A class of dc free, synchronization error correcting codes

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Abstract **- We present a class of binary codes that combine spectral shaping properties with additive error and synchronization error correcting capabilities, provided that only one of the above classes of errors occur in a word. These codes are constructed by finding certain balanced subcodes of the single synchronization error correcting Levenshtein block codes. These subcodes have a minimum Hamming distance of four, thus single additive errors can be corrected and double additive errors can be detected. By further constraining the codes, we obtain codes for which the second derivative of the power spectrum at 0 Hz is zero and the power spectrum has a spectral null at half the normalized frequency. The rates of these codes are determined and presented for wordlengths less than 25 bits.**

I. INTRODUCTION

In this paper we present a new class of codes for use in magnetic recording systems. The new codes form a subset of the Levenshtein codes [1] which are binary block codes that are capable of correcting the deletion or insertion of a binary symbol in a random position in the code word. In the construction of the codes we add additional constraints on the binary code word in such a way that the second derivative of the power spectrum at 0 Hz is zero. This is also termed as a first order null at dc [2]. In magnetic and optical recording systems it is preferable to have a dc free code due to the physical properties of the recording media as well as the nature of the recording system. As an example, in optical coding a dc free code is used to reduce interaction between the data written on the track and the servo system employed. By using a code that has a first order null at dc, a much higher suppression of the dc component is achieved. Another property of the new codes is that they have a spectral null at half the normalized frequency. This is also called the minimum bandwidth property because only half of the bandwidth need be used to transmit the code message without errors in a noiseless system. This allows a doubling of the possible code transmission rate for a certain fixed bandwidth.

These new codes also have single additive error correcting capabilities, i.e. the substitution of a binary symbol by its counterpart can be corrected. These codes are also able to detect two random additive errors and hence a single peak shift error.

II. CODE CONSTRUCTION

We next present a construction method for the new codes. The new codes form a subset of the Levenshtein codes [1] which are single synchronization error correcting codes, i.e. the loss or gain of a binary symbol due to synchronization errors can be corrected. These Levenshtein codes are described by

$$\sum_{i=1}^{n} ix_i \equiv a \bmod (n+1), \quad x_i \in (0,1) \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)$ is a binary word, n is the length of x and a is an integer corresponding to the code partition [1].
Investigation of the Levenshtein codes [3] revealed relationships between the Hamming distance and weight of the Levenshtein codes. Of special interest was the observation that the balanced subcodes (i.e. having a weight of n/2) of a Levenshtein code have a minimum distance $d_{min} = 4$, thus these codes are able to correct a single additive error and detect two additive errors.
Relations describing the spectral shaping of binary codes can be found in [2]. From [2] a first order spectral null at 0 Hz (i.e. the second derivative of the power spectrum at 0 Hz is zero), is found when the code complies to the following criterium :

$$\sum_{i=1}^{n} ix_i = 0, \quad x_i \in (-1,+1), \tag{2}$$

and the variables i and n are defined for (1), we assume a mapping from {0,1} onto {-1,1} and furthermore n is a multiple of 4.
Another property that can be incorporated into the new class of codes is a spectral null at the Nyquist frequency. The condition from [4], known as the Running Alternate Sum (RAS), for this property is stated in equation (3)

$$\sum_{i=1}^{n} (-1)^i x_i = 0, \tag{3}$$

and all the variables are the same as in (2). Codes that comply to the RAS have minimum bandwidth and require half the normalized bandwidth. Utilizing this property it is possible to double the transmission rate of the information bits with zero probability of error over a bandwidth restricted noiseless channel.

III. EXAMPLE

To illustrate the new class of codes, we present a specific example. We consider the set of all binary words of length 8 and which have a weight of $n/2 = 4$. There are $\binom{8}{4} = 70$ such words. From this set, words are rejected which do not satisfy equations (1) to (3). The surviving set of words is the valid code with the properties discussed in section I. For the case of $n = 8$ we find 8 valid code words, shown in Table I. It is thus possible to map 3 information bits onto the 8 code bits, giving an information rate $R = 3/8$.

TABLE I

CODE BOOK FOR A RATE R = 3/8 CODE

00111100	10010110
01011010	10011001
01100110	10100101
01101001	11000011

In general the number of surviving code words is not a power of two. In this case some words are arbitrarily discarded until the number of code words is a power of two.

The calculated power spectrum for the $R = 3/8$ code is shown in Fig. 1. In Fig.2 we present information symbols vs. word length for the new class of codes for $n \leq 24$.

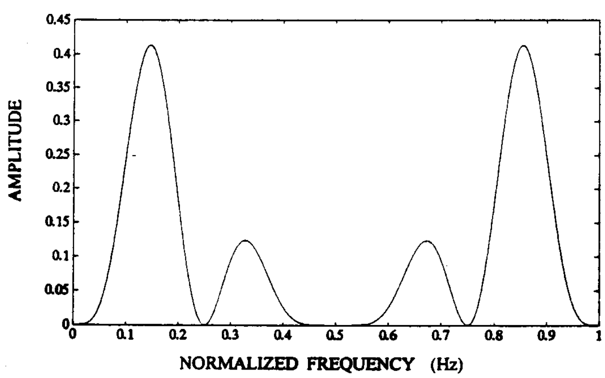

Fig. 1. Normalized power spectrum of example

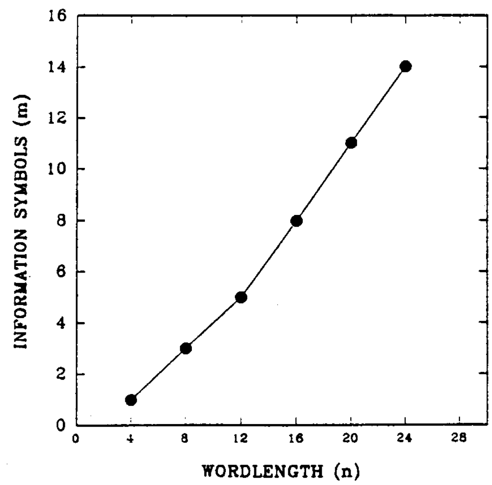

Fig. 2. Information symbols vs. word length

IV. CONCLUSION

The new class of codes is an attractive alternative to some recording codes currently used such as the Hedeman codes [5]. The Hedeman codes have dc-free and minimum bandwidth spectral properties. The new class of codes presented here has better dc suppression than the Hedeman codes and in addition also offers single additive error correcting and double additive error detecting capabilities, which implies that a single peak shift can also be detected, along with the property that the loss or gain of a single bit due to synchronization errors can be corrected.

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