

ON THE MULTIPLE ACCESS CHANNEL.

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Abstract. We consider the two-user Multiple Access Channel (2-AC). For the only two non-trivial binary input 2-AC's we discuss the capacity regions. Then, we give some examples of M-ary input 2-AC's. For two specific classes we develop an encoding strategy such that both users can operate at the total cooperation rate.

Introduction.

We consider the communication situation where 2 senders transmit simultaneously to a single receiver via a discrete memoryless deterministic multiple-access channel. Both senders are synchronized and have complete knowledge of the channel output Y via a noiseless feedback link. The channels accept M-ary input signals X_k , $k=1,2$, and emit a single output Y in accordance with a prescribed conditional probability distribution $P(y|x_1, x_2)$, see Fig.1, where $M=2$.

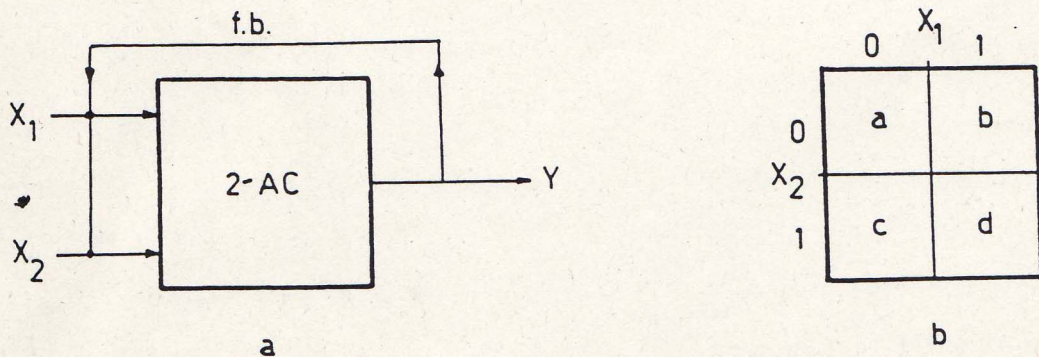


Fig.1. Two-Access Channel (2-AC) with input/output representation.

Cover and Leung [1] found an achievable rate region for the discrete memoryless MAC with feedback, using superposition coding. Subsequently, Willems [2], showed that for MAC's for which at least one input is a function of the output Y and the other input (MAC's in class D), the region found by Cover and Leung is the feedback capacity region. The feedback capacity of MAC's in class D is given by

$$R_{CL}^d \triangleq \{(R_1, R_2) : 0 \leq R_1 \leq H(X_1|U)$$

$$0 \leq R_2 \leq I(X_2; Y|X_1, U) = H(X_2|U)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y) = H(Y) \text{ for}$$

$$P(u, x_1, x_2, y) = P(u)P(x_1|u)P(x_2|u)P(y|x_1, x_2)$$

$$\text{and } |U| \leq \min \{|X_1| \cdot |X_2| + 1, |Y| + 2\},$$

where $|U|$, $|X_1|$, and $|X_2|$ are the alphabet cardinalities of the random variables U, X_1 and X_2 , respectively.

Binary input 2-AC

The different types of binary input 2-AC's follow from specification of the set $\{a, b, c, d\}$. The number of different elements in the set is denoted by the cardinality $|Y|$. For instance, for the set $\{a, a, b, c\}$ the cardinality $|Y| = 3$. It is easy to verify that this is the only interesting case with binary inputs. Furthermore, one can show that all possible ternary output channels can be converted into one of the two channels of Fig.2, where we use the symbols 0, 1 and 2 instead of the letters a, b and c.

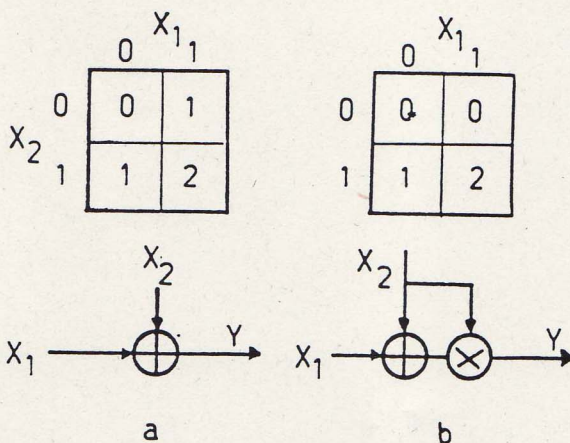


Fig. 2. Two binary input/ternary output 2-AC's.

Note that channel 2a is the well known binary adder channel. In channel 2b, user X_2 -switches the connection between X_1 and Y on and off. By selecting specific switch moments, X_2 transmits information. The capacity region for channel 2b. is given below.

$$R_{2b.} = \{(R_1, R_2); R_1 \leq \sum_i P_i \alpha_i h(\beta_i) \leq \bar{\alpha}$$

$$R_2 \leq \sum_i P_i h(\alpha_i) \leq h(\bar{\alpha})$$

$$R_1 + R_2 \leq \bar{\alpha} + h(\bar{\alpha})\},$$

where $|U| = 5$, $h(\cdot)$ is the binary entropy function and $\alpha_i = P(X_2=1|U=i)$, $\beta_i = P(X_1=0|U=i)$, $P_i = P(U=i)$. The capacity region is given in Fig.3,

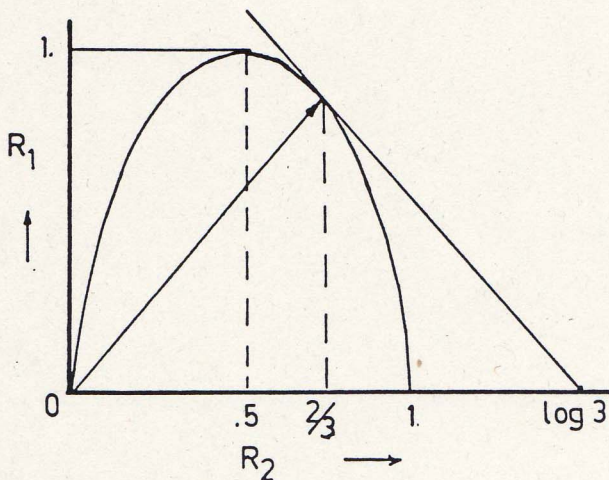


Fig. 3. Capacity region for channel 2b.

For $|U| = 1$, we achieve the same region. Hence, one does not need the feedback information here.

The capacity region for channel 2a is not completely known in numerical form. From the non-feedback region the rate pairs $(R_1, R_2) = (1, 0.5)$ and $(0.5, 1)$ are known to be achievable. Furthermore Willems [3] proved optimality of the point $R_1 = R_2 = 0.7911$ with $|U| = 2$. Thus far no other asymmetric optimal rate pairs are known. From computations we suspect that $|U| = 2$ is sufficient to generate the whole capacity region.

An encoding strategy.

We now consider M -ary input 2-AC's. In [4], we define two specific channel models. The output of the first channel indicates which subset of input symbols occurred, i.e.

$$Y = \{X_1, X_2\}, X_i \in \{0, 1, \dots, M-1\}.$$

The second channel gives as an output the algebraic sum of the input letters, i.e.

$$Y = X_1 + X_2, X_i \in \{0, 1, \dots, M-1\}.$$

In Fig. 4, we give the input/output relations for both channels when $M=3$.

$X_1 \backslash X_2$	0	1	2
0	{0}	{0,1}	{0,2}
1	{0,1}	{1}	{1,2}
2	{0,2}	{1,2}	{2}

$Y = \{X_1, X_2\}$

$X_1 \backslash X_2$	0	1	2
0	0	1	2
1	1	2	3
2	2	3	4

$Y = X_1 + X_2$

Fig. 4. Input/output relation for the ternary erasure MAC and the ternary adder MAC.

The output cardinalities $|Y| = \frac{M(M+1)}{2}$ and $(2M-1)$, respectively. Note that for $M=2$ both channels reduce to the well known binary erasure MAC.

For both channels, we proved [4], that in symmetrical operation the Cover-Leung capacity equals total cooperation. In particular, we give probability distributions $p(u, x_1, x_2, y)$ such that $H(X_k | Y) > \frac{1}{2}H(Y)$, $k=1,2$, and $H(Y) = \log |Y|$.

Gaarder and Wolf [5] showed that the capacity region for the erasure MAC is enlarged by using feedback. They developed a two stage coding strategy. During the first stage the channel accepts N independent input digits from both users. Ambiguous receptions are known to both users, due to the presence of the feedback links. After a block of N transmission both encoders transmit at the cooperative channel capacity of $\log |Y| = \log \frac{M(M+1)}{2}$ bits/transmission, in order to resolve the receiver's uncertainty.

We use the concept of superposition coding. We describe an encoding strategy such that both users transmit at the total cooperation rate. The

key role in the encoding is played by the random variable U .

Let both users represent a selected message θ_k , $k=1,2$ as location of a point on the unit interval $(0,1)$, see also Schalkwijk [6]. The interval $(0,1)$ is divided into M subintervals $0 \leq i \leq M-1$. The length of a specific subinterval equals the probability $P(X_k=i|U=j)$, for a specific value of U . If θ_k lies in interval l , then encoder k gives l as output. Both users are supposed to have the same u for each transmission. After a transmission each encoder normalizes its specific subinterval to length 1. The average amount of information, R_k , provided by the outcome X_k , about the location of θ_k is equal to $H(X_k|U)$. Suppose that each transmission both users select together the same value of U with $P(U=j)=1/M$. Then, the probabilities $P(x_k|u)$ specify a channel $P(y|u)$. This channel is used to transmit common information from both users to the receiver Y . Now, encoding of the messages θ_1 and θ_2 is done in blocks of length N , with a two stage superposition strategy.

In the initial fraction $(1-\alpha)$ of the first block of N transmissions both users use $U=0$ for their respective message encoding. The total average amount of uncertainty due to ambiguous receptions is equal to $(1-\alpha)NH(X_1X_2|Y,U=0) = (1-\alpha)NH(X_k|Y,U=0)$, $k=1$ or 2 . This uncertainty is known to both users and is considered to be common information. In the last fraction α , the users may transmit part of the common information in total cooperation, say R_t bits per transmission, by selecting one of the possible outputs of Y . The remaining uncertainty is transmitted in the initial fraction $(1-\alpha)$ of the next block via the imaginary channel from U to Y . This is possible with a vanishing small error probability for large N if

$$\begin{aligned} (1-\alpha)NH(X_k|YU) &\leq (1-\alpha)NI(U;Y) + \alpha NR_t \\ &= (1-\alpha)N(H(Y) - H(Y|U)) + \alpha NR_t \\ &= (1-\alpha)N(R_t - H(X_1|U) - H(X_2|U) + H(X_k|YU)) + \alpha NR_t \\ &= (NR_t - 2(1-\alpha)NH(X_k|U) + (1-\alpha)NH(X_k|YU)) \end{aligned}$$

For the M -ary erasure and adder MAC, we give input distributions [4], such that

$$\begin{aligned} H(X_k|U) &> \frac{1}{2}H(Y), \\ &= \frac{1}{2}R_t. \end{aligned}$$

Hence, the above condition is satisfied if α is chosen such that

$$(1-\alpha)H(X_k|U) = \frac{1}{2}R_t.$$

From the above we conclude that both families of channels can be used at the total cooperation rate in symmetrical operation.

Conclusions

We discuss the capacity region for two non trivial binary input two-access channels. Then we describe an encoding strategy for M-ary input Multiple-Access Channels such that both transmitters can operate at the total cooperation rate.

References

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