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Coded Modulation with a Constraint on the Minimum Channel Symbol Duration

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***Abstract*—A coded modulation scheme is described which improves the error performance of constant envelope data transmission without requiring more bandwidth. This is achieved by controlling the minimum channel symbol duration by run-length limited (RLL) encoding. It is shown that the same amount of information can be transmitted within the same minimum channel symbol duration with a coding gain of 4 dB compared with the uncoded modulation. The bandwidth is analyzed using the power spectra of channel signals. As a practical application, we consider bandwidth limited transmission with a restriction on the maximum transmitted power. Following a discussion of a coded modulation principle, constructed RLL codes and the evaluation of their code performances are provided, and the simulation results for BPSK and 2-FSK modulations are presented. Comparisons are strictly made on the basis of equal information rate and the same minimum channel symbol duration.**

***Index Terms*- coded modulation, constant envelope signaling, RLL codes, the power spectra of RLL codes, Reed-Solomon code.**

I. Introduction

Coding for bandwidth limited channels has been a very dynamic research area, since multilevel coding and trellis-coded modulation were introduced [1], [2], [3]. The principle of coded modulation is the interpretation of coding and modulation as a single entity in order to save transmitter power on a data-bit basis without changing the minimum channel symbol duration. The considerable gains in terms of SNR can be achieved with respect to the uncoded situation, sacrificing neither the information rate nor the bandwidth as it was shown by Ungerboeck [3]. The scheme is based on the set partitioning principles. The key point is finding subsets of the constellation in such a way that the signal points inside each partition are maximally separated. With $M = 2^m$ different signal points one can transmit m information bits per channel symbol without coding. In trellis coded modulation $(m-1)$ information bits are transmitted and a $(m-1)/m$ trellis code is used to generate sequences with high Euclidean distances. The uncoded system with constellation size of $2^{(m-1)}$ is used as a reference system transmitting at the same information rate. Note that the minimum channel symbol duration in the uncoded system is kept the same as for the coded system.

In this correspondence, we present a coded modulation scheme based on controlling the minimum channel symbol duration in a set of block waveforms defined in a constant time. Since the transmission bandwidth is related to the minimum channel symbol duration, coding gains can be achieved without requiring more bandwidth. The practical applications of the proposed method can be bandwidth limited communication systems with constant envelope PSK and FSK modulations. The application for the narrowband powerline communication (PLC) with the FSK modulation can be found in [5].

The paper is organized as follows. In Section II, the coded modulation principle based on run-length limited (RLL) coding is presented. This is followed by the construction of efficient RLL codes and the evaluation of their code performances. In Section V, we discuss the bandwidth efficiency of the proposed coded modulation scheme showing the power spectra of channel signals. Section VI addresses the simulated BER performance of coded and uncoded transmission of a concatenated coding scheme for the additive white Gaussian noise (AWGN) channel.

II. Coded Modulation principle

Without loss of generality, we consider an uncoded system with BPSK modulation, where the information has a minimum bit duration of τ seconds. We transmit k information bits in time $T = k\tau$ with a total energy kE_b. This corresponds to the set of $2^{T/\tau}$ block waveforms. If channel coding is used with an efficiency $R_c = k/n$, we transmit n code symbols with a minimum code symbol duration of $\widehat{\tau} = \tau\frac{k}{n}$ seconds in the same time T. For this speed we need to expand the bandwidth with a factor of $\frac{n}{k}$. The energy per symbol is $E_s = k/nE_b$. To avoid the bandwidth expansion, a binary RLL encoder with $R_r = n/m$ is applied after the channel encoder. The set of all possible sequences which the RLL encoder can generate satisfies the $(d+1)$ constraint. Here, $(d+1)$ indicates the minimum run of the same symbols, and it can be used to control the spectra of RLL sequences. For more information about RLL codes, we refer to [4]. The time interval T is partitioned into RLL symbols with duration τ' seconds. This corresponds to the set of $2^{TR_cR_r/\tau'}$ block waveforms for k information bits, which is illustrated in Fig. 1. Since the comparisons are made on the basis of the same minimum channel symbol duration, we choose $\tau' = \frac{\tau}{(d+1)}$ seconds. Thus, a number of $TR_cR_r(d+1)/\tau$ input symbols can be mapped onto the set of $2^{TR_cR_r(d+1)/\tau}$ block waveforms. For $R_cR_r(d+1) = 1$, the same amount of information with channel coding can be transmitted within the same minimum

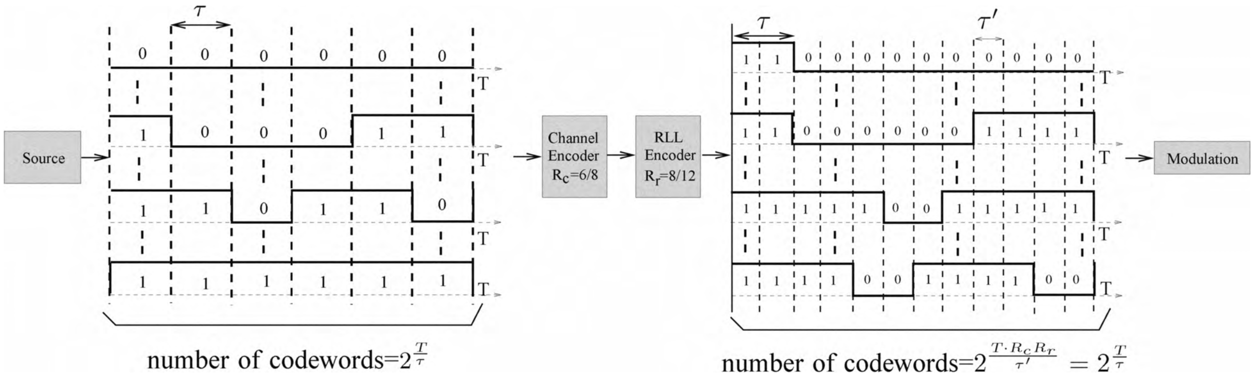

Fig. 1. Binary waveforms of a set of RLL-RS coded sequences, $(d+1)=2$.

channel symbol duration $\tau' = \frac{\tau}{(d+1)}$ and within the same bandwidth (since the bandwidth is related to the minimum channel symbol duration) compared with uncoded modulation. It should be noted that the sampling clock in the proposed coded modulation scheme runs $(d+1)$ times faster than the clock in the uncoded modulation. The energy per RLL symbol is reduced to $E_s = E_b/(d+1)$, and the total energy in time T is equal to kE_b. In general, in M-ary PSK, the efficiencies of the channel- and RLL encoder are chosen as

$$R_c R_r (d+1) = \log_2 M. \quad (1)$$

In Section V, we analyze the bandwidth using the power spectra of channel signals.

The maximum value of R_r that can be attained for values of $(d+1)$ was provided by Shannon [7] as the asymptotic information rate of RLL sequences. The asymptotic information rate, denoted by C_d, is defined as the number of information bits per channel bit that can maximally be carried by the constrained sequence. For the magnetic recording systems, Wolf [8] discussed how the factor $(d+1)C_d$ influences the recording density. He stated that, in the absence of noise, for increasing $(d+1)$ a larger amount of information can be stored at a minimum channel symbol duration, see Table I. We extend the idea reported in [8] to reduce the bit error rate for communication systems with constant envelope modulation. The evaluation of the RLL code performance depending on d over the AWGN channel with intersymbol interference-free signaling is provided in Section IV.

The basic transmitter consists of the serial concatenation of a Reed-Solomon (RS) code as the outer code with an RLL code as the inner code followed by a signal modulation. The RLL code is a short block code with a minimum distance $d_{\min}$. An advantage of this scheme is that we use block-wise RLL decoding without error propagation. A symbol oriented error correcting code is used to correct the remaining RLL block errors. We should note that a Bliss scheme [4] can also be used by changing the position of the RLL and the RS codes. This leads to a construction of long RLL encoded data streams with a high efficiency. The disadvantage is that the RS decoder has to perform hard decision decoding on symbols with a reduced symbol energy. However, it is shown in Section III that the constructed RLL code efficiencies are very close to the related capacities for $d_{\min} = 1$.

TABLE I
C_d AND $(d+1)C_d$ AS A FUNCTION OF d

	d=0	d=1	d=2	d=3	d=4	d=5
C_d	1	0.6942	0.5515	0.4650	0.4057	0.3620
$(d+1)C_d$	1	1.388	1.654	1.860	2.028	2.172

TABLE II
CODE TABLE FOR THE RLL$(d+1=2)$, $R_r = 3/5$

Source	Codeword	Source	Codeword
0	(00011)	4	00001 or 00110
1	(00111)	5	11110 or 11001
2	(11000)	6	01111 or 10011
3	(11100)	7	10000 or 01100

III. CONSTRUCTION OF EFFICIENT RLL CODES

The problem to be solved is to construct good RLL codes with error correction capabilities for $R_r(d+1) > \log_2 M$. In [6], Hollmann proposed a finite look-ahead code construction approach for designing efficient RLL codes. In his work, he described an efficiency $8/12$, $(d+1=2)$ code with a $d_{\min}$ equal to one. In this paper, we extend the methods reported in [6] to construct binary and M-ary, $d+1 = 2,3,4$ codes which have $d_{\min}$ larger than one. These codes have simple encoder tables and a fixed length block-wise decoder. For example, Table II lists the encoding procedure for eight source words of 3 bits. The first four source words are uniquely represented by a single codeword. The remaining source words are represented by two codewords. The choice depends on the last sent channel bit or the upcoming source word. It can easily be verified that these codewords can be concatenated without violating the run length constraint, that is, solitary "ones" or "zeros" will never occur. In other words, the ability of looking ahead and looking back enables the encoder to decide on one alternative codeword which maintains the $(d+1)=2$ condition in concatenation.

RLL codes with $d_{\min}$ properties can be constructed by extending the construction rule in [6] as follows. Given the number of input bits, n, and the constraint $(d+1)$, choose the

TABLE III
CONSTRUCTED RLL CODES FOR $R_c \cdot R_r \cdot (d+1) = \log_2 M$

M	d	R_r	$d_{\min}$	$(d+1)R_r$	R_c
2	1	8/12=0.66	1	1.32	0.7576 (191/255)
2	1	8/14=0.57	2	1.14	0.8772 (223/255)
2	1	9/18=0.5	3	1	-
2	2	8/15=0.53	1	1.59	0.6289 (159/255)
2	2	8/17=0.47	2	1.41	0.7092 (181/255)
2	2	8/21=0.38	3	1.14	0.8772 (223/255)
2	2	6/18=0.33	4	1	-
2	3	8/18=0.444	1	1.77	0.5649 (145/255)
2	3	8/21=0.38	2	1.52	0.6578 (167/255)
2	3	6/18=0.33	3	1.33	0.7518 (47/63)
2	3	6/20=0.3	4	1.2	0.8333 (53/63)
2	3	6/24=0.25	5	1	-
3	1	10/11=0.90	1	1.80	0.87
3	1	10/12=0.83	2	1.66	0.95
4	1	8/7=1.142	1	2.28	0.87
4	1	12/11=1.09	2	2.18	0.91

number of output symbols, m, such that there exists $C \subseteq C_p$ and $|C| \geq 2^n$, and $d_{\min}(C) \geq 2$, where C_p is a set of potential look-ahead codewords. The Gilbert-Vashamov type of construction method is applied for the exhaustive search for good codes with desirable $d_{\min}$ and $(d+1)$. The results are presented in Table III, see also Table I for the code capacities. Note that the minimum distance allows us to apply soft decoding at the level of RLL decoding.

The code, shown in Table IV, exemplifies that the RLL encoder can be efficiently realized. The eight source words are labeled by 0,...,7. Four source words are uniquely represented by a single codeword. The remaining source words are represented by two codewords depending on the last RLL channel bit sent, c_8 and the most significant bit (MSB) of the upcoming source word, b_1. It should be noted that as the parameter d of the RLL code increases, the sampling speed also increases which leads to more complex hardware implementations.

IV. RLL CODE PERFORMANCE DEPENDING ON d

The RLL codeword error probability, P_{RLL}, of the system depends mainly on the $d_{\min}$ and the $(d+1)$ parameters of the RLL codes. We can compute an upper bound of the RLL codeword error probability using a union bound, which sums the contributions of the pairwise error probabilities (PEP) over all error events. The PEP is the probability that, in the decision between RLL codewords $c \in C$ and $c' \in C$, c' is erroneously decoded given that c was transmitted. We assume that every RLL codeword is transmitted equally likely with the probability $Pr(c) = 1/|C|$. BPSK modulation is considered, and intersymbol interference-free signaling with AWGN with zero mean and power spectral density $N_0/2$ is assumed. A matched filter detector is utilized. Maximum-likelihood (ML) minimum Euclidean distance decoding of the RLL codewords is applied to avoid loss of information. Using the fact that the codewords with minimum distance have the highest error

TABLE IV
CODE TABLE FOR THE RLL($d+1$=3), $d_{\min} = 2$ AND $R_r = 3/8$

Input	$b_1 = 0, c_8 = 0$	$b_1 = 1, c_8 = 0$	$b_1 = 0, c_8 = 1$	$b_1 = 1, c_8 = 1$
0	00000111	00000111	00000111	00000111
1	00011111	00011111	00011111	00011111
2	00111000	00111000	11000111	11000111
3	00011100	11100011	00011100	11100011
4	11111000	11111000	11111000	11111000
5	11100000	11100000	11100000	11100000
6	01111111	01111111	10000000	10000000
7	01110000	01110000	10001111	10001111

probability, we get the PEP of the RLL coding

$$P(c \to c') \leq Q(\sqrt{\frac{d_{\min} 2E_s}{N_0}}), \quad (2)$$

where E_s is the RLL bit energy in a duration τ'. Since the system is designed such that $R_c R_r(d+1) = 1$, the information bit energy E_b in a symbol duration τ can be calculated as $E_s = E_b R_c R_r = E_b/(d+1)$. Thus, (2) can be rewritten as

$$P(c \to c') \leq Q(\sqrt{\frac{d_{\min}}{(d+1)} \frac{2E_b}{N_0}}). \quad (3)$$

Note that compared with uncoded BPSK modulation, the same amount of information is transmitted within the same block transmission time $T = k\tau$. Since the total energy in a block time T does not change, there is no efficiency loss compared with uncoded BPSK modulation in (3). Another observation can be as follows. The PEP of the RLL coding is dependent on the minimum distance $d_{\min}$ and the $(d+1)$ parameter of the code. Large values of d could give larger improvements in $(d+1)C_d$, see Table I. However, codes with larger values of d can be constructed at the expense of a lower efficiency and an increased complexity due to the increased RLL code length in the code design. The loss due to the $(d+1)$ can be compensated by using higher minimum distance $d_{\min} > 1$. However, construction of RLL codes with desired $d_{\min}$ needs an exhaustive search among constructed candidate RLL codes (see Table III) which also leads to a lower efficiency and an increased complexity. Using a union bound, the RLL codeword error probability can be upper bounded as

$$P_{\text{RLL}} \leq |C| Q(\sqrt{\frac{d_{\min}}{(d+1)} \frac{2E_b}{N_0}}). \quad (4)$$

Simulation results will be discussed in Section VI.

V. SPECTRAL ANALYSIS OF TRANSMITTED SEQUENCES

The power spectra of transmitted sequences offer a measure of the bandwidth occupation. Since the computation of the spectra of RLL sequences generated by the constructed codes is a difficult task, we first study the maxentropic (with maximum information content) RLL sequences. The calculation of the power spectrum has been derived in [9] for the transmitted sequence of binary symbols whose run-lengths $L_j \in (d+1, \ldots)$

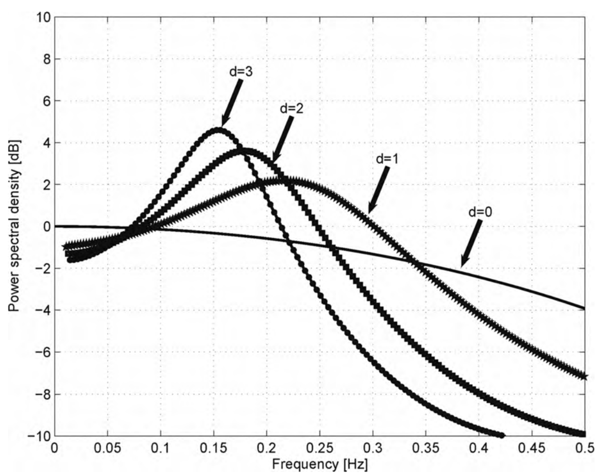

Fig. 2. Spectral density of maxentropic RLL codes (same information rate).

are independently emitted with probability $Pr(L_j)$. Using the run-length probability distribution in [4] for the maxentropic RLL sequences, the power spectra of baseband RLL sequences for various values of $(d+1)$ can be obtained. If $(d+1)$ code bits are transmitted in τ seconds, then the maximum information content for the RLL codes is expressed as $(d+1)C_d$ bits/τ.

In Fig. 2, we normalize the frequency scale such that the information rate stays constant, in which case different codes with different d parameters have different minimum channel symbol duration. The power spectra of the baseband BPSK modulation is represented as $d=0$. As d increases, the frequency occupation of the main lobe and the spectral density at zero frequency decreases. Furthermore, the peak in the spectrum increases and shifts in the direction of lower frequencies. However, it has been mentioned by Anderson [10] that in some narrowband coding schemes, side lobes should also be considered to measure the bandwidth usage.

Fig. 3 compares the simulated power spectra of RLL coded BPSK modulation with the uncoded BPSK modulation for the same minimum channel symbol duration. We conclude that the frequency occupation stays almost unchanged for the same minimum channel symbol duration. Note that with RLL coding, channel coding with an efficiency $R_c = 0.8772$ can be used in the same bandwidth, see Table III.

VI. RS-RLL Concatenated Coding Scheme

In the preceding sections, we have presented how the same amount of information can be transmitted within the same bandwidth with channel coding. In this work, we have studied the concatenation of a RS code with the RLL code. In this case, the number of bits per RS symbol is chosen as the length of the input block of the RLL encoding. All of the simulation results are obtained for the AWGN channel with parameters $M = 2$ and $R_c \cdot R_r \cdot (d+1) = 1$. The code efficiencies are chosen according to Table III.

Fig. 4 depicts the RLL codeword error probability as a function of E_b/N_0 for varying d and $d_{\min}$. E_b is the energy per information bit (energy in the minimum channel symbol duration τ). The cardinality of the codes we used for the simulation is given in Fig. 4. As we discussed in Section IV, the RLL codeword error probability, depends mainly on the minimum distance, $d_{\min}$, and the $(d+1)$ parameter of the RLL codes. For $(d+1) = d_{\min}$, the same error performance can be observed.

Fig. 5 depicts the bit error rate (BER) as a function of E_b/N_0. It is apparent that the transmission with the concatenation of RS and RLL coding has a significant BER advantage over the uncoded BPSK transmission within the same bandwidth. The improvement achieved by the proposed system is 4 dB at BER of 10^{-5}. Large coding gains are obtained by using a RLL code as an inner ECC. The RS(255,223)-RLL($d+1 = 3$) with $d_{\min} = 3$ has the best performance due to its optimized RLL codetable.

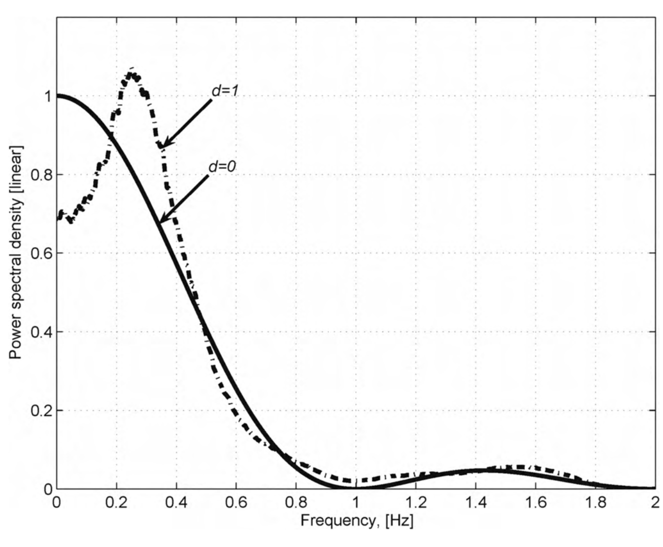

Fig. 3. Simulated spectral density of RLL coded ($(d+1) = 2$ and $d_{\min} = 2$) and uncoded BPSK modulation with the same minimum channel symbol duration.

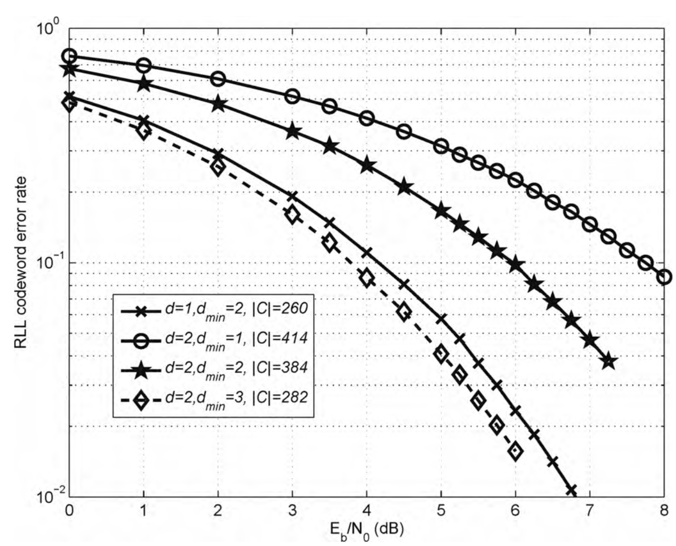

Fig. 4. RS-RLL coded BPSK transmission with varying d and $d_{\min}$ (the same minimum channel symbol duration).

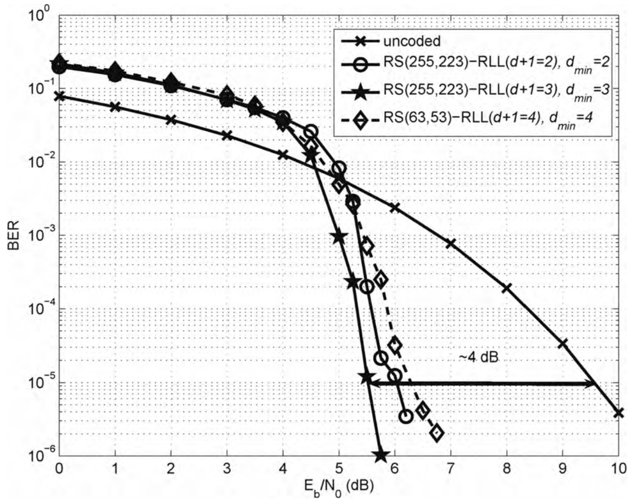

Fig. 5. RS-RLL coded BPSK transmission with varying d and $d_{\min}$ (the same minimum channel symbol duration).

Fig. 6 illustrates the BER performance for varying $d_{\min}$ parameters. The curve with $d_{\min} = (d+1)$ has the best BER performance. The RLL codewords are decoded using soft decision. The uncorrectable errors can be decoded by the RS decoder using hard decision.

In Fig. 7, we plotted the simulated BER performance of coherent 2-FSK transmission with the same minimum channel symbol duration. Since the FSK transmission has a constant envelope modulation, the RS-RLL coded FSK modulation scheme can be suitable for communication systems where there is a limitation on the maximum transmitted power and bandwidth.

VII. Conclusion

A coded modulation technique was described which improves the error performance of constant envelope data transmission without requiring more bandwidth. The same amount of information can be transmitted within the same minimum channel symbol duration with a coding gain of 4 dB compared with the uncoded BPSK and 2-FSK modulation. RLL coding is used to keep the minimum channel symbol duration unchanged. Since the bandwidth is related to the minimum channel symbol duration, we showed using the power spectra of channel signals that the bandwidth stays almost the same compared with the uncoded modulation.

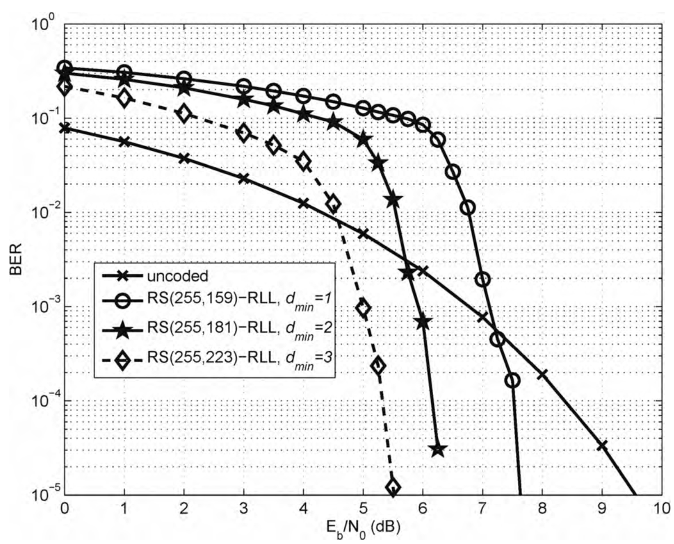

Fig. 6. RS-RLL coded BPSK transmission with varying $d_{\min}$ for $(d+1) = 3$ (the same minimum channel symbol duration).

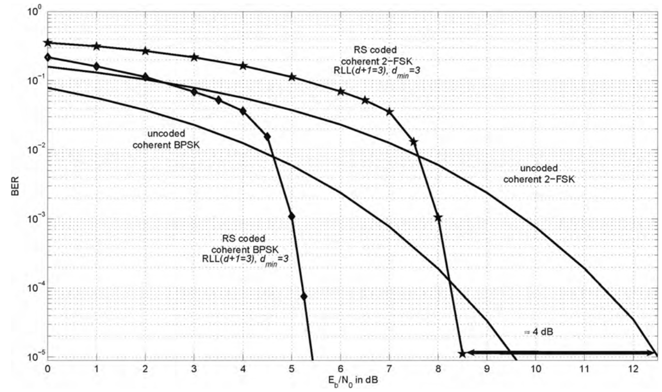

Fig. 7. RS-RLL coded coherent 2-FSK and BPSK transmission (the same minimum channel symbol duration).

In this paper, we designed several block-wise run-length limited look-ahead codes with error correcting capabilities. These codes have simple encoder tables, and they can be decoded by a maximum-likelihood decoder on a block basis. A serial concatenation of the RS code as the outer code with a RLL code as the inner code is considered. Furthermore, the scheme can be generalized to multilevel transmission.

The synchronization problem has not been studied in this paper. As the parameter d of the RLL code increases, the sampling speed also increases which can be a limiting factor as in magnetic recording.

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