

# Low Complexity Equalization with an Efficient Coding Scheme for Broadband Mobile Radio Systems

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## Abstract

In this paper, different low complexity equalization schemes applied to mobile radio channels are investigated and compared. Using Monte Carlo simulations for the bit error rates the performance losses of the different low complexity equalization schemes in comparison to ideal equalization represented by the matched filter bound are determined. It is shown, that considerable complexity reduction is possible with a performance loss of less than 1 dB under the assumption that the appropriate receiver structure is chosen. Moreover, a single-parity-check encoding scheme with low complexity soft decoding is presented which achieves a coding gain of about 2.8 dB at a bit error rate of  $P_b = 10^{-4}$ .

## 1 Introduction

Due to the effects of multipath propagation it is advantageous to employ broadband transmission schemes rather than narrowband transmission schemes in digital mobile radio communications. Considering mobile radio channels, a transmission scheme is referred to as *broadband* if the bandwidth  $B$  or equivalently the corresponding symbol rate  $R_s = 1/T_s$  is greater than the coherence bandwidth  $(\Delta f)_c$  of the mobile radio channel, i.e.

$$B > (\Delta f)_c = \frac{1}{T_m} \quad , \quad (1)$$

where the multipath spread  $T_m$  is equal to the length of the finite channel impulse response. Therefore, mobile radio channels are frequency-selective for broadband transmission schemes [1]. Whereas narrowband signals suffer heavily from fading ("Rayleigh fading"), broadband signals can exploit multipath diversity.

The main drawback of broadband transmission schemes for mobile radio channels is the intersymbol interference (ISI) at the receiver. The actual transmitted bit is distorted by the  $L$  preceding and the  $L$  following bits, where  $L$  is the memory length of the mobile radio channel

$$L = \left\lceil \frac{T_s + T_m - 1}{T_s} \right\rceil \quad , \quad (2)$$

and  $\lceil x \rceil$  denotes the smallest integer  $\geq x$ . The ISI should be removed at the receiver in order to obtain a low bit error rate (BER). This can be achieved, for example, by applying

a Viterbi equalizer [1, 2]. Since the number of states grows exponentially in  $L$  for the Viterbi equalizer a considerable detection complexity results even for a moderate memory length  $L$ . Thus, equalization with reduced complexity is desirable.

In this paper, different low complexity equalizers for mobile radio channels are investigated – a simple decision feedback equalizer (DFE), a reduced state Viterbi equalizer (RSVE) obtained through channel impulse response truncation, and a reduced state sequence estimator which is the special case of the delayed decision feedback sequence estimator (DDFSE) proposed in [3] for finite impulse response (FIR) channels. The performance losses of the different low complexity equalizers in comparison to the full state Viterbi equalizer (FSVE) and the matched filter bound (MFB) are determined through Monte Carlo simulations. The MFB is a tight analytical lower bound on the BER in mobile radio channels with ideal equalization [4]. Moreover, a single-parity-check (SPC) encoding scheme with low complexity soft decoding is proposed. Simulation results show that this simple coding scheme overcompensates the performance loss caused by low complexity equalization.

## 2 Communications System

The block diagram of the communications system used throughout this paper is depicted in Figure 1. The binary symbols  $b(k) \in \{-1, +1\}$  are assumed to be equally likely, ideally interleaved, and their symbol rate  $R_s = 1/T_s$  is chosen to be  $R_s = 500$  kbit/s. For convenience in carrying out the performance simulations, the binary symbols  $b(k)$  denote uncoded as well as coded symbols and the same symbol rate  $R_s$  is used for both the uncoded and the coded transmission.

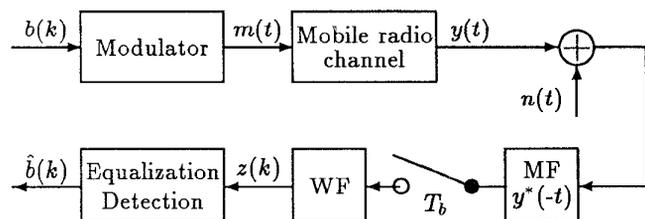


Figure 1: Block diagram of the communications system

The modulator is a BPSK modulator which generates the transmission signal  $m(t)$ . The mobile radio channel models used are those suggested by the Groupe Spécial Mobile (GSM) [5] with the four different power delay profiles ‘rural area’ (RA), ‘typical urban area’ (TU), ‘bad urban area’ (BU), and ‘hilly terrain’ (HT). The signal  $y(t)$  after the mobile radio channel is corrupted by additive white Gaussian noise  $n(t)$  with the one-sided power spectral density  $N_0$ . The receiver structure is the whitened matched filter (WMF) as proposed by Forney [6]. The coloured Gaussian noise after the matched filter (MF) is decorrelated by use of the whitening filter (WF). Therefore, the Viterbi based equalizers can apply the Euclidian distance for their path metric calculations. The WMF is assumed to be ideally realized, i.e. the channel impulse response is estimated exactly at the receiver. The signal  $z(k)$  after the WMF is passed to an equalization and detection unit where an estimate  $\hat{b}(k)$  of the transmitted symbol  $b(k)$  is determined.

The transmission system can be described using an equivalent discrete-time FIR-model  $U(z)$  [1]. Due to the MF receiver the impulse response  $u(k)$  of the equivalent discrete-time FIR-model has the properties of an autocorrelation function. Especially, the following equation is valid

$$U(z) = F(z) \cdot F^*(1/z^*) \quad , \quad (3)$$

i.e. the  $2L$  zeros of  $U(z)$  are pairwise symmetric with respect to the unit circle. It can be shown [1] that the transfer function  $H_{WF}(z)$  of the WF is given by

$$H_{WF}(z) = \frac{1}{F^*(1/z^*)} \quad , \quad (4)$$

i.e. the WF removes one half of the pairwise symmetric zeros of  $U(z)$ . Since, in principle, no restrictions for the distribution of the  $2L$  zeros of  $U(z)$  between  $F(z)$  and  $F^*(1/z^*)$  exist, there are  $2^L$  possible realizations for the WF. Especially, the distribution can be done in such a way that all the  $L$  zeros within the unit circle are assigned to  $F(z)$ . This leads to an overall equivalent discrete-time FIR-model  $F(z)$  with impulse response  $f(k)$  including the WF (Fig. 2) which is minimumphase. This minimumphase system is outstanding

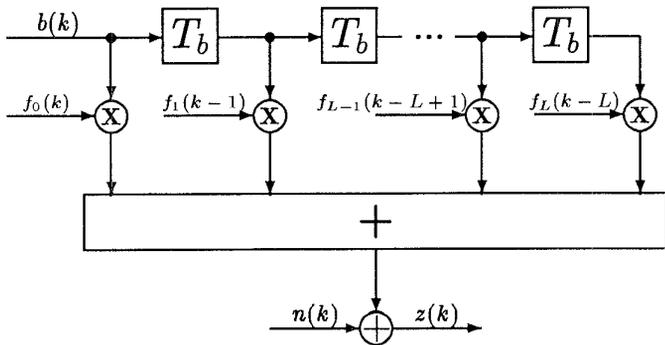


Figure 2: Equivalent overall discrete-time FIR-model

compared to all other realizations, because its energy series  $W_m(k)$  is always greater than or equal to the energy series  $W(k)$  of any other realization [7]

$$W_m(k) = \sum_{l=0}^k |f_m(l)|^2 \geq W(k) = \sum_{l=0}^k |f(l)|^2 \quad \forall k \leq L. \quad (5)$$

Thus, the minimumphase realization causes the energy of the impulse response  $f(k)$  of the overall discrete-time FIR-model to be concentrated in the first samples. This is advantageous if suboptimal detection such as low complexity equalization is employed.

### 3 Low Complexity Equalization

Three different low complexity equalization schemes for mobile radio channels are investigated – a simple DFE, a RSVE obtained through channel impulse response truncation, and a DDFSE which utilizes the whole channel impulse response for the path metric calculations.

The DFE produces an estimate  $\hat{b}(k)$  of the current symbol considering the incoming signal  $z(k)$  and the already obtained last  $L$  estimates  $\hat{b}(k-i)$ ,  $i = 1, \dots, L$  of the transmitted symbol sequence. A threshold  $T(k)$  is calculated

$$T(k) = \sum_{i=1}^L \hat{b}(k-i) \cdot f_i \quad , \quad (6)$$

where the  $f_i$  are the exactly estimated tap coefficients of the mobile radio channel. The detection rule

$$\delta = z(k) - T(k) \quad \begin{cases} > 0 & \implies \hat{b}(k) = +1 \\ < 0 & \implies \hat{b}(k) = -1 \end{cases} \quad (7)$$

is applied in order to obtain the best estimate  $\hat{b}(k)$  of the current symbol.

The whole channel memory  $L$  of the mobile radio channel is taken into account if the full state Viterbi algorithm is used for equalization. Whereas, using a RSVE the channel memory is truncated to length  $L' < L$  and the Viterbi algorithm with only  $2^{L'}$  states is applied for equalization. Since the tail of the channel impulse response is disregarded completely, the RSVE suffers from residual ISI leading to a significant performance loss in comparison to the FSVE.

A more promising equalization strategy is to use the DDFSE. This is a modified RSVE which also considers only  $2^{L'}$  states, but takes the whole channel impulse response into account for the path metric calculations. This can be achieved by combining decision feedback principles with the RSVE. Consider the step from time  $(k-1)$  to time  $k$  in the reduced trellis with  $2^{L'}$  states. Since a binary transmission is assumed there are two possibilities for the extension of each state in the trellis. Thus, the number  $N_{pm}$  of path metrics  $M_j(k)$  which have to be calculated is twice the number of reduced states, i.e.  $N_{pm} = 2^{L'+1}$ . The actual path metric  $M_j(k)$  for path  $j$



algorithm is applied in order to achieve ML decoding. An error event starts at that row  $r = \rho$  where a parity error is detected and ends after  $L$  error free rows, where  $L$  is the memory length of the channel. Thus, the length  $l_e^\rho$  of the error vector  $\underline{e}^\rho$  starting at row  $r = \rho$  is bounded by

$$R \geq l_e^\rho \geq L + 1 \quad (11)$$

Having defined the error vector, the ML decoding scheme can be described after introducing vector notations for the received signals  $z(k_\gamma^\rho)$

$$\underline{z}_\gamma^\rho = [z(k_\gamma^\rho)z(k_\gamma^{\rho+1}) \dots z(k_\gamma^{\rho+l_e^\rho-1})]^T, \quad (12)$$

the estimated symbols  $\hat{b}(k_\gamma^\rho)$

$$\hat{\underline{b}}_\gamma^\rho = [\hat{b}(k_\gamma^\rho)\hat{b}(k_\gamma^{\rho+1}) \dots \hat{b}(k_\gamma^{\rho+l_e^\rho-1})]^T, \quad (13)$$

the corrected symbols  $\tilde{b}(k_\gamma^\rho)$

$$\tilde{\underline{b}}_\gamma^\rho = \hat{\underline{b}}_\gamma^\rho \odot \underline{e}^\rho, \quad (14)$$

the path weights  $\hat{w}(k_\gamma^\rho) = \sum_{i=0}^L \hat{b}(k_\gamma^{\rho-i})f_i$  corresponding to the estimated symbols  $\hat{b}(k_\gamma^\rho)$

$$\hat{\underline{w}}_\gamma^\rho = [\hat{w}(k_\gamma^\rho)\hat{w}(k_\gamma^{\rho+1}) \dots \hat{w}(k_\gamma^{\rho+l_e^\rho-1})]^T \quad (15)$$

and the path weights  $\tilde{w}(k_\gamma^\rho) = \sum_{i=0}^L \tilde{b}(k_\gamma^{\rho-i})f_i$  corresponding to the corrected symbols  $\tilde{b}(k_\gamma^\rho)$

$$\tilde{\underline{w}}_\gamma^\rho = [\tilde{w}(k_\gamma^\rho)\tilde{w}(k_\gamma^{\rho+1}) \dots \tilde{w}(k_\gamma^{\rho+l_e^\rho-1})]^T. \quad (16)$$

The notation ‘ $\odot$ ’ in Equation (14) denotes elementwise multiplication. Applying an FSVE to every column of the decoding matrix leads to the maximum log-likelihood  $L^\rho$  for each row  $r = \rho$

$$L^\rho = \sum_{c=1}^C \log P(\underline{z}_c^\rho | \hat{\underline{b}}_c^\rho). \quad (17)$$

Error correction is done according to Equation (14) by modifying all vectors  $\hat{\underline{b}}_c^\rho$ ,  $c = 1, \dots, C$  using the error vector  $\underline{e}^\rho$  and determining that modified vector  $\tilde{\underline{b}}_\gamma^\rho$  for which the new log-likelihood  $\tilde{L}_\gamma^\rho$

$$\tilde{L}_\gamma^\rho = \sum_{\substack{c=1 \\ c \neq \gamma}}^C \log P(\underline{z}_c^\rho | \hat{\underline{b}}_c^\rho) + \log P(\underline{z}_\gamma^\rho | \tilde{\underline{b}}_\gamma^\rho) \quad (18)$$

is nearest to the maximum log-likelihood  $L^\rho$  of row  $r = \rho$ . The vector  $\hat{\underline{b}}_\gamma^\rho$  in the corresponding column  $c = \gamma$  is replaced by the modified vector  $\tilde{\underline{b}}_\gamma^\rho$ . Instead of considering the log-likelihoods  $L^\rho$  and  $\tilde{L}_\gamma^\rho$  the corresponding distances  $D^\rho$

$$D^\rho = \sum_{c=1}^C d(\underline{z}_c^\rho, \hat{\underline{w}}_c^\rho) \quad (19)$$

and  $\tilde{D}_\gamma^\rho$

$$\tilde{D}_\gamma^\rho = \sum_{\substack{c=1 \\ c \neq \gamma}}^C d(\underline{z}_c^\rho, \hat{\underline{w}}_c^\rho) + d(\underline{z}_\gamma^\rho, \tilde{\underline{w}}_\gamma^\rho) \quad (20)$$

can be considered, where  $d(\underline{x}, \underline{y})$  denotes the Euclidian distance between the vectors  $\underline{x}$  and  $\underline{y}$ . In this case the soft value  $\delta_\gamma^\rho$  is defined as the difference between the modified distance  $\tilde{D}_\gamma^\rho$  and the minimum distance  $D^\rho$  of row  $r = \rho$

$$\delta_\gamma^\rho = \tilde{D}_\gamma^\rho - D^\rho = d(\underline{z}_\gamma^\rho, \tilde{\underline{w}}_\gamma^\rho) - d(\underline{z}_\gamma^\rho, \hat{\underline{w}}_\gamma^\rho) \geq 0, \quad (21)$$

where  $\delta_\gamma^\rho$  can be simplified to

$$\delta_\gamma^\rho = \sum_{i=0}^{l_e^\rho-1} 2z(k_\gamma^{\rho+i})[\hat{w}(k_\gamma^{\rho+i}) - \tilde{w}(k_\gamma^{\rho+i})] + [\hat{w}^2(k_\gamma^{\rho+i}) - \tilde{w}^2(k_\gamma^{\rho+i})]. \quad (22)$$

The search of the log-likelihood which is closest to the maximum log-likelihood is equivalently to the determination of that column  $c = \gamma$  for which  $\delta_\gamma^\rho$  is minimum. This procedure has to be repeated for all determined error vectors  $\underline{e}^\rho$ .

The described decoding scheme is ML under the assumption that no overlapping error events occur in the decoding matrix, i.e. if there is an error vector which indicates more than one bit error these errors are all assumed to be located in the same column of the decoding matrix. This decoding scheme, termed ‘conventional’ in the sequel, was introduced in [8] and used for the decoding of a convolutional SPC concatenated code. Considering the mobile radio channel as a special kind of convolutional encoder one main difference to a real convolutional encoder has to be taken into account. In the case of a convolutional code the error event with the minimum distance contains normally more than one bit error leading to the well-known burst error behaviour of convolutional codes. However, this fact does not hold in the case of a mobile radio channel viewed as convolutional encoder. In almost all cases the time-variant impulse response of the mobile radio channel leads to minimum distance paths which only correspond to one bit error [4]. With regard to this fact a modified decoding scheme which is in fact a simplification of the conventional decoding scheme should result in a better performance. The length  $l_e^\rho$  of the error vector  $\underline{e}^\rho$  is kept fix to the shortest possible length  $l_e^\rho = L+1$  resulting in an error vector which contains only one bit error. Following this approach, the longer error events are assumed to be composed of overlapping error vectors and therefore are split up into several error vectors of shortest possible length which can be located in different columns.

## 5 Simulation Results

The simulation results presented in this section are restricted to the mobile radio channels BU and HT since the results for the other channel models exhibit the same quantitative behaviour. For the given symbol rate of  $R_s = 500$  kbit/s the memory lengths of the BU and HT channel are  $L = 6$

and  $L = 11$ , respectively. With respect to the detection effort no FSVE is applicable to the HT channel and even the DDFSE and the RSVE are restricted to reduced memory lengths  $L' \leq 6$ . The Figures 4 - 7 show the resulting BERs with respect to the signal-to-noise ratio (SNR) defined as  $E_s/N_0$ . All statements about performance losses and gains for both the uncoded and coded transmission are given in  $E_b/N_0$  and refer to a BER of  $P_b = 10^{-4}$  until otherwise specified.

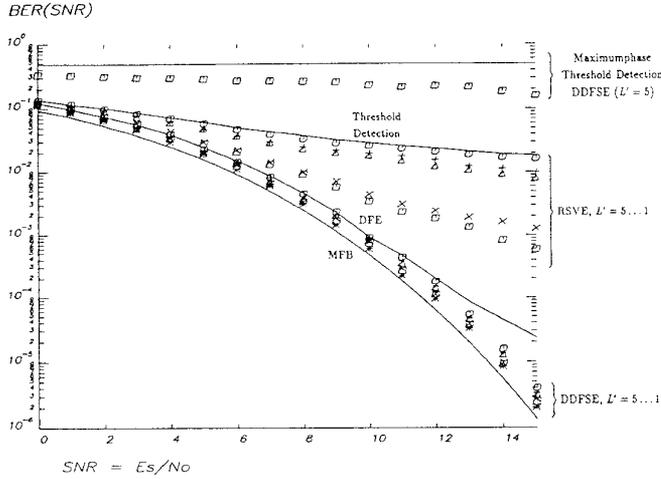


Figure 4: Simulation results for the different equalizers for BU channel and uncoded transmission: FSVE (\*), DDFSE with  $L' = 5(\square)/4(x)/3(\triangle)/2(+)/1(o)$ , RSVE with  $L' = 5(\square)/4(x)/3(\triangle)/2(+)/1(o)$ , and DDFSE with  $L' = 5(\square)$  (maximumphase realization of FIR-model)

The Figures 4 and 5 show the simulation results for the uncoded transmission over the BU and HT channel, respectively, utilizing a DDFSE, an RSVE, a DFE or simple threshold detection at the receiver. Moreover, the results for a DDFSE with reduced memory length  $L' = L - 1$  and the simple threshold detector in the case of a maximumphase realization of the FIR-model as well as the MFBs are depicted. From both figures it is verified that a non-minimumphase realization of the FIR-model leads to catastrophic performance results with BERs  $P_b > 0.1$ . Considering the minimumphase realization the results become better. However, simple threshold detection and the RSVE still perform bad. In the case of the threshold detector the ISI leads to severe disturbance. Looking at the RSVE, the performance increases with the number of states taken into account since the residual ISI decreases. Though, using the RSVE a certain amount of ISI remains leading to an error floor above  $P_b = 10^{-4}$  for BU and  $P_b = 10^{-3}$  for HT. Considerable improvement is achieved applying the DFE and the DDFSE. For all  $L'$  the DDFSE performs better than the DFE. The performance losses in comparison to ideal equalization (MFB) are about 1.25 dB (BU) and 1.13 dB (HT) for the DFE, whereas the losses for the DDFSE with  $L' = 1$  are only about 0.92 dB (BU) and 0.96 dB (HT). Moreover, the

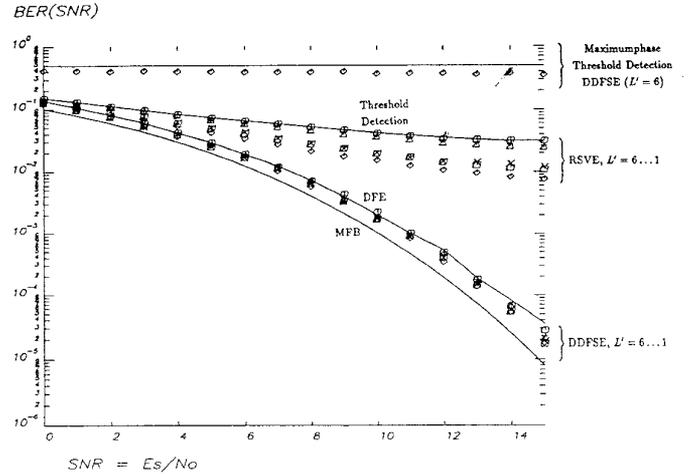


Figure 5: Simulation results for the different equalizers for HT channel and uncoded transmission: DDFSE with  $L' = 6(\diamond)/5(\square)/4(x)/3(\triangle)/2(+)/1(o)$ , RSVE with  $L' = 6(\diamond)/5(\square)/4(x)/3(\triangle)/2(+)/1(o)$ , and DDFSE with  $L' = 6(\diamond)$  (maximumphase realization of FIR-model)

performance curves of the DDFSE possess slopes comparable to the slope of the MFB, whereas the performance curves of the DFE flatten out at higher SNRs leading to more severe performance losses at higher SNR. Thus, only a very low performance loss of less than 1 dB is caused by low complexity equalization with a DDFSE even if  $L' = 1$  and the channel memory length is  $L = 11$  (HT).

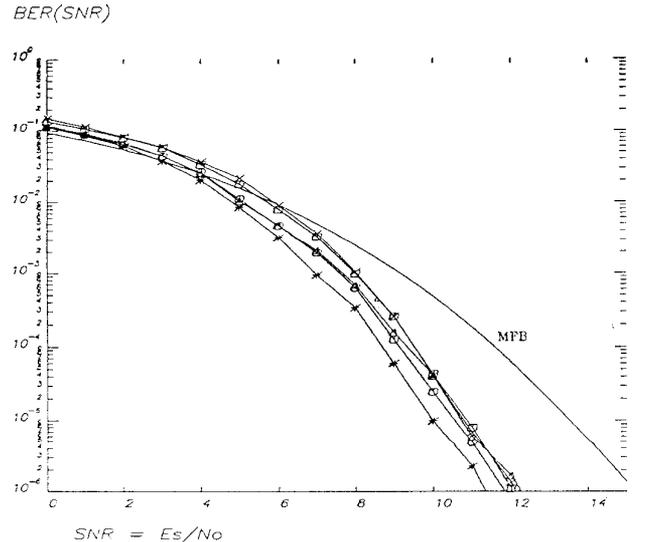


Figure 6: Simulation results for the different equalizers for BU channel and coded transmission ( $r = 7/8$ ): DFE ( $\triangle$ ), FSVE with conventional decoding scheme ( $x$ ), FSVE with modified decoding scheme (\*), DDFSE with conventional decoding scheme and  $L' = 1(\square)$ , and DDFSE with modified decoding scheme and  $L' = 1(o)$

The performance loss can easily be overcompensated using the SPC encoding scheme with low complexity soft decoding described in Section 4 with code rate  $r = 7/8$  (15/16). Figure 6 shows the simulation results for the coded transmission over the BU channel. Surprisingly, the performance of the DFE is as good as the performance of both the FSVE and the DDFSE with  $L' = 1$  if the conventional decoding scheme is applied. This is due to the fact that this decoding scheme is not adapted to the error structure of the special kind of convolutional encoder the mobile radio channel can be viewed as. The FSVE and the DDFSE with  $L' = 1$  become better than the DFE if the modified decoding scheme is used. The performance gain with respect to ideal equalization (MFB) is 1.67 dB for the DFE, whereas 1.88 dB are achieved for the DDFSE with  $L' = 1$ . The maximum achievable gain is 2.34 dB defined through the performance curve of the FSVE. Thus, the coding gain for the SPC encoding scheme with low complexity soft decoding is about 2.8 dB for DFE, DDFSE, and FSVE. Further simulations show (Fig. 7) that even with less redundancy, e.g.  $r = 15/16$ , the performance loss of low complexity equalization can be overcompensated since the coding gain is about 2.7 dB. Moreover, considering the behaviour of this coding scheme for small SNR values, i.e.  $E_s/N_0 < 4$  dB, there is only a slight degradation with respect to the uncoded transmission (Fig. 6 / 7), since in the case of wrong error correction only a small number of bit errors are generated.

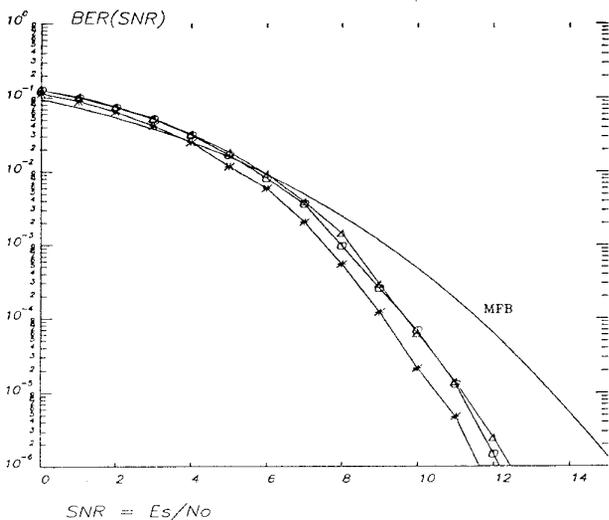


Figure 7: Simulation results for the different equalizers for BU channel and coded transmission ( $r = 15/16$ ): DFE ( $\Delta$ ), FSVE with modified decoding scheme ( $*$ ), and DDFSE with modified decoding scheme and  $L' = 1$  ( $\circ$ )

## 6 Conclusions

The simulation results of the previous section prove that equalization schemes with a considerable complexity reduction can be applied to mobile radio channels if a properly

chosen WMF at the receiver is used. The resulting overall impulse response  $f(k)$  is minimumphase leading to the necessary concentration of the impulse response energy within the first samples.

Complexity reduction through simple truncation of the impulse response (RSVE) results in a very poor performance, whereas the DFE and the DDFSE for all considered mobile radio channel models exhibit only small performance losses in comparison to ideal equalization (MFB). Even the least complex DDFSE with  $L' = 1$  applied to a mobile radio channel with long impulse response ( $L = 11$ , HT) causes a performance loss of less than 1 dB at a BER of  $P_b = 10^{-4}$ . The described SPC encoding scheme with low complexity soft decoding is very efficient since it requires only little redundancy ( $r = 7/8$ ) to achieve a coding gain of about 2.8 dB at a BER of  $P_b = 10^{-4}$ . In the case of Viterbi based equalizers the proposed modified decoding scheme should be used since this scheme is adapted to the error structure generated by mobile radio channels.

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