - + -; - + - - + - - -; - - - + - - + -; + - + + + - - -; + + + - - - + -; - + - - + - - -)

(7) The 7th flock:
(+ - + + + - - -; + + + - + + - +; + - + + - + + +; + + + - - - + -; - + - - - + + +; - - - + - - + -; + - + + - + + +; + + + - - - + -)

(8) The 8th flock:
(+ - + + + - - -; - - - + - - + -; + - + + - + + +; - - - + + + - +; - + - - - + + +; + + + - + + - +; + - + + - + + +; - - - + + + - +)

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Hsiao-Hwa Chen received his B.Sc. and M.Sc. degrees from Zhejiang University, China, and Ph.D. from the University of Oulu, Finland, in 1982, 1985, and 1990, respectively, all in electrical engineering. He worked with Academy of Finland for the research on spread spectrum communications as a research associate during 1991–1993 and the National University of Singapore as a Lecturer and then a Senior Lecturer from 1992 to 1997. He joined Department of Electrical Engineering, National Chung Hsing University, Taiwan, as an associate Professor in 1997 and was promoted to a full-professor in 2000. In 2001 he moved to National Sun Yat-Sen University, Taiwan, as the founding director of the Institute of Communications Engineering of the University. Under his leadership the institute was ranked the 2nd place in the country in terms of SCI journal publications and National Science Council funding per faculty in 2004. He has been a visiting professor to Department of Electrical Engineering, University of Kaiserslautern, Germany, in 1999, the Institute of Applied Physics, Tsukuba University, Japan, in 2000, and Institute of Experimental Mathematics, University of Essen, Germany in 2002. He is a recipient of numerous Research and Teaching Awards from the National Science Council and Ministry of Education, Taiwan, from 1998 to 2001. He has authored or co-authored over 120 technical papers in major international journals and conferences, and three books and several book chapters in the areas of communications. He served as a TPC member and symposium chair of major international conferences, including IEEE VTC, IEEE ICC, and IEEE Globecom, etc. He served or is serving as member of the Editorial board or Guest Editor for IEEE Communications Magazine, IEEE JSAC, IEEE Transaction on Wireless Communications, Wireless Communications and Mobile Computing (WCMC) Journal and International Journal of Communication Systems, etc. He has been a Guest professor of Zhejiang University, China, since 2003.

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His other activities include Chair of Asian Pacific Areas Assembly for Communications Systems sponsored by IECEJ since 1988. He authored numerous papers for both the IEEE and IECEJ transactions and conferences as well as four books in communications, written in Japanese.

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