On the Channel Capacity of a European-style Residential Power Circuit

Olaf G. Hooijen[*] and A.J. Han Vinck[**]

[*] Signaal Communications, P.O.Box 88, 1270 AB Huizen, the Netherlands
Phone +31 35 52 48 335, Fax +31 35 52 48 343, Email o_hooijen@signaal.nl
[**] Institute for Experimental Mathematics, University of Essen, Ellernstrasse 29, 45326 Essen, Germany,
Phone +49 201 320 64 49, Fax +49 201 320 64 63, Email vinck@exp-math.uni-essen.de

Abstract — **In this paper, based on the results of an extensive measurement campaign executed in the city of Amsterdam, a channel model for the residential power line channel (RPLC) used as a digital communications channel is presented. From this model bounds for the channel capacity of such a channel, used under European regulations are calculated. It is concluded that it is indeed theoretically possible to us the RPLC to solve the local loop problem in a second, fixed line telephone network.**

I. Introduction

During the last few decades electricity companies have invested a lot in setting up a telecommunication infrastructure in parallel with their power lines. The main goal of this telecommunication network was signalling, allowing communication between a central point (i.e. the control station) and elements making up the electricity transportation network (e.g. transformers and circuit-breaking switches). Furthermore, by means of these telecom-lines the generators inside separate energy-plants can be kept in perfect 50 Hz synchronization.

Meanwhile, thousands of kilometers of high-quality glass-fibre (primarily concentrated to the high-voltage transportation network) and copper twisted pair connections (in parallel to the medium voltage distribution lines) are available. Because of this network, most electricity companies now have a modern telecommunication network at their disposal, with high quality glass-fibre back-bones spanning entire regions and even countries and twisted pair lines reaching almost all distribution transformers. Since in most European countries a very high percentage of the customers live within a 400 m radius of such a distribution transformer, only these last 400 m have to be bridged in order to set up a second fixed line telecommunication network, an issue which has gained a lot of attention due to the fact that in Europe on January 1ST 1998 the national PTT's lose their monopoly position on fixed line telephony.

However, although it seems simple to bridge this last gap between the distribution transformer and a costumer, a distance of typically less than 400 m, reality is somewhat different. Not the distance is the main problem, but the number of lines that would be necessary. In a city like Amsterdam about 500,000 connections would have to be realized, meaning a total of more than 50,000 km of copper wire! Therefore, although it seems like the largest portion of the infra-structure for a fixed telephony network is already present, more than 50% of the total costs still has to be made.

When it would be possible to use the RPC-lines (which are already available, connecting every household with a distribution transformer) for digital speech communications, the "missing link" could be realized on a very low-cost basis. However, due to the fact that only very little knowledge existed concerning the parameters that make up the RPC-channel (like noise, signal-attenuation, etc.), until now no clear statements could be made with respect to the (theoretical) possibility of setting up an RPC-based second fixed telephone network.

In this paper, based on a model of a European style low voltage RPC resultings from an extensive measurement campaign executed in the city of Amsterdam, the channel capacity of such a network is calculated.

II. RPC-related Regulations in Europe

A typical European-style low voltage power-line circuit has the following main characteristics:

- 3 phase system, 400 V between phases; loads are typically connected between a phase and zero (—> 240V). Heavy loads are connected between two phases. In certain older RPC's the voltage between phases is 240 V. In this case loads are

PLC'98 ISBN:90-74249-18-3

connected between two phases.
- 50 Hz power cycle.
- Typically 400 houses are connected to a single distribution transformer in a city environment; these houses can be found in a circle with an average radius of 400 m.

For Western-Europe (i.e. the countries forming the European Union plus Norway and Switzerland) regulations concerning RPC-communications are described in EN 50065, entitled "Signalling on low-voltage electrical installations in the frequency range 3 kHz to 148.5 kHz". This European Norm replaces the individual standards that existed per country. In part 1 of this EN-standardization-paper, entitled "General requirements, frequency bands and electromagnetic disturbances" [1], the allowed frequency band and output voltage for communications over the RPC are indicated.

The *frequency range* which is allowed for communications ranges from 3 to 148.5 kHz and is subdivided into five sub-bands:
- Frequency band from 3 to 9 kHz:
  The use of this frequency band is limited to energy providers; however, with their approval it may also be used by other parties inside the consumers premises.
- Frequency band from 9 to 95 kHz:
  The use of this frequency band is limited to energy providers and their concession-holders. This frequency band is often referred to as the "*A-band*" (due to the fact that the frequency band from 3 to 9 kHz was defined in a later stage, no "letter" description exists for it).
- Frequency band from 95 to 125 kHz:
  The use of this frequency band is limited to the energy providers' costumers; no access-protocol is defined for this frequency band. This frequency band is often referred to as the "*B-band*".
- Frequency band from 125 to 140 kHz:
  The use of this frequency band is limited to the energy providers' costumers; in order to make simultaneous operation of several systems within this frequency band possible, a carrier-sense multiple access-protocol using a center frequency of 132.5 kHz was defined. Details concerning this protocol can be found in [1]. This frequency band is often referred to as the "*C-band*".
- Frequency band from 140 to 148.5 kHz:
  The use of this frequency band is limited to the energy providers' costumers; no access-protocol is defined for this frequency band. This frequency band is often referred to as the "*D-band*".

The *maximum allowed transmitter output voltage* is defined as follows:
- For the frequency band from 3 to 9 kHz:
  The transmitter should be connected to a 50 Ω//(50 μH + 1.6Ω) RPC-simulation-circuit. In principle the transmitter output voltage should not exceed 134 dB(μV) ≡ 5V.

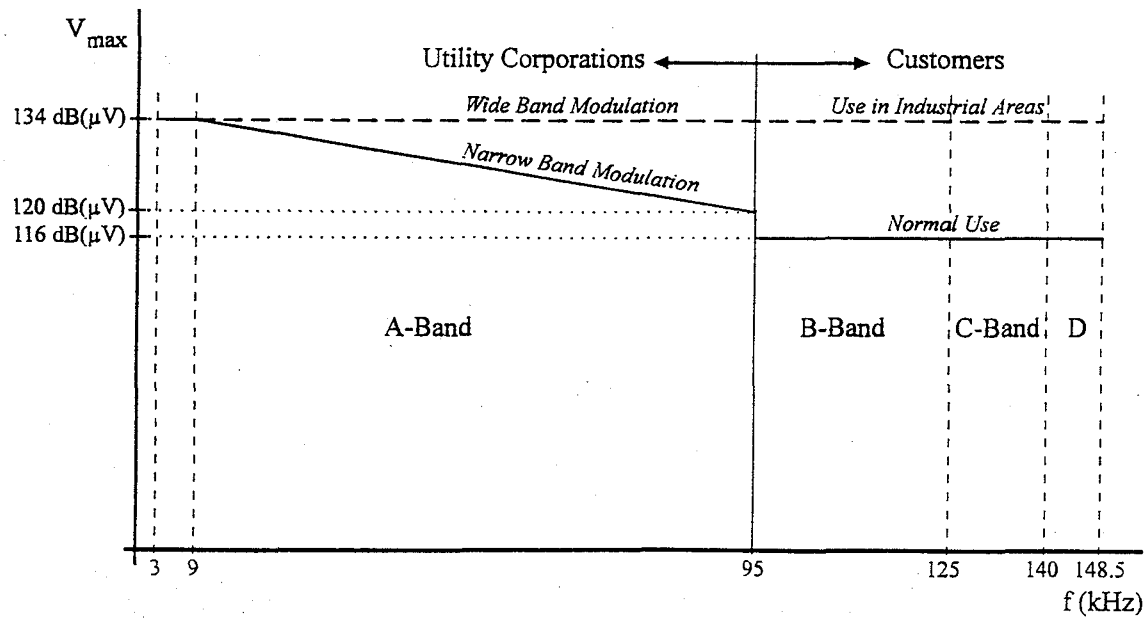

Fig. 1 Overview of European RPLC regulations

- For the frequency band from 9 to 95 kHz:
  The transmitter should be connected to a 50 Ω//(50 μH + 5 Ω) RPC-simulation-circuit. Different maximum transmitter output voltages apply for narrow-band (i.e. a 20-dB bandwidth of less than 5 kHz in width) and broad-band transmitters:
  - Narrow-band signals:
    The maximum allowed peak voltage at 9 kHz equals 134 dB(μV) ≡ 5 V, exponentially decreasing to 120 dB(μV) ≡ 1 V at 95 kHz.
  - Broad-band signals:
    The maximum allowed peak voltage equals 134 dB(μV).
    Furthermore, in any 200 Hz wide frequency band the maximum transmitter output voltage should not exceed 120 dB(μV).
- For the frequency band from 95 to 148.5 kHz:
  The transmitter output voltage should not exceed 116 dB(μV) ≡ 0.63 V. In certain cases an exception can be made allowing 134 dB(μV).

III. A Model for a European-style RPC

The earliest RPC-model that could be found in literature was presented in [2]. Basically, it is a time-variant linear filter channel model, moderated to incorporate the effects of 60 Hz synchronous (Canadian RPC) signal- and noise-fading that were reported. In [3] an RPC-channel model is presented, based on a set of time-variant linear filter channels, one for each frequency under consideration. A more practical RPC-channel model is given in [4]. Basically, this model again is the standard time-variant linear filter model. However, the additive noise in this model is recognized to originate from many different sources, each one with it own specific transmission path (and thus with its own time-variant linear filter) to the receiver. This allows to take into account a finite number (i.e. the most important) of noise sources. The model is therefore specifically suited to describe the channel between a given transmitter- and receiver-location pair. All noise-sources in that area can then be taken into account, leading to a very accurate channel description. This accuracy, however, makes the model less suited as a general channel model, i.e. applicable to describe the ensemble of all possible RPC-channels.

The relatively large differences in channel parameters (i.e. noise and signal attenuation) as a function of time and transmitter-/receiver location, which were measured in the RPC of the city of Amsterdam and reported in [5], already imply that for a *general* RPC channel model statistical description methods have to be used. With a general model here a model is meant that is applicable for "all" possible sets of transmitter/receiver- location pairs, where the word "all" is put between quotes because the author is well aware that it will probably be possible to find an RPC-channel somewhere, which falls outside of the proposed general model; the word "all" should therefore be read as "a very large percentage (e.g. >98%) of all possible RPC-channels". It should furthermore be noted that the quantitative aspect of the model proposed hereafter is only valid for the CENELEC A-band (i.e. the frequency band from 9 to 95 kHz) and primarily based on the measurement results obtained in the city of Amsterdam. This is a fully meshed, underground RPC, with large distribution transformers supplying typically around 400 households each. For other RPC-network types the quantitative part of the proposed model may differ. However, it is expected from the results reported in other publications (e.g. [6]) that the qualitative nature of the proposed model (i.e. the model as displayed in Fig. 2) is also applicable to other RPC-types.

Because the measurement results discussed in [5] showed no indication for a correlation between noise-power and signal-attenuation, like the RPC-channel models mentioned before, the RPC-channel model proposed here is based on the linear, time-variant filter channel with additive noise. A schematic representation of the model is given in Figure 2.

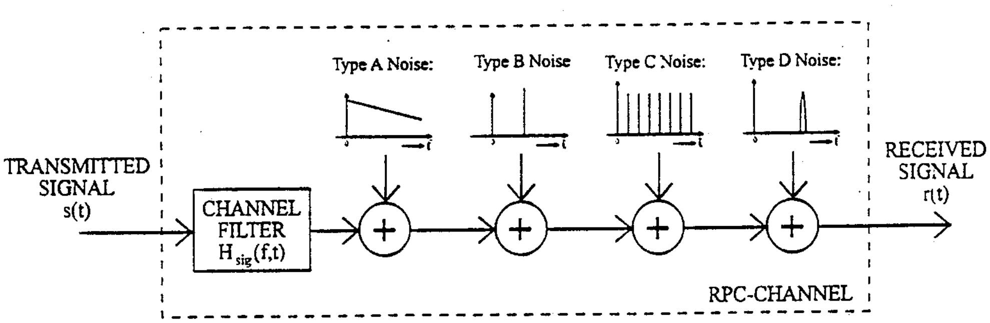

Fig. 2 The proposed RPC-channel model

Concerning the two main elements of the model, i.e. the linear, time-variant filter and the additive noise, the following can be said:

- The linear, time-variant filter:

$$H(f,t) = e^{-a(f,t)\ -\ jb(f,t)} \ . \qquad (1)$$

As the general expression for a filter- or transfer function $H(f,t)$ given in equation (1) shows, the transfer function can be split up in a real part $e^{-a(f,t)}$ equal to the signal attenuation and an imaginary part $e^{-jb(f,t)}$ equal to the phase-shift introduced to the signal. With respect to these two elements, the following can be stated:

- Signal attenuation:
The measurements [5] show that the amount of attenuation introduced to a signal transmitted over an RPC-line is approximately frequency- and time-independent for a given location. The level of attenuation increases as the distance from the transmitter increases. The analysis of signal attenuation along an RPC-line and the topology of an RPC-network seem to support a more or less exponential relation between the signal attenuation level a and the distance d between transmitter and receiver. The attenuation part of equation (1) can therefore be written as $e^{-a(d)}$. The measurements show that attenuation levels range from around 40 dB/km to more than 100 dB/km. Equation (1) then becomes:

$$H(f,t,d) = \sqrt{10^{-a*d}} * e^{-jb(f,t)} , \qquad (2)$$

with a assuming values in the range from 0.004 ($\equiv$ 40 dB/km) to 0.01 ($\equiv$ 100 dB/km). Measurements were performed on too few sites to make accurate statements concerning the actual probability density function of a over the range from 0.004 to 0.01.

- Phase shift:
The phase shift measurements [5] showed that the phase shift introduced to a signal *at a given frequency* is more or less time-independent. The "more or less" points to the constant jitter of up to 10° that was measured with respect to the reference sine-wave. Due to this phase-jitter, simple phase modulation techniques (like e.g. BPSK) can be expected to work without a problem, but more complex ones (like e.g. 16-PSK or 64-QAM) may not. No measurements were performed with respect to the phase-shift introduced to the signal as a function of frequency. It is therefore unknown whether $b(f)$ increases linearly with increasing frequency. However, due to the good results that have been achieved with respect to DS/PSK transmission schemes without equalization filters, such a linear increase is likely. We therefore postulate (given the remark concerning the phase-jitter mentioned above):

$$H(f,d) = \sqrt{10^{-a*d}} * e^{-jbf} \ . \qquad (3)$$

- Additive RPC-Noise:
From the around 700 noise-spectra that were obtained and to some extent analog to [6], the noise on the RPC can be considered to be a summation of four noise types:

a. Background noise (noise type a):
That portion of the noise that remains when subtracting all other three noise types from the total noise measured at a certain location. Unlike the other noise types it is always present.

Overall, back-ground noise power decreases for increasing frequencies. From the ensemble of 700 measured noise spectra, it could be shown that the background noise at a certain location, at an arbitrary time for frequencies within the CENELEC A-band in a good approximation is equal to:

$$N(f) = 10^{(K\ -\ 3.95\ 10^{-5} f)} \qquad [W/Hz], \qquad (4)$$

where K is normally distributed with an average μ = -8.64. The standard deviation σ of K equals 0.5.

b. Impulse noise, also referred to as noise type b:
All those disturbances on the RPC which last for a very small fraction of time, typically (but not necessarily) less than 100 μs.

The bulk of the impulses occurring on the RPC have a width that is typically less than 100 μs [7]. Impulse amplitudes typically lie more than 10 dB above the average background noise level and can exceed 40 dB roughly seems to be confirmed by the measurements [7].

Impulses originate from many different and uncorrelated sources. Therefore, although not confirmed by our measurements or literature, it seems fair to assume that noise impulse arrival has (approximately) a Poisson distribution with arrival rates in the order of $0 \leq \lambda \leq 0.5$ impulses per second.

c. Noise synchronous to the power system frequency, also referred to as noise type *c*:
A train of noise impulses in the time domain, arriving every $1/(k{*}f_{net})$ seconds, with k an integer, usually $k = 1$ or $k = 2$.

Separate impulses are not so powerful and only play a role when type *a*-noise levels are low. Furthermore, the chance that type *c*-noise is found at a certain moment at a given location is relatively small.

d. Narrow-band noise, also referred to as noise type *d*:
Noise confined to a narrow portion of the frequency band. Possible at any frequency within the CENELEC A-band, but most likely at television related frequencies (i.e. 31, 47, 62, 78 and 94 kHz).

Although exceptions exist, normally type *d*-noise only plays a role when type *a*-noise levels are low.

IV. RPC-Channel Capacity Bounds using the Shannon-Hartley Theorem

The title of this section might seem a bit strange at first glance: channel capacity in itself is a bound and the title of section might therefore be interpreted as "finding bounds for a bound". In fact this interpretation is correct, but does need some explanation:

As is well know from information theory, channel capacity (as it was first defined by Shannon is his famous 1948-paper [8]) is a figure for the maximum number of bits per second that can be transmitted error-free over a certain channel. It thus represents a bound for *that* channel, i.e. a channel with fixed properties. Now, as was shown in the previous section, the RPC-channel can not be treated as a single channel with fixed properties. A given RPC-channel represents one out of an ensemble of (infinitely) many possible channels. It is therefore impossible to calculate the channel capacity for *the* RPC-channel. The approach taken here therefore is to first find acceptable bounds for the ensemble of all possible RPC-channels (in the form of a worst-case and a best-case channel) and then to calculate the channel capacity for those bounding channels. The channel capacity of a certain RPC-channel will then lie somewhere between these bounds.

In this section rough bounds for the channel capacity are obtained by bounding the best case and worst case noise levels by additive white Gaussian noise. This allows the use of the Shannon-Hartley capacity theorem, given by:

$$C = W \log_2\left(1 + \frac{S}{N_0 W}\right) \quad [bit/s]. \tag{5}$$

Since W in equation (5) is fixed by the CENELEC-regulations to 86,000 Hz (i.e. the bandwidth of the A-band, 9-95 kHz), the only bounds that have to be found to complete calculation of (5) are bounds for S, the total received signal power, and N_0, the <u>single-sided</u> noise power spectral density of the white noise at the receiver.

As was indicated in section II, the CENELEC-specifications define a maximum transmitter output voltage rather then a maximum output power. For a broad-band transmitter this voltage should not exceed 134 dB(μV) = 5.0 Volt. Because of the fact that the RPC-impedance is by no means constant, as was shown in [5], the output power of a transmitter that tries to transmit at maximum allowed voltage can also vary over a wide range. Since the channel impedance can assume extremely low values, the transmitter output power is normally upper limited by the maximum power the transmitter is designed for, rather then by $S_{TR} = 25/|Z_{RPC}|$ where 25 is the square root of 5 V and $|Z_{RPC}|$ the RPC-channel impedance. In fact, unless the transmitter is able to transmit at very high output levels (i.e. $S_{TR} > 50$ W), generally the transmitter output power will be equal to the maximum transmitter output power due to the very low network impedances.

In the following the transmitter output power will therefore be assumed to be equal to 25 W. It can be shown that the outcome of the following analysis would only differ slightly, for other practical transmitter power levels, $10 \leq S_{TR} \leq 50$ W, primarily due to the fact that S_{TR} is located under the *log* sign in equation (5).

We assume that no coupling losses occur (i.e. a transmitter output impedance equal to 0 Ω). The transmitter output power therefore remains 25 W, independent of the RPC-impedance.

On the channel the transmitted signal is attenuated by a factor which, according to the channel model presented in the previous section for a best case scenario is equal to 40 dB/km and for a worst case scenario is equal to 100 dB/km. According to (3) this leads to a received signal power equal to:

$$S_{re,best}(d) = 25 * 10^{-0.004 * d} \quad [W] , \qquad (6)$$
$$S_{re,worst}(d) = 25 * 10^{-0.010 * d} \quad [W] . \qquad (7)$$

In equations (6) and (7) d represents the distance between transmitter and receiver in meters.

According to the channel model presented in the previous section, based on the measurements performed in Amsterdam, the noise that occurs on the power-line channel is a summation of background noise (type a-noise) and impulse noise (type b-noise), in some cases complemented by 50 Hz synchronous noise (type c) and fixed frequency noise (type d).

- In a *best case channel* only type a-noise is present. Due to the fact that the level of type a-noise according to equation (4) has a Gaussian distribution, in principle the best case type a-noise level is zero, although the chance that this level is ever reached is infinitely small. Of course, such a best case level results in unworkable results (the best case capacity would be equal to infinity, the worst case capacity zero). Therefore we define the best case type a-noise level to be equal to equation (4) with $K = \mu - 2\sigma = -9.64$. What results is the best case single sided noise power spectral density:

$$N_{re,best}(f) = 10^{(-9.64 - 3.95\ 10^{-5} f)} \quad [W/Hz] . \qquad (8)$$

  Due to the proposed choice of K, 97.72% of the channels in the ensemble of all possible channels has type a-noise levels equal or higher than equation (8).

- In a *worst case channel* apart from worst case type a-noise, also the other three noise types are present. As defined in the channel model, however, type c- and type d-noise only play a role of importance in case type a-noise levels are relatively low. Since in this case type a-noise is assumed to be maximum, type c- and type d-noise can safely be neglected. Also a large portion of the impulse noise (noise type b) can be neglected for the same reason. Only the very strong noise impulses come through and dominate the noise at the moment of their occurrence. Due to the fact that the impulse arrival rate of such strong impulses is very small compared to the targeted bit-rates and in order to simplify the calculations that are to follow, also type b is neglected in the following.

  Analog to the best case noise, we define the worst case RPC-channel noise to be equal to the worst case type a-noise, with $K = \mu + 2\sigma = -7.64$ in equation (4). What results is the worst case single sided noise power spectral density:

$$N_{re,worst}(f) \approx 10^{(-7.64 - 3.95\ 10^{-5} f)} \quad [W/Hz] . \qquad (9)$$

  Again, due to the proposed choice of K, 97.72% of the channels in the ensemble of all possible channels has type a-noise levels equal to or lower than equation (9).

Both the best-case (8) as well as the worst-case (9) noise levels as a function of frequency are depicted in Figure 3. Due to the choice of $K = \mu \pm 2\sigma$, from the ensemble of all possible channels at least 95% fit within these best- and worst-case levels.

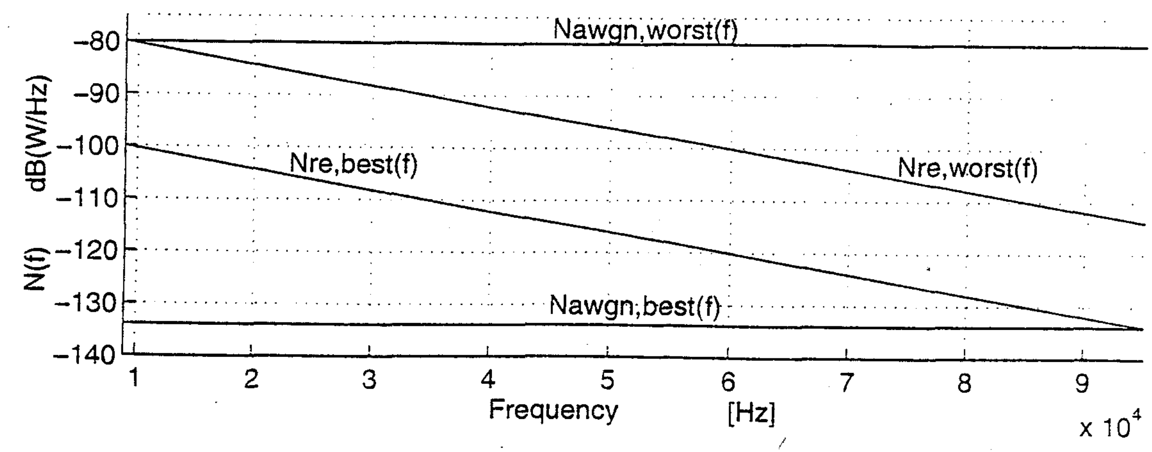

Fig. 3 Best- and worst case RPC-channel noise levels and the AWGN-noise levels that serve as bounds

In a first approximation, in order to be able to apply the Shannon-Hartley formula as given by equation (5), we bound the RPC-noise bounds given by equations (8) and (9) by appropriate additive white Gaussian noise (AWGN-)levels:

$$N_{AWGN,\,best}(f) = N_{re,\,best}(95000) = 4.1 \cdot 10^{-14}, \qquad (10)$$

$$N_{AWGN,\,worst}(f) = N_{re,\,worst}(9000) = 1.0 \cdot 10^{-8}. \qquad (11)$$

Finally, in order to obtain an expression for the best case channel capacity, equations (6) and (10) should be filled in in equation (5). Similar, for the worst case channel capacity, equations (7) and (11) should be introduced. Since the received signal power S_{re} in equations (6) and (7) are a function of the distance between transmitter and receiver d, also the capacity bounds are a function of distance.

$$C_{best}(d) = W \log_2\left(1 + \frac{S}{N_0 W}\right) = 86000 \log_2\left(1 + \frac{25 * 10^{-0.004 * d}}{4.1\ 10^{-14} * 86000}\right) \ [bits/s], \qquad (12)$$

$$C_{worst}(d) = W \log_2\left(1 + \frac{S}{N_0 W}\right) = 86000 \log_2\left(1 + \frac{25 * 10^{-0.01 * d}}{1.0\ 10^{-8} * 86000}\right) \ [bits/s]. \qquad (13)$$

The capacity bounds corresponding to equations (12) and (13) are depicted as a function of distance between transmitter and receiver in Figure 4.

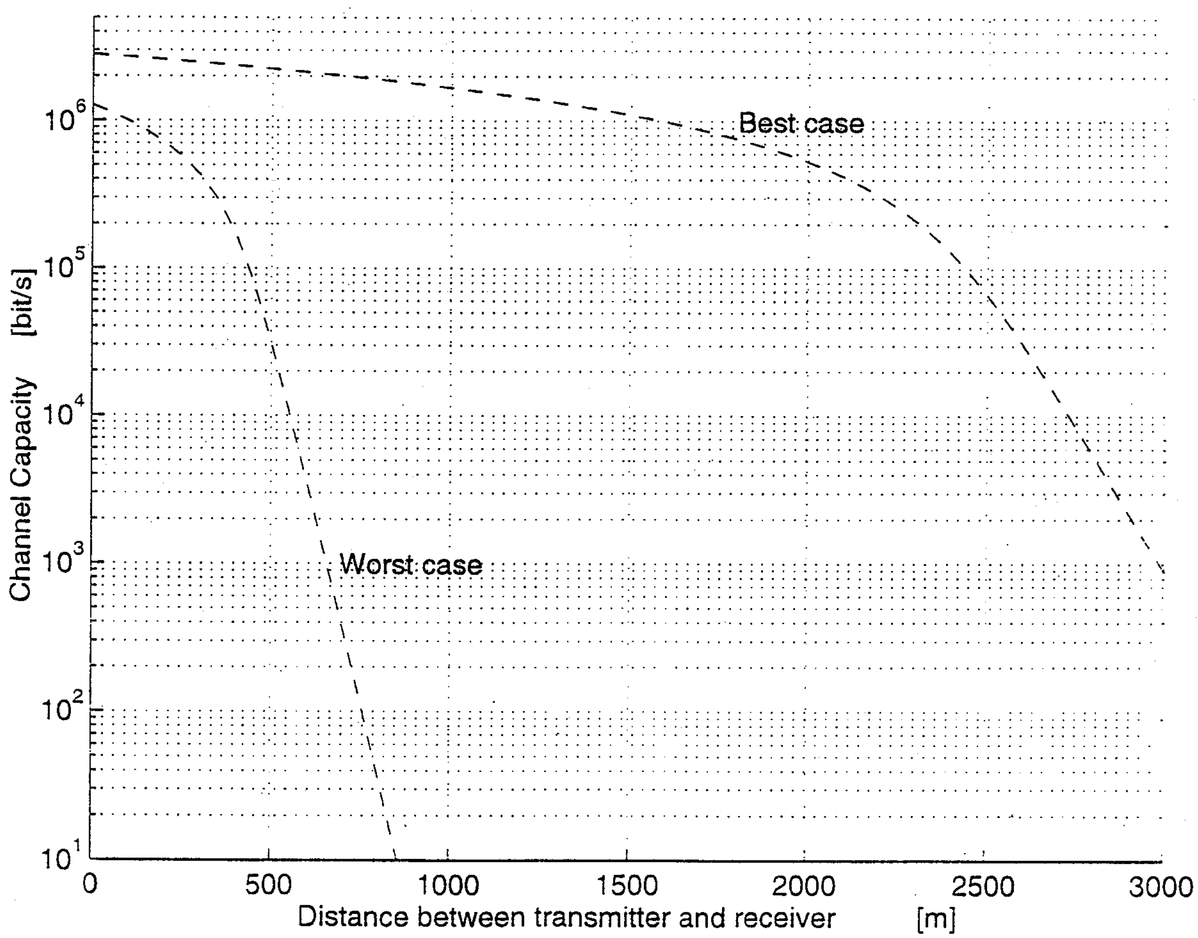

Fig. 4 RPC-channel capacity bounds, based on white noise approximation

V. RPC-Channel Capacity Bounds using the "Water-filling" Approach

Equations (12) and (13) already give some basic insight in the channel capacity of an RPLC-channel. The main problem with equations (12) and (13), however, is the fact that the white-noise approximation which was used to come to these equations led to relatively rough bounds. The actual bounds will lie somewhere between the best- and worst case curves displayed in Figure 4, but where is still unclear.

To come to more accurate bounds a calculation method should be used, in which equations (8) and (9) can be introduced directly. Such a method was found in the so called "water-filling" approach, which will be briefly explained hereafter and then applied to the RPLC-channel model as proposed in section III. For a very elaborate treatment of the "water-filling" approach the reader is referred to [9].

The water-filling approach is applicable for any linear filter channel with additive Gaussian noise (i.e. white or non-white), an example of which is shown in Figure 5.

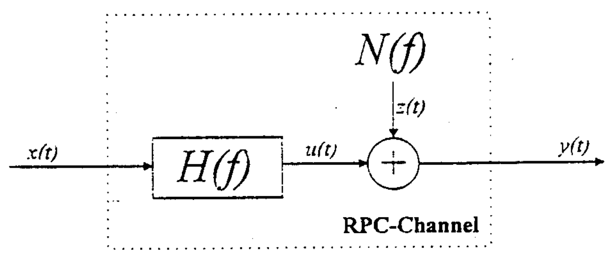

Fig. 5 Linear filter channel with additive noise

For the linear filter channel with additive noise as displayed in Figure 5 it can be shown that the channel capacity is equal to [9]:

$$C = \int_{f \in F_B} 1/2 \log_2 \left[\frac{|H(f)|^2 B}{N(f)} \right] df \,, \qquad (14)$$

where F_B is the range of f for which:

$$\frac{N(f)}{|H(f)|^2} \leq B \,, \qquad (15)$$

B being the solution to:

$$S = \int_{f \in F_B} \left[B - \frac{N(f)}{|H(f)|^2} \right] df \,. \qquad (16)$$

The example shown in Figure 6 clarifies how equations (14) to (16) should be interpreted.

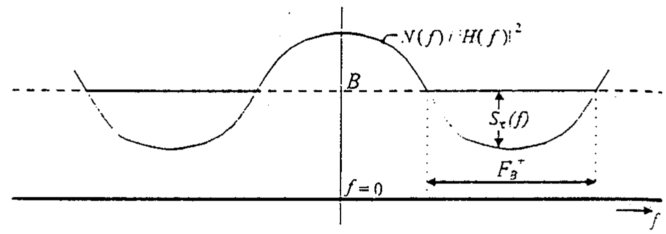

Fig. 6 Interpretation of input power for capacity [GAL68]

Figure 6 clearly shows that the power spectral density for the channel input $x(t)$ that achieves capacity is equal to:

$$S_x(f) = \begin{cases} B - \dfrac{N(f)}{|H(f)|^2} \,; & f \in F_B \\ 0 \,; & f \notin F_B \end{cases} \qquad (17)$$

Figure 6 and equation (17) make clear why the above often is referred to as "the water-filling approach": it is as if into a container with a bottom described by the function $N(f)/|H(f)|^2$ an amount S of "water" is poured in. The resulting water-level in the container is equal to B. The way in which the "water" is distributed over F_B (i.e. the area were the water level is larger than 0) then gives an indicating how the input-power should be distributed over the frequency band F_B in order to achieve capacity.

Since the RPC-channel according to the model presented in section III is a linear filter channel with additive noise, the water-filling approach can be used to come to tighter bounds for the RPC-channel capacity than the ones calculated in the previous chapter.

As a first step we define the channel filter, equal to the CENELEC A-band:

$$H(f) = \begin{cases} 1 & 9000 \leq |f| \leq 95000 \\ 0 & elsewhere. \end{cases} \qquad (18)$$

With respect to the noise power spectral density it is important to notice that $N(f)$ in equations (14) to (17) refers to a double sided noise power spectral distribution, whereas the $N_{re}(f)$ in equations (8) and (9) refer to single sided spectra. Therefore a transition of equations (8) and (9) is necessary in order to be able to apply equations (14) to (17):

$$\ddot{N}_{re,best}(f) = 1/2 * 10^{(-9.64\ -\ 3.95\ 10^{-5}f)} = 10^{(-9.94\ -\ 3.95\ 10^{-5}f)} \quad [W/Hz]\,, \qquad (19)$$

$$\ddot{N}_{re,worst}(f) = 1/2 * 10^{(-7.64\ -\ 3.95\ 10^{-5}f)} = 10^{(-7.94\ -\ 3.95\ 10^{-5}f)} \quad [W/Hz]\,. \qquad (20)$$

When introducing equation (18) and equation (19) or (20) into equation (16), B can be calculated as soon as F_B is known. With respect to this F_B, taking into account the monotonous decreasing character of (19) and (20) in principle the problem of solving equation (16) can be split into two different solution regions:

- Region 1:
  F_B = [-95000,-9000] ∪ [9000, 95000];
  Solution region 1 is valid for small transmitter/receiver distances d. Here the received signal power is still relatively large, resulting in enough power to fill the entire bottom of the container in the water-filling interpretation.

- Region 2:
  F_B = [-95000,-x] ∪ [x, 95000]; $9000<x<95000$;
  Solution region 2 is valid for large(r) transmitter/receiver distances d. Here (due to the signal attenuation over the RPC-line) no longer enough power is left to fill the entire bottom of the container in the water-filling approach. The water (read "power") gathers in the deepest parts of the container.

In Figure 7 the two solution regions have been drawn in a similar way as the example given in Figure 6.

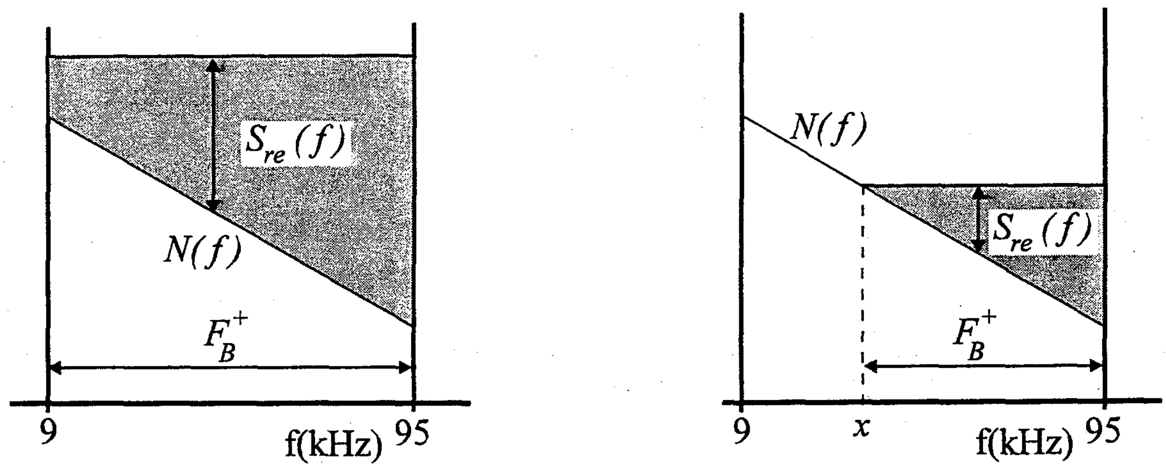

Fig. 7 Solution regions (left complies to region 1) in the water-filling interpretation for the calculation of the RPC-channel capacity

Solution Region 1:

- For a *worst case RPC-channel*, in equation (16) B should be the solution to:

$$S = 25 \cdot 10^{-0.01\,d} = 2 * \int_{9000}^{95000} [B - N(f)]\, df$$

$$= 2Bf\Big|_{9000}^{95000} - 10^{-7.64} \int_{9000}^{95000} 10^{-3.95\,10^{-5} f}\, df$$

$$= 172000B + \frac{10^{-7.64}}{3.95\,10^{-5}\ln(10)} 10^{-3.95\,10^{-5} f}\Big|_{9000}^{95000}$$

$$= 172000B - 1.1\,10^{-4} \tag{21}$$

$$\Rightarrow B = \frac{25 \cdot 10^{-0.01d} + 1.1\,10^{-4}}{172000}$$

$$= 1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10}. \tag{22}$$

As can be seen in Figure 7 on the transition point between solution region 1 and solution region 2, the following is valid:

$$S_x(9000) = B - N(9000) = 0. \tag{23}$$

From (23) it follows that the largest value for d for which solution region 1 is still applicable is equal to:

$$B = 1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10} = N(9000) = 10^{-7.94 - 3.95 \cdot 10^{-5} \cdot 9000}$$

$$\Rightarrow 10^{-0.01d} = 3.05 \cdot 10^{-5}$$

$$\Rightarrow d = 452\ [m].$$

The worst case channel capacity as a function of distance between transmitter and receiver in the area d = [0,452] can then finally be found from equation (14):

$$C = \int_{9000}^{95000} \log_2\left(\frac{1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10}}{10^{-7.94 - 3.95\,10^{-5} f}}\right) df$$

$$= \int_{9000}^{95000} \log_2(1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10})\, df - \int_{9000}^{95000} \log_2\left[2^{(\log_2 10)(-7.94 - 3.95\,10^{-5} f)}\right] df$$

$$= 86000 \log_2(1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10}) + \log_2 10 \cdot \int_{9000}^{95000} (7.94 + 3.95\,10^{-5} f)\, df$$

$$= 86000 \log_2(1.45 \cdot 10^{-4} \cdot 10^{-0.01d} + 6.46 \cdot 10^{-10}) + 2.9\,10^{6}. \tag{24}$$

This has been plotted in Figure 8 (worst case line, for $0 \leq d \leq 452$).

- For the *best case RPC-channel*, calculation of the channel capacity goes in a very similar way. In this case B is equal to:

$$B = 1.45 \cdot 10^{-4} \cdot 10^{-0.004d} + 6.46 \cdot 10^{-12}. \tag{25}$$

The largest value for d for which solution region 1 still applies in this case is equal to 1629 m. From equation (14) the best case RPC-channel capacity for distances d in the area [0,1629] can then be shown to be equal to:

$$C_{best}(d) = 86000 \log_2 (1.45 \cdot 10^{-4} \cdot 10^{-0.004d} + 6.46 \cdot 10^{-12}) + 3.4\ 10^6. \tag{26}$$

This has been plotted in Figure 8 (best case line, for $0 \leq d \leq 1629$).

SOLUTION REGION 2:

As can be seen in Figure 7, in the calculations concerning solution region 2 a new variable x is introduced, indicating the frequency range for which equation (15) is valid, i.e. F_B = [-95000, $-x$] ∪ [x, 95000], $9000 < x < 95000$. Due to this extra variable x, calculating the channel capacity as a function of d analytically, as was done in solution region 1, is no longer possible. However, both the channel capacity C as well as the distance between transmitter and receiver d can be expressed as a function of x. From this, a graph for $C(d)$ can be easily drawn, using x as a mutual parameter.

- For a *worst case RPC-channel*, it is already known from the calculations concerning solution region 1 that solution region 2 is valid for transmitter/receiver-distances $d > 452$ m.

  Since $N(f)$ is a monotonous decreasing function of f, at the lowest positive frequency which is still part of F_B, i.e. $f = x$ the following always holds:

$$S_x(x) = B - N(x) = 0 \qquad 9000 < x < 95000$$
$$\Rightarrow \quad B = N(x) = 10^{-7.94 - 3.95 \cdot 10^{-5} \cdot x}. \tag{27}$$

  Using equation (27) the distance between transmitter and receiver d as a function of x can be found by solving equation (16):

$$S = 25 \cdot 10^{-0.01d} = 2 * \int_x^{95000} [B - N(f)]\, df$$
$$= \int_x^{95000} \left(10^{-7.64} 10^{-3.95\ 10^{-5} x} - 10^{-7.64} 10^{-3.95\ 10^{-5} f} \right) df$$

$$\Rightarrow 10^{-0.01d} = 9.2\ 10^{-10} \int_x^{95000} \left(10^{-3.95\ 10^{-5}x} - 10^{-3.95\ 10^{-5}f}\right) df$$

$$= 9.2\ 10^{-10}\left[10^{-3.95\ 10^{-5}x}(95000 - x) + \frac{10^{-3.95\ 10^{-5}\cdot 95000} - 10^{-3.95\ 10^{-5}x}}{3.95\ 10^{-5}\ln(10)}\right]$$

$$= 9.2\ 10^{-10}\left(-x\cdot 10^{-3.95\ 10^{-5}x} + 8.4\ 10^{4}\ 10^{-3.95\ 10^{-5}x} + 1.94\right)$$

$$\Rightarrow d(x) = -100\log_{10}\left[9.2\ 10^{-10}\left(-x\cdot 10^{-3.95\ 10^{-5}x} + 8.4\ 10^{4}\ 10^{-3.95\ 10^{-5}x} + 1.94\right)\right]. \qquad (28)$$

By combining equations (14), (18) (20) and (27) an expression for the channel capacity C as a function of x can be found:

$$C(x) = \int_x^{95000} \log_2\left(\frac{10^{-7.94 - 3.95\ 10^{-5}x}}{10^{-7.94 - 3.95\ 10^{-5}f}}\right) df$$

$$= \int_x^{95000} \log_2\left(10^{-3.95\ 10^{-5}x}\right) df - \int_x^{95000} \log_2\left(10^{-3.95\ 10^{-5}f}\right) df$$

$$= (95000 - x)\log_2\left(10^{-3.95\ 10^{-5}x}\right) + \log_2 10\cdot\int_x^{95000} 3.95\ 10^{-5} f\ df$$

$$= (95000 - x)\log_2\left(10^{-3.95\ 10^{-5}x}\right) + 6.56\ 10^{-5}\left(95000^2 - x^2\right). \qquad (29)$$

In Figure 8 the combination of equations (28) and (29) using x as a mutual parameter has been plotted (worst case line, for $d \geq 452$).

- For a *best case RPC-channel*, it is already known from the calculations concerning solution region 1 that solution region 2 is valid for transmitter/receiver-distances $d > 1629$ m.

Similar to equation (27) for the best case channel B can be shown to be equal to:

$$B = 10^{-9.94 - 3.95\cdot 10^{-5}\cdot x}. \qquad (30)$$

The distance d as a function of x in this case is equal to:

$$d(x) = -250\cdot\log_{10}\left[9.2\ 10^{-12}\left(-x\cdot 10^{-3.95\ 10^{-5}x} + 8.4\ 10^{4}\ 10^{-3.95\ 10^{-5}x} + 1.94\right)\right] \qquad (31)$$

and the channel capacity C as a function of x is exactly the same as equation (29)

In Figure 8 the combination of equations (29) and (31) using x as a mutual parameter has been plotted (best case line, for $d \geq 1629$).

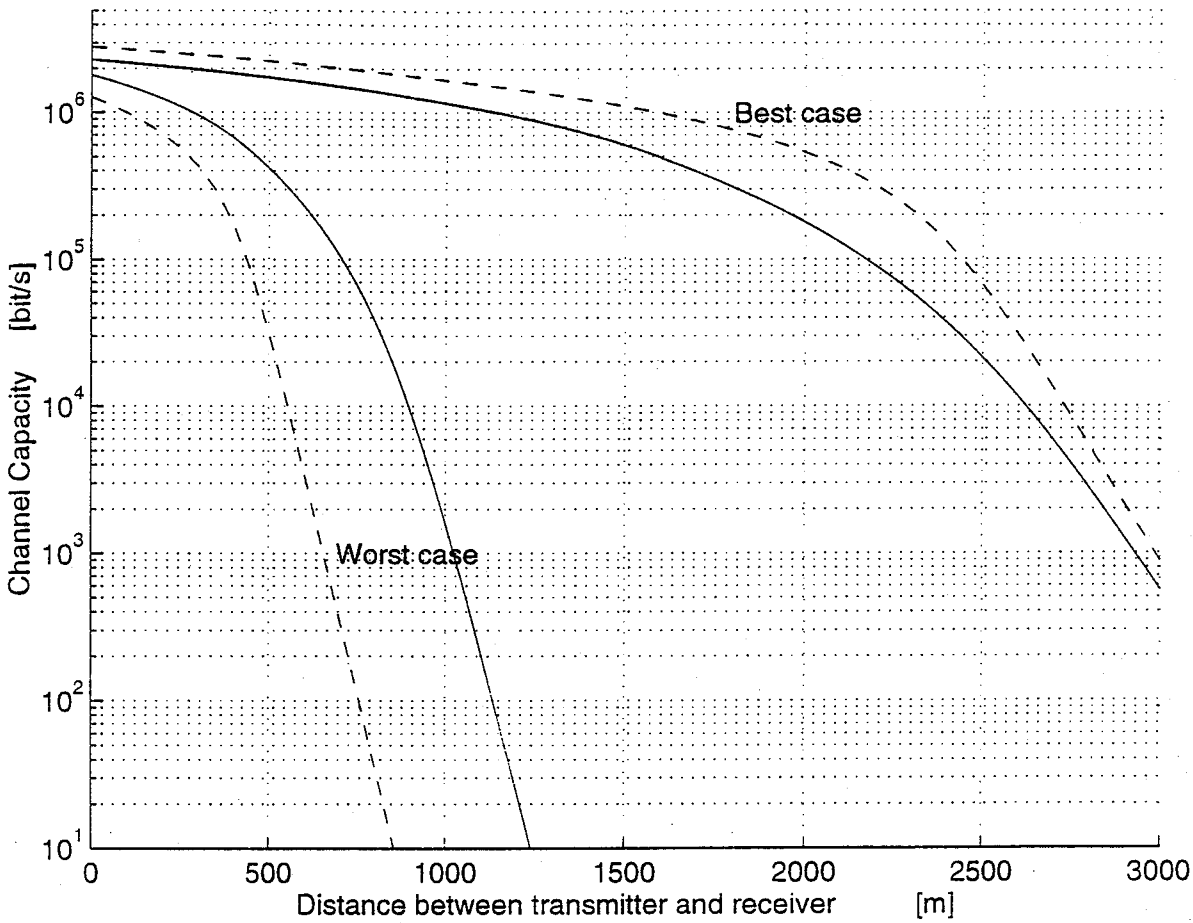

Fig. 8 RPC-channel capacity bounds based on "water-filling" (solid lines) and white noise approximation (dashed lines) [from Figure 4]

VI. Conclusions

As was mentioned in the introduction to this paper, most energy providers in Europe already have a telecommunications network that reaches all the way down to the medium-/low voltage transformers. When using the RPC to bridge the remaining gap between the transformer and the customer, the following two points should be noted:

- In a typical RPC in a city like Amsterdam, each medium-/low voltage transformer supplies power to around 400 households;
- These households are normally located within a distance of 400 to 500 m from the medium-/low voltage transformer.

Both of these points speak in favor of using some kind of master-slave architecture when setting up an RPC-based telephone network, the master (often also referred to as "the concentrator") located close to the transformer and the slave-units (e.g. in the form of telephone-equipment) inside the consumers premises. In this way distances that have to be bridged are usually smaller than 500 m, which according to Figure 8 still ensures a worst case channel capacity of $3{\cdot}10^5$ bit/s.

The main question now is whether a channel with a capacity of at least $3{\cdot}10^5$ bit/s is sufficient to provide a reliable telephone connection for 400 households. In principle with modern vocoders speech can be transmitted at near-toll-quality at rates of 4.8 kbps. This means that theoretically it is possible to provide $n = 62$ speech channels per medium-/low voltage transformer, which should be shared by 400 households. When allowing the probability that all channels are occupied, (i.e. that an arriving call will be blokked) to be equal to $B(n,a) = 0.5\%$ the Erlang B formula

$$B(n, a) = \frac{a^n/n!}{\sum_{k=0}^{n} a^k/k!}, \tag{32}$$

shows that a traffic intensity a of 46.5 Erlang can be supported over distances up to 500 m. Since the traffic intensity $a = \lambda/\mu$ (i.e. the product of the average number of calls per second offered to the group of channels λ times the mean service time per call in seconds $1/\mu$), it can be shown that with an average call holding time of 3 minutes 930 calls coming in per hour can be supported. This must be considered to be enough for normal purposes.

The discussion above shows that setting up an RPC-based telephone system theoretically is possible. However, this does not mean that such a system can also be realized in a practice:

- The fact that the phase-shift introduced to a signal transmitted over the RPC in a practical system is not exactly constant in time (it was assumed to be in the calculations) causes a decrease in the actual channel capacity;
- Telephone is a real-time application; only small delays are acceptable. This puts a constraint on the length of code-words that can be used and therefore decreases the practical channel capacity.

It is therefore clear that still a lot of investigation is necessary before operational RPC-based fixed line telephone networks may become a common sight.

References

[1] EN 50 065-1, "Signalling on low voltage electrical installations in the frequency range 3 kHz to 148.5 kHz," *CENELEC*, Brussels, 1991.

[2] J.O. Onunga and R.W. Donaldson, "Personal Computer Communications on Intrabuilding Power Line LAN's Using CSMA with Priority Acknowledgments," IEEE Journal on Selected Areas in Communications, vol. 7, no. 2, pp. 180-191, February 1989.

[3] H. Ruse, *Spread-Spectrum-Technik zur störsicheren digitalen Datenübertragung über Stromversorgungsnetze*. VDI Fortschritt-Berichte, Reihe 10, Nr. 185, Düsseldorf, 1991 (in German).

[4] G. Threin, *Datenübertragung über Niederspannungsnetze mit Bandspreizverfahren*. VDI Fortschritt-Berichte, Reihe 10, Nr. 156, Düsseldorf, 1991 (in German).

[5] O.G. Hooijen, "A Channel Model for the Low-Voltage Power-Line Channel; Measurement and Simulation Results," *Proceedings of the 1997 International Symposium on Power Line Communications and its Applications (ISPLCA '97)*, Essen, Germany, April 1997, pp. 51-56.

[6] R.M. Vines et al., "Noise on residential power distribution circuits," *IEEE Transactions on Electromagnetic Compatability*, vol. 26, no. 4, pp. 161-168, November 1984.

[7] M.H.L. Chan and R.W. Donaldson, "Amplitude, Width and Interarrival Distribution for Noise Impulses on Intrabuilding Power Line Communication Networks," *IEEE Transactions on Electromagnetic Compatibility*, vol. 31, no. 3, pp. 320-323, August 1989.

[8] C.E. Shannon, "A Mathematical Theory of Communication," *Bell Syst. Tech. Journal*, vol. 27, pp. 623-657, October 1948.

[9] R.C. Gallager, *Information Theory and Reliable Communication*. Wiley and Sons, NY, 1968.