Constructing Coset Codes With Optimal Same-Symbol Weight for Detecting Narrowband Interference in M-FSK Systems

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***Abstract*—Narrowband interference can cause undetected errors when M-FSK data is encoded with an algebraic code containing the all-e codewords. This is due to the fact that the narrowband interferer will cause the output of the M-FSK demodulator to correspond to one of the all-e codewords. One possible solution is to use a coset code of a code containing the repetition code. The choice of the coset leader should be such that the resulting coset code has minimum same-symbol weight. We give a general construction for generating coset codes with minimum same-symbol weight and present results where an optimal coset code for an (n, k) Reed-Solomon code is applied in an M-FSK environment with narrowband interference. From the results it is evident that the optimal coset codes outperform linear codes when narrowband interference is present.**

***Index Terms*—Coset codes, minimum same-symbol weight, Reed-Solomon codes.**

I. Introduction

On many communication channels narrowband noise compromises the integrity of the data transmitted over the channel (see [1] and [2]). One such an environment is the Power Lines Communications (PLC) channel. In the presence of narrowband noise, algebraic codes which contain the all-e codewords in conjunction with M-FSK modulation suffers from undetectable errors.

In [3], a method was described to detect and correct the presence of narrowband noise in M-FSK systems by utilizing certain Reed-Solomon codes and coset Reed-Solomon codes. In this paper we give a general algebraic construction for coset codes that perform optimally in the presence of narrowband interference. We also apply the construction to an (n, k) Reed-Solomon code and simulate and compare the performance of the constructed coset code with normal Reed-Solomon codes in the presence of narrowband noise and additive white Gaussian noise. The optimal coset codes outperform Reed-Solomon codes when narrowband interference is present due to errors which the Reed-Solomon codes cannot detect.

The remainder of this paper is structured as follows. Section II provides a brief overview on the effect of narrowband noise on M-FSK modulated transmission. In Section III, we give a construction for finding the optimal coset code of an algebraic code. Section IV discusses how encoding and decoding can be achieved. In Section V, the results are presented and we conclude in Section VI.

Manuscript received August 26, 2008; revised July 23, 2010. Date of current version November 19, 2010. This work was supported by Telkom SA Ltd. and by the South African National Research Foundation under Grant 2053408. The material in this paper was presented (in part) at the Ninth International Symposium on Communication Theory and Applications (ISCTA'07), Ambleside, U.K., July 2007.

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Communicated by I. Dumer, Associate Editor for Coding Theory.

Digital Object Identifier 10.1109/TIT.2010.2079013

II. M-FSK and Narrow-Band Noise

We refer the reader to [4], [5], and [6] for the normal AWGN channel for M-FSK modulation. We use the same one-to-one mapping from the field $\mathrm{GF}(2^m)$ onto the $M = 2^m$ distinct frequencies of the M-FSK modulator as is done in [4] and [5].

In essence, noncoherent M-FSK detection chooses from a set of M frequencies the one with the highest energy present at a sampling instance T, assuming that the desired frequency was transmitted with energy $E_{\mathcal{S}}$. Furthermore, the SNR for such a system is given as $\mathrm{SNR} = E_{\mathcal{S}}/N_0$ (refer to [6]). Practically noncoherent M-FSK detection is implemented by using a bank of $2M$ correlators, with a quadrature pair for each frequency. For each quadrature pair the output is added together using the square law to produce a metric for the corresponding frequency candidate. The most likely transmitted symbol for sampling instance T is determined based on these M metrics. For normal envelope detection, the symbol corresponding to the metric with the highest value is chosen as the candidate [6, p. 258].

The Viterbi threshold ratio test detector [7] performs a rudimentary form of soft-decision detection. As such, for sampling instance T, the output S is given as

$$S = \begin{cases} S_j, & m_1 \times \lambda > m_2 \\ \mathcal{E}, & \text{otherwise} \end{cases} \tag{1}$$

where $S_j \in \mathrm{GF}(2^m)$ is the symbol corresponding to the transmitted frequency f_j. For sampling instance T, m_1 is the largest metric (corresponding to frequency f_j) and m_2 is the second largest metric. The threshold $0 < \lambda \leq 1$ is decided on before transmission. If the inequality $m_1 \times \lambda > m_2$ is not satisfied, the output is flagged as an erasure ($\mathcal{E}$).

Let $E_{\mathcal{N}}$ denote the energy detected at the demodulator due to a narrowband noise source. If $E_{\mathcal{N}}$ exceeds $E_{\mathcal{S}}$, the demodulator will have a symbol that is "always on". This poses a problem, since the all-e vector (that is, the vector with all its entries equal to e, where $e \in \mathrm{GF}(2^m)$) is a valid codeword for most linear codes. Therefore, the presence of narrowband noise disturbances will result in undetected errors.

Thus, some modifications should be made to the detectors or the coding scheme in order to detect the presence of narrowband noise. We consider the use of a coset algebraic code, i.e., adapt the coding scheme to exclude the all-e vectors and thereby detect the presence of narrowband noise.

As we have seen, when coding to detect narrowband interference, the all-e codewords should be avoided. Choi [8] observed that a coset code of a binary BCH code is comma-free and used these codes to achieve synchronization. With comma-free codes the all-e vector ($e \in \mathrm{GF}(2^m)$) cannot be in the code, otherwise the definition of comma-freedom will be violated. Due to the strict constraints imposed by comma freedom, the coset codes constructed by Choi cannot achieve high rates. However, these stringent conditions do not have to be met when coding for narrowband noise. As such we can use coset codes which do not impact the rate of the code.

We next consider the notion of optimal coset codes. In order to define an optimal coset code, we introduce the following two definitions.

Definition 1 (Same-Symbol Weight of a Codeword): The same-symbol weight of a codeword c is the maximum number of times any element of $\mathrm{GF}(2^m)$ appears in c.

Definition 2 (Same-Symbol Weight of a Code): The same-symbol weight of a code C is the maximum same-symbol weight of the codewords in C.

As an example, the same-symbol weight of the all-zero codeword of an (n,k) RS code is equal to n. Also, the same-symbol weight of the all-e codeword ($e \in \mathrm{GF}(2^m)$) is equal to n. Thus, the same-symbol weight of an (n,k) RS codebook containing the all-e codewords is equal to n.

The fewer times any given symbol e appears in a valid codeword, the more errors are necessary for the word to be received as an all-e word. Therefore minimum same-symbol weight leads to minimum probability of an all-e word being received. In this sense a code with minimum same-symbol weight will be optimal.

III. Constructing Coset Codes With Minimum Same-Symbol Weight

In what follows is a general construction of optimal coset algebraic codes.

Proposition 1: Let C_0, C_1 and C_2 be linear codes of length n defined over some field such that $C_0 \subset C_1 \subset C_2$. Let C_0 be the repetition code (i.e., the 1-D code consisting of the all-e codewords), and let $\dim(C_2) = 1 + \dim(C_1)$. Let d_1 and d_2 denote the minimum Hamming weights of codewords in C_1 and C_2 respectively, and suppose $d_2 < d_1$. If $x \in C_2 \setminus C_1$, then $\mathrm{ssw}(x + C_1) = n - d_2$, that is, the same-symbol weight of the coset $x + C_1$ is $n - d_2$.

Proof: Since $x \notin C_1$, it follows from the dimensions that x and C_1 span all of C_2. Hence, if y is a word in C_2 of minimal weight d_2, then we can write $y = ax + y_1$, where a is in the alphabet field and $y_1 \in C_1$. If $a = 0$, then $y \in C_1$, which is impossible, since $\mathrm{wt}(y) = d_2$ and no word in C_1 has weight less than d_1. Thus, $a \neq 0$, so we can write $a^{-1}y = x + a^{-1}y_1 \in x + C_1$. Since $\mathrm{wt}(a^{-1}y) = \mathrm{wt}(y) = d_2$, it follows that $a^{-1}y$ is a word in $x + C_1$ that contains $n - d_2$ entries equal to 0. Thus, $\mathrm{ssw}(x + C_1) \geq n - d_2$. On the other hand, suppose w is a word in $x + C_1$ of maximal same-symbol weight say k, and suppose w contains k entries equal to the symbol c from the alphabet field. Then $\mathrm{wt}(w - (c, c, \ldots, c)) = n - k$, and $w - (c, c, \ldots, c) \in C_2$ since $C_0 \subset C_1 \subset C_2$. Thus, $n - k \geq d_2$ by definition of d_2, so $k \leq n - d_2$, that is, $\mathrm{ssw}(x + C_1) \leq n - d_2$, which completes the proof. ■

In a systematic (n,k) code C, every message word (of length k) is a substring of its encoded word (of length n). The all-e words of length k are valid message words (since all $(2^m)^k$ words are possible), and are substrings of their encoded words, which therefore have same-symbol weight at least k. Thus, $\mathrm{ssw}(C) \geq k$. The same inequality applies to a non-systematic code having the same codebook as C. (For example, Reed-Solomon codes based on a polynomial $g(z)$, whether systematic or nonsystematic, have the same codebook, namely the set of all polynomials divisible by $g(z)$.)

IV. Encoding and Decoding for Narrow-Band Noise

The construction of Section III can be applied to many algebraic codes including Reed-Solomon codes and Hermitian codes. Encoding the coset-code is accomplished by adding the codeword x after the information $m(z)$ is encoded with the encoder of C_1 (which can be systematic or non-systematic). Any choice for x will suffice, as long as the coset-encoder and coset-decoder agrees on the same x. A choice for x is to use the generator polynomial $g_2(z)$ of C_2. Fig. 1 shows a block diagram of non-systematic algebraic encoding for C_1 [Fig. 1(a)] and how coset encoding is achieved [Fig. 1(b)].

Fig. 2 outlines the general demodulation and decoding procedure. The demodulator records the set of M metrics for each timeslot $T_{S_0}, T_{S_1}, \ldots, T_{S_{n-1}}$ and stores the metrics in a $M \times n$ matrix D. Note that the matrix D is constructed in such a way that the stored metrics corresponds to a single received vector $r(z)$ of length n, i.e., the metrics contained in column D_i, recorded at timeslot T_{S_i}, determines the ith coefficient of $r(z)$. The first step is to determine whether a narrowband interferer was present during the transmission of $r(z)$. This is achieved by noting for each frequency ω, the number of times the metrics $D_{\omega,i}, i = \{0, 1, \ldots, n-1\}$ exceeds $\mu \cdot E_{\mathcal{S}}$ for some $\mu \leq 1$. Let the frequency ω_m be the frequency with the maximum number of times the metrics $D_{\omega_m,i}$ exceeds $\mu \cdot E_{\mathcal{S}}$, and let χ be the count (i.e., the number of times $D_{\omega_m,i}$ exceeds $\mu \cdot E_{\mathcal{S}}$).

Narrowband interference is assumed to be present if $\chi > \tau$, where

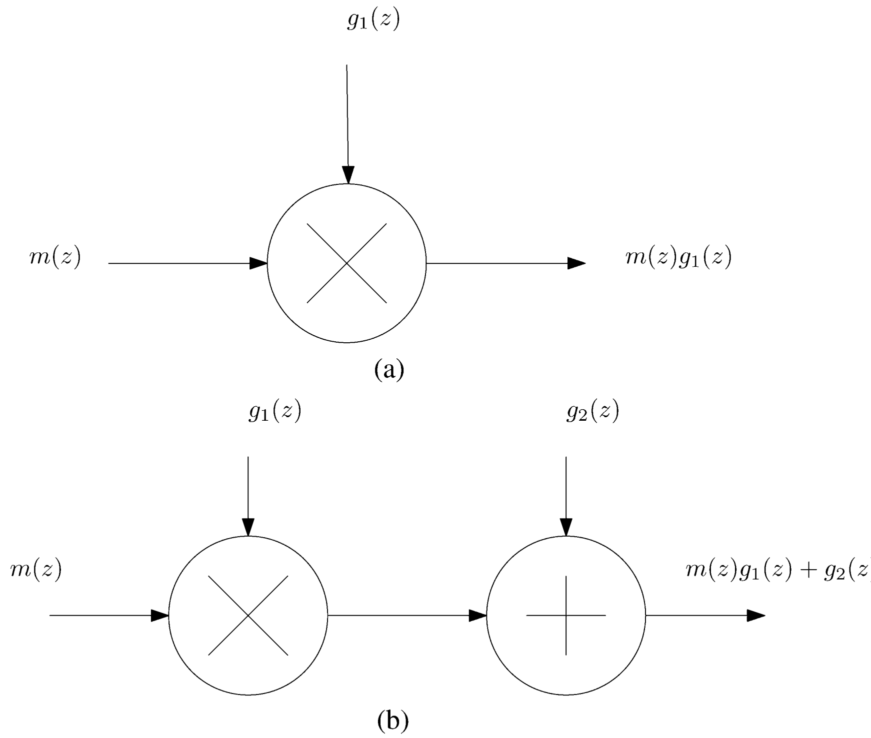

Fig. 1. Encoding. (a) Algebraic encoding. (b) Algebraic coset encoding.

$$\tau = \left\lfloor \frac{n + \mathrm{ssw}(C_1 + g_2(z))}{2} \right\rfloor . \tag{2}$$

If narrowband interference is detected, the frequency of the narrowband interferer is assumed to be ω_m. Subsequently, we set the metrics $D_{\omega_m,i}, i = \{0, 1, \ldots, n-1\}$ equal to zero, in order to cancel the narrowband interferer.

After this step, normal M-FSK demodulation follows, where the appropriate rule (envelope detection, Viterbi threshold ratio test, etc) is applied to column D_i in order to determine the ith coefficient of $r(z)$. Note that when $\chi \leq \tau$, the matrix D is passed on unaltered to the demodulation process. The demodulator produces the polynomial $r(z) = e(z)+x(z)+c(z)$. After the coset header $x(z)$ has been subtracted, any decoder for C_1 can be used to perform decoding.

Error correction capabilities of the coset code $(x + C_1)$ is the same as for the code C_1, i.e., the coset code can correct any combination of $d_1 > 2t+\varepsilon$ errors (t) and erasures (ε). However, due to narrowband interference, another error mechanism acts on the transmitted codeword. If the codeword $c \in (x + C_1)$ is transmitted with symbol $\eta \in GF(2^m)$ occurring $\xi \leq \mathrm{ssw}(x + C_1)$ times in c and the narrowband interferer corresponds to the frequency mapped to η, ξ errors or erasures will be introduced. The errors are introduced when the entries of the row corresponding to η of the matrix D is set to zero. Whether errors or erasures are introduced, depends on the detector being used for demodulation.

V. Results

We now demonstrate Proposition 1 by way of an example. Let the alphabet field be $\mathrm{GF}(2^3)$ generated by the primitive element α such that $\alpha^3 = \alpha + 1$. Let C_1 be the (7, 4) RS code generated

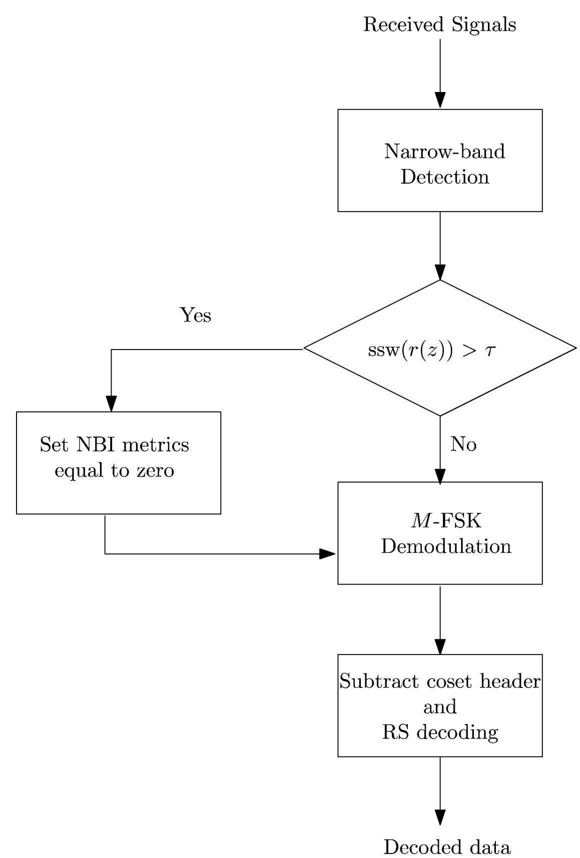

Fig. 2. Combined demodulation and decoding for narrowband interference.

by $g_1(z) = (z - \alpha^1)(z - \alpha^2)(z - \alpha^3)$. Similarly, C_2 is the (7, 5) RS code generated by $g_2(z) = (z - \alpha^1)(z - \alpha^2)$. Suppose a certain message polynomial is encoded (systematically or otherwise) as the code polynomial $c_1(z) = \alpha^6 + \alpha^1 z + \alpha^1 z^2 + \alpha^5 z^3 +$

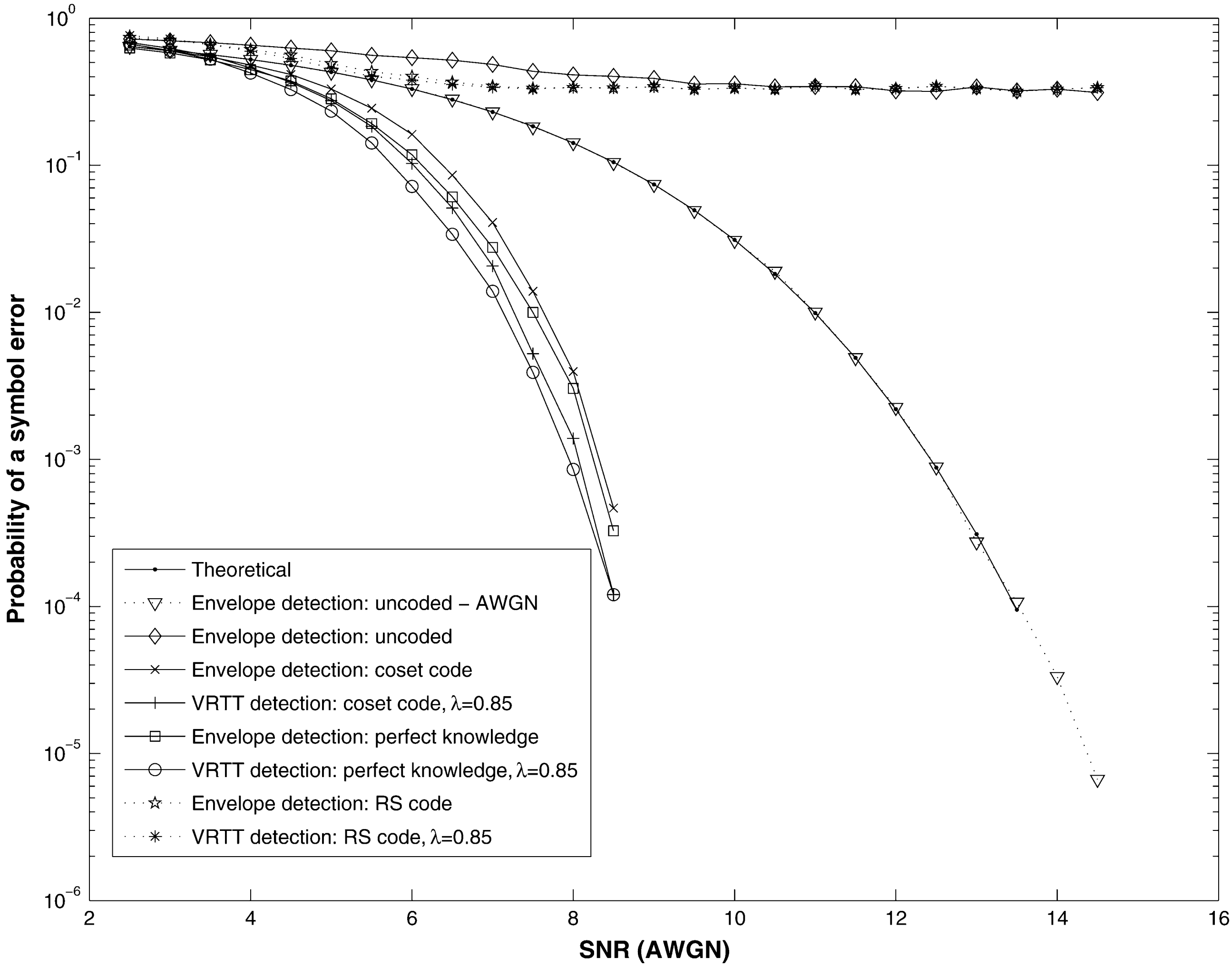

Fig. 3. Modified M-FSK detection using a (15, 3) coset Reed-Solomon code and $E_{\mathcal{N}} = 4E_{\mathcal{S}}$.

$\alpha^4z^4+\alpha^1z^5+\alpha^6z^6$ in C_1. This could be transmitted directly, but since $\mathrm{ssw}(C_1) = n = 7$, it would not be possible to detect narrowband interference. Instead, we use the coset code $g_2(z)+C_1$ (which has ssw $= 4$), and we therefore add $g_2(z)$, and transmit $c_1(z)+g_2(z) = \alpha^4+\alpha^2z+\alpha^3z^2+\alpha^5z^3+\alpha^4z^4+\alpha^1z^5+\alpha^6z^6$.

If narrowband interference corresponding to the frequency of α^j occurs, then after M-FSK demodulation the received polynomial will be $\alpha^j(1 + z + z^2 + \cdots + z^n)$ which is not in $g_2(z) + C_1$, so the narrowband interferer has been detected. This is the important property of the coset code. The received polynomial is again demodulated, ignoring the frequency corresponding to α^j.

Now the received polynomial is of the form $c_1(z) + g_2(z) + e(z)$, where $e(z)$ is a possible error polynomial due to other noise processes, for example if $e(z) = \alpha z^3$, $c_1(z) + g_2(z) + e(z) = \alpha^4 + \alpha^2z + \alpha^3z^2 + \alpha^6z^3 + \alpha^4z^4 + \alpha^1z^5 + \alpha^6z^6$.

We now subtract $g_2(z)$ to obtain $c_1(z) + e(z) = \alpha^6 + \alpha^1z + \alpha^1z^2 + \alpha^6z^3 + \alpha^4z^4 + \alpha^1z^5 + \alpha^6z^6$. This can now be decoded with any decoder, for example the Massey-Berlekamp algorithm, to obtain $c_1(z)$.

The performance of the coset Reed-Solomon codes was investigated using the narrowband noise channel discussed in [3] using two M-FSK demodulator techniques, namely envelope detection [6] and the Viterbi threshold ratio test [7].

In Figs. 3 and 4 we simulate a 16-FSK channel where, in addition to AWGN noise, a narrowband disturbance is present with probability $\rho = 0.1$, duration $\delta \times n$, $\delta \leq 10$ and energy $E_{\mathcal{N}} = 4 \times E_{\mathcal{S}}$. The narrowband detection threshold μ is set to 0.5. (Fig. 3 is for a (15, 3) coset RS code and Fig. 4 is for a (15, 9) coset RS code.) Theoretical performance when only AWGN noise is present ("Theoretical") is calculated using the approach in [6] and is benchmarked against a simulation where only AWGN noise is present with no coding performed ('Envelope detection: uncoded—AWGN'). The graph 'Envelope detection: uncoded' depicts the performance of the communication system when no coding is performed in the presence of narrowband interference. The graphs "Envelope detection: coset code" and "VRTT detection: coset code, $\lambda = 0.85$" depict the performance of a communication system deploying envelope detection in conjunction with the proposed coset coding method and a communication system deploying the Viterbi ratio threshold test ($\lambda = 0.85$) in conjunction with the proposed coset coding method, respectively. The graphs "Envelope detection: perfect knowledge" and "VRTT detection: perfect knowledge, $\lambda = 0.85$" both depict the performance of a communication

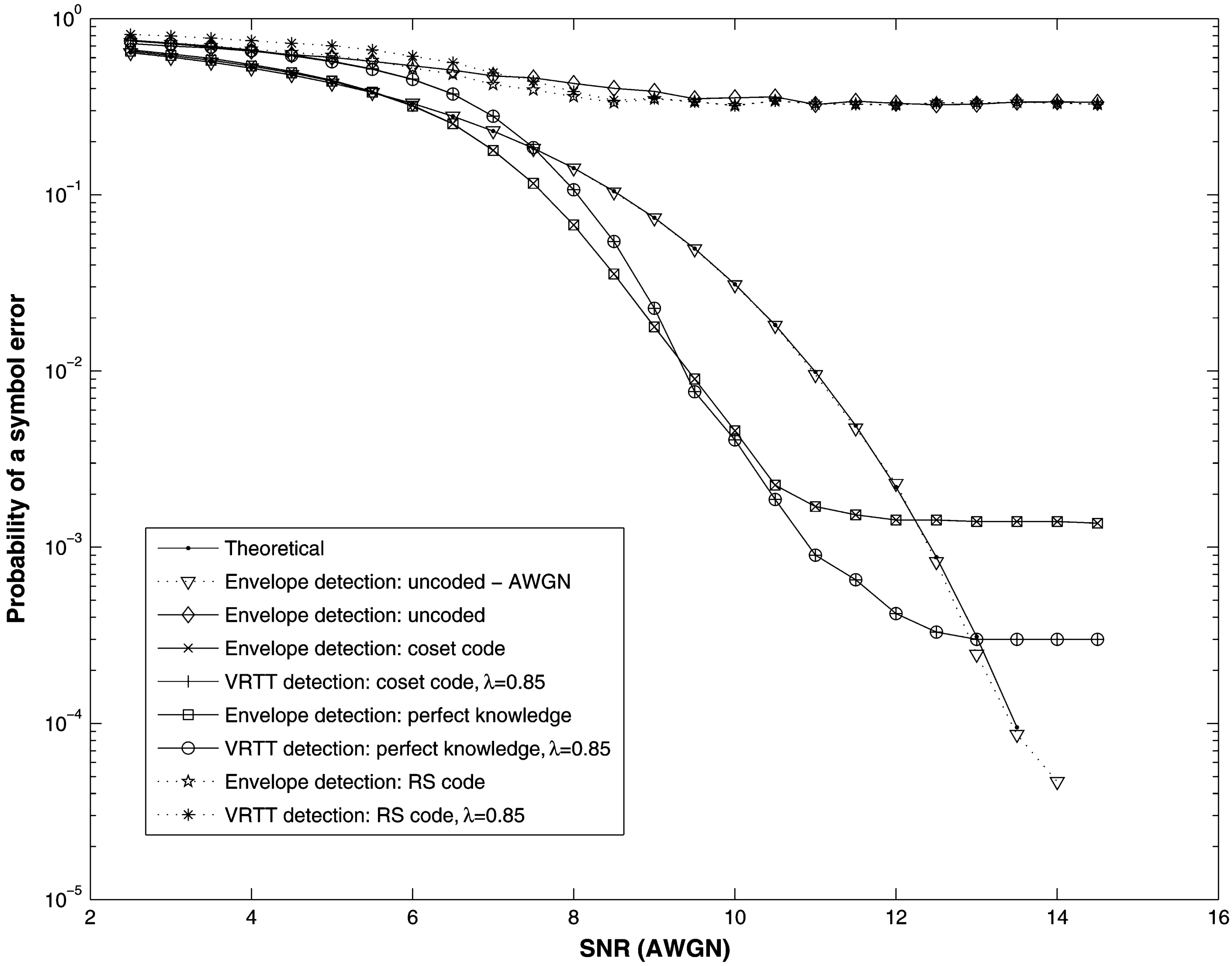

Fig. 4. Modified M-FSK detection using a (15, 9) coset Reed-Solomon code and $E_{\mathcal{N}} = 4E_{\mathcal{S}}$.

system where the detector receives perfect knowledge of the positions of symbols in error due to narrowband interference for the respective detector schemes. The graphs "Envelope detection: RS code" and "VRTT detection: RS code, $\lambda = 0.85$" depict the performance of systems where normal Reed-Solomon coding is used.

As can be seen, the presence of narrowband interference causes a catastrophic failure when normal Reed-Solomon coding is used (depicted by 'Envelope detection: RS code' and 'VRTT detection: RS code, $\lambda = 0.85$'). It is evident from the figures that when the proposed coset coding scheme is deployed, it greatly improves the system's performance in the presence of narrowband interference. Almost the same performance is realised with the modified detection scheme in the presence of narrowband interference as when only AWGN noise was present, i.e., a performance increase of almost $10\log_{10}\frac{k}{n}$ dB is achieved (which relates to 7 dB for the (15, 3) code and 2.2 dB for the (15, 9) code) due to the reduced code rate. Furthermore, the proposed scheme yields similar performance compared to the systems which have perfect knowledge of the positions of symbols affected by narrowband interference. (More complex narrowband detection schemes might improve the performance to match that of the perfect knowledge schemes.) In the simulations, a clear error floor is noticeable for high code rate systems, even at high signal-to-noise ratios. This can be attributed to the fact that the power level of the narrowband noise remains constant—even when the SNR increases—and that the proposed scheme introduces additional errors or erasures when the frequency of the narrowband interferer coincides with the frequency of one or more transmitted symbols. Since the SSW of a codeword is equal or less than $n - d_2$, and for a Reed-Solomon code $d_{\min} = n - k + 1$, we have that when $k \leq n/3$ we have $d_1/2 > n - d_2$ and no error floor occurs. For the (15, 3) code $d_{\min} = 13$ and the same-symbol weight is 3. For the (15, 9) code the $d_{\min} = 7$ and the same-symbol weight is 8. Hence, for the (15, 9) code we expect an error floor, at a level depending on the value of ρ and δ.

Fig. 5 consider the effect of various values for $E_{\mathcal{N}}$ on the proposed narrowband detection scheme's performance for both the envelope detector the Viterbi ratio threshold test detector when a (15, 3) Reed-Solomon code is used. From Fig. 5 it is evident that the value of $E_{\mathcal{N}}$ has no significant effect on the proposed scheme's performance. Furthermore, the Viterbi ratio threshold test detector yields a better performance than the envelope detector. This can be attributed to the fact that the envelope detector introduces errors, whereas the Viterbi detector introduces

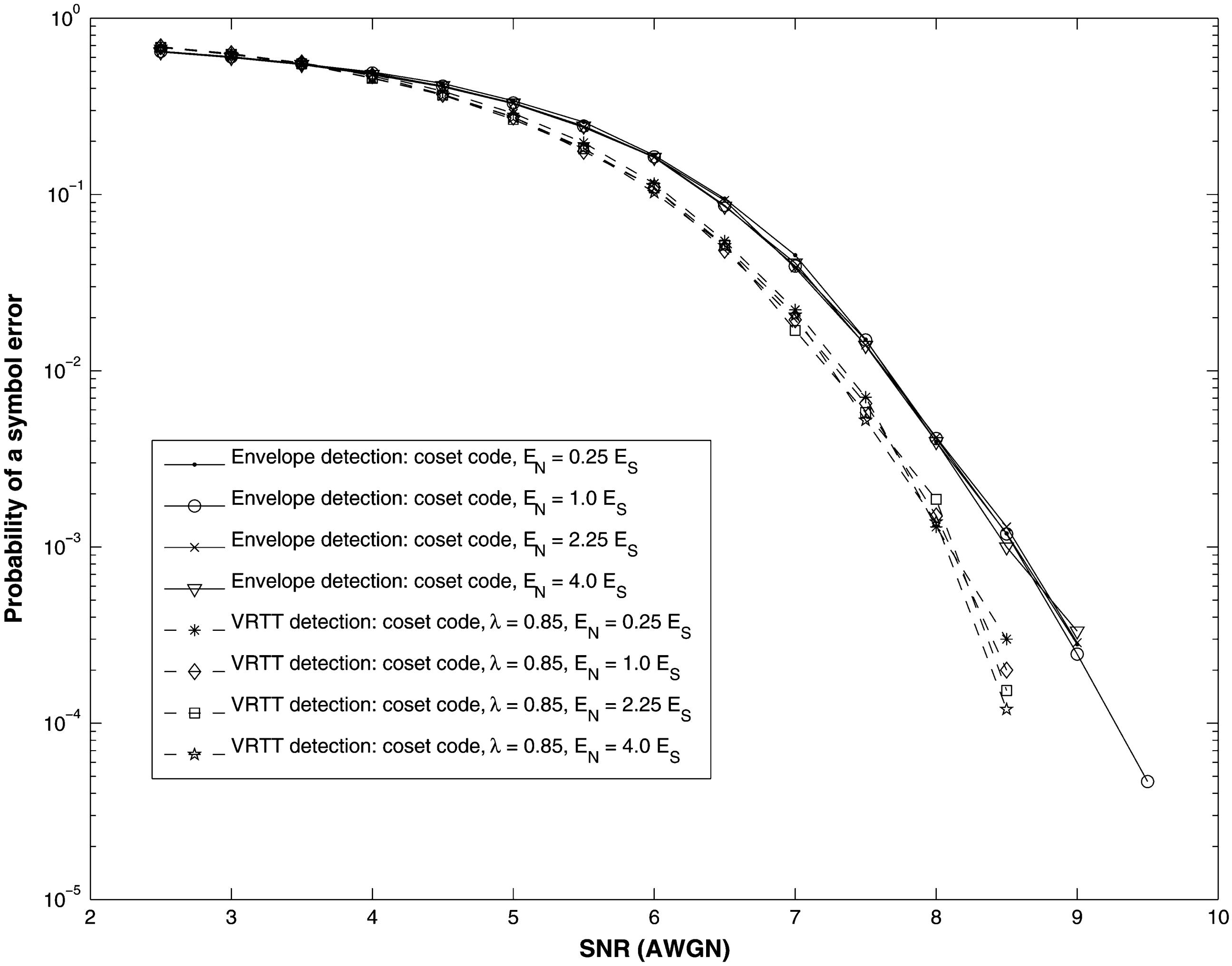

Fig. 5. Performance of modified envelope detection and modified Viterbi ratio threshold detection for different values of $E_{\mathcal{N}}$, (15, 3) code.

erasures whenever a codeword element agrees with the narrowband noise position.

VI. Conclusion

We gave a construction method for finding coset leaders resulting in coset codes with the minimum same-symbol weight. We also showed that the minimum same-symbol weight can be calculated as $n-d_2$. The coset codes outperform the linear codes from which they are constructed when used with M-FSK modulation in an environment where narrowband interference occurs. This point has been illustrated by simulations.

Acknowledgment

This material is based on work that was performed by D. J. J. Versfeld while he was on a DAAD scholarship in Germany.

The authors would like to thank Prof. J. Lahtonen for his help with Proposition 1. They would also like to acknowledge Prof. D. Sarwate, Prof. R. Pellikaan, and Dr. T. G. Swart for their invaluable inputs. The comments from the anonymous reviewers were also very helpful.

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