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On Synchronization for Burst Transmission*

A.J. Han VINCK† *and* A.J. van WIJNGAARDEN†, *Nonmembers*

SUMMARY We consider methods to locate sync words in packet or frame transmission over the additive white Gaussian noise channel. Our starting point is the maximization of the probability of correctly locating the sync word. We extend Massey's original result to the specific synchronization problem, where the sync word is prefixed to the data stream and each packet is preceded by idle transmission or additive white Gaussian noise. We give simulation results for several interesting sync words such as Barker sequences of length 7 and 13 and a sync word of length 17 with good cross-correlation properties. One of the conclusions is that the newly derived formula for the probability of correctly locating the sync word enables the reduction of the false sync detection probability.

key words: *synchronization, packet synchronization, maximum likelihood detection*

1. Introduction

In communication systems, where data packets are transmitted continuously the sender inserts a sync word at the beginning of each packet. This allows the receiver to determine the location of the packets in the detector output sequence by searching for the sync word using a correlator as a pattern recognizer. For AWGN channels, we first derive the optimum decision rule for the detection of the sync words, where, as indicated in [1], the random data surrounding the sync word has to be taken into account. Each packet, of fixed length N, starts with a sync word $c_0c_1\dots c_{L-1}$ of length L, followed by the data sequence $d_0\dots d_{N-L-1}$ of length $N-L$. We assume that the data bits d_i are statistically independent random variables satisfying $\Pr[d_i=-1]=\Pr[d_i=+1]=\frac{1}{2}$. At the receiver side, a span of N consecutive outputs $\boldsymbol{\rho}=\rho_0\rho_1\dots\rho_{N-1}$ from the detector is considered, where $\rho_i=\pm\sqrt{E_s}+n_i$, and E_s denotes the signal energy per symbol. Without loss of generality we choose $E_s=1$ in the remaining of the paper. The components n_i are independent zero-mean Gaussian random variables with variance $\sigma^2=N_o/2$. The optimum decision rule maximizes the probability of correctly locating the sync word, which is equivalent to choosing, as an estimate of the correct synchronization position m, $0\leq m<N$, the value μ which maximizes

$$\Pr[m=\mu|\boldsymbol{\rho}]=\frac{p(\boldsymbol{\rho}|m=\mu)\cdot\Pr[m=\mu]}{p(\boldsymbol{\rho})},$$

where $p(x)$ denotes the density function of the random variable x. We assume that there is no synchronization information available from previous packets, i.e., $\Pr[m=\mu]=1/N$. The received components following the sync word are statistically independent from the synchronization word and thus, using Bayes rule, the *optimum rule* for locating the sync word is obtained by maximizing

$$\frac{p(\boldsymbol{\rho}|m=\mu)}{p(\boldsymbol{\rho})}=\frac{p(\rho_\mu\rho_{\mu+1}\cdots\rho_{\mu+L-1}|m=\mu)}{p(\rho_\mu\rho_{\mu+1}\cdots\rho_{\mu+L-1})}.$$

For AWGN channels, this equation is simplified, and the constant terms which do not contribute to the decision rule are left out. The decision variable P_O to be used becomes

$$\begin{aligned}P_O(\mu)&=\prod_{i=0}^{L-1}\frac{2e^{-(\rho_{i+\mu}-c_i)^2/2\sigma^2}}{P_D(\rho_{i+\mu})}\\&=\prod_{i=0}^{L-1}\frac{2e^{\rho_{i+\mu}c_i/\sigma^2}}{e^{\rho_{i+\mu}/\sigma^2}+e^{-\rho_{i+\mu}/\sigma^2}},\end{aligned}\tag{1}$$

where

$$P_D(\rho_i)=e^{-(\rho_i-1)^2/2\sigma^2}+e^{-(\rho_i+1)^2/2\sigma^2}$$

denotes the likelihood term for the random data part.

Although the derivation of P_O is different from that given by Massey [1], the result is the same. The advantage of our approach is that we are able to extend the result to one-shot or burst transmission.

Another common rule, often used because of its simplicity, is the *correlation rule*, for AWGN channels defined as the maximization of

$$P_C(\mu)=\prod_{i=0}^{L-1}e^{\rho_{i+\mu}c_i/\sigma^2}.\tag{2}$$

For burst transmission we consider packets of length N, consisting of a sync word of length L, followed by a data word of length $N-L$. In the *burst mode*, packets are generated with a small probability of occurrence.

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†The authors are with the Digital Communications Group of the Institute for Experimental Mathematics, University of Essen, Ellernstr. 29, 45326 Essen, Germany.

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The packets are surrounded by additive white Gaussian noise with zero mean and variance σ^2. The synchronizer calculates for each possible packet starting position μ the a posteriori probability

$$P''_B = \frac{p(\boldsymbol{\rho}|m=\mu)}{p(\boldsymbol{\rho})}\ \Pr[m=\mu].$$

Since all starting positions are equally likely, we leave out $\Pr[m=\mu]$ and obtain for $\mu = 0$ the modified decision variable

$$P'_B = \frac{p(\rho_0\dots\rho_{L-1}|m=0)\cdot p(\rho_L\dots\rho_{N-1}|m=0)}{p(\rho_0\rho_1\dots\rho_{N-1})} = \frac{\prod_{i=0}^{L-1} e^{-(\rho_i-c_i)^2/2\sigma^2}\cdot \prod_{i=L}^{N-1}\frac{1}{2}P_D(\rho_i)}{\prod_{i=0}^{N-1}\left(\frac{1}{2}P_k\cdot P_D(\rho_i)+(1-P_k)\cdot e^{-\rho_i^2/2\sigma^2}\right)},$$

where P_k is the probability that a certain received symbol is part of a transmitted packet. For small P_k, we reduce P'_B for an arbitrary starting position μ to

$$P_B(\mu) = \prod_{i=0}^{L-1} e^{\rho_{i+\mu}c_i/\sigma^2}\cdot \prod_{i=L}^{N-1}\left(e^{\rho_{i+\mu}/\sigma^2}+e^{-\rho_{i+\mu}/\sigma^2}\right). \tag{3}$$

Hence, we have a normal correlation term multiplied with a correction term that depends on the entire received packet. At incorrect sync positions, the received values outside the packet do not contribute to P_B. It has been verified by simulations that this has a big impact on the false sync probability for short sync words.

To give an impression of both correlation rules P_C and P_B, packets of length 42 have been transmitted over an AWGN channel for which the signal-to-noise ratio (SNR) is equal to 0 dB. The packet occurrence probability is $P_P = 0.01$. Each packet contains the Barker sequence 1110010 of length 7, followed by 35 data bits. Figures 1 and 2 show the correlation $P_C(\mu)$ and the corresponding optimal correlation $P_B(\mu)$ for the first 1000 time instants. Packets occur at times 76, 162, 258, 301, 354, 646, and 700. One clearly sees the influence of using the received signal energy.

For practical implementations we need a detection method using a threshold. In Sect. 2 we consider the implementation of the detection rules, and derive bounds on the performance. In Sect. 3 we discuss the transmission of packets in burst mode, where packets have a small probability of occurrence. We give simulation results and comparisons with the classical correlation rule P_C.

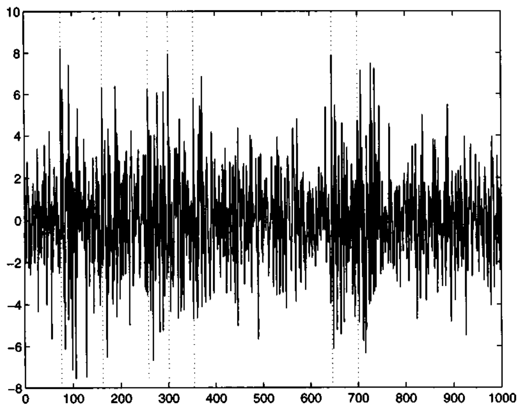

Fig. 1 Correlation $P_C(\mu)$, where $0 \leqq \mu < 1000$, for packets of length 42 and sync word 1110010 of length 7 at a signal-to-noise ratio of 0 dB.

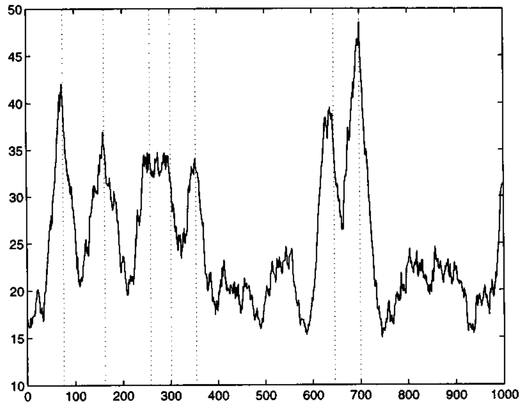

Fig. 2 Correlation $P_B(\mu)$, where $0 \leqq \mu < 1000$, for packets of length 42 and sync word 1110010 of length 7 at a signal-to-noise ratio of 0 dB.

2. Implementation of the Detection Rules

We describe a practical detection method based on P_C and P_B, respectively. We assume packets of length N to be transmitted over an AWGN channel. Every packet contains a fixed prefix or sync word of length L followed by $N-L$ information bits. We use sync words which have good autocorrelation properties, in particular Barker sequences [5]. As in [4], we do not consider bit-stuffing, or precoding [6], and thus allow single or multiple occurrences of the sync word within the packets. For the ordinary correlation based on P_C, we adopt the following detection method. For every possible synchronization time instant μ we calculate

$$L_C(\mu) = \sum_{i=0}^{L-1} \rho_{i+\mu}\cdot c_i. \tag{4}$$

If $L_C(\mu) \geqq T_C$, we synchronize at position μ. The value of T_C, a predefined threshold, will be discussed later. The following $N-1$ positions are discarded as possible packet starting positions. Fixing this specific sync position for N time instants is necessary, because the sync word may occur within the packets.

The detection method based on P_B is described as follows. For every possible synchronization time instant μ and threshold T_B, we calculate

$$L_B(\mu) = \sum_{i=0}^{L-1} \rho_{i+\mu} \cdot c_i + \sigma^2 \cdot \ln \prod_{i=L}^{N-1} 2\cosh(\rho_{i+\mu}). \tag{5}$$

If $L_B(\mu) \geqq T_B$, we assume correct synchronization *unless* for the $N-1$ following time instants a higher value of L_B occurs. We then assume correct synchronization for this latter position. For the detection methods based on P_C and P_B we assume that packets are separated by at least N or L idle transmissions, respectively. This assumption simplifies the performance calculations.

The performance of both methods can be expressed by the *false detection rate* and the *undetected sync rate*. The false detection rate is defined as the fraction of positions erroneously detected as being correct. The undetected sync rate is defined as the fraction of correct sync positions that are not detected as such.

We first consider the detection method based on P_C. During idle periods, the probability $\Pr[L_C(\mu) \geqq T_C]$ is equal to the probability that the sum of L independent Gaussian distributed random variables, each with mean 0 and variance σ^2, exceeds the threshold T_C. For $T_C = L/2$, we may bound $\Pr[L_C(\mu) \geqq T_C]$ as

$$\Pr[L_C(\mu) \geqq L/2] < \exp\left(-\frac{L}{8\sigma^2}\right). \tag{6}$$

For the detection method based on P_C, an undetected synchronization error at time μ occurs whenever $L_C(\mu) < T_C$ or whenever $L_C(\mu - i) \geqq T_C$, $0 < i < N$. We then exclude the correct sync position as a possible candidate for synchronization.

For the correct sync position, the expected value for $L_C(\mu) = L$ and the probability $\Pr[L_C(\mu) < L/2]$ is thus equal to the probability that the sum of L independent Gaussian distributed random variables, each with mean 0 and variance σ^2, exceeds $L/2$. For $0 < i < L$, $L_C(\mu - i)$ depends on the structure of the sync word. For sync words with an aperiodic autocorrelation of the form

$$\sum_{i=0}^{L-j-1} |c_i \cdot c_{i+j}| = 0 \qquad \text{for } 0 < j < L, \tag{7}$$

(6) remains valid. The Barker sequences, well-known for having the best autocorrelation properties, are very close to this condition. For $L \leqq i < N$, we may use (6) to bound $\Pr[L_C(\mu - i) \geqq L/2]$.

With respect to the undetected sync rate, the optimum value of threshold T_C for high signal-to-noise ratios is $L/2$.

For the method based on P_B, the false detection rate can be estimated using the Chernoff bounding technique. We only consider idle periods for the false detection rate, since the packet occurrence probability is very low. During idle periods, (5) can be written as

$$\begin{aligned} L_B(\mu) &= \sum_{i=0}^{L-1} n_{i+\mu} + \sigma^2 \\ &\quad \cdot \sum_{i=L}^{N-1} \ln(e^{-n_{i+\mu}/\sigma^2} + e^{n_{i+\mu}/\sigma^2}) \\ &< \sum_{i=0}^{N-1} |n_{i+\mu}| + \sigma^2(N-L)\ln(2). \end{aligned}$$

Hence, during idle periods,

$$\begin{aligned} &\Pr[L_B(\mu) \geqq T_B] \\ &< \Pr\left[\left(\sum_{i=0}^{N-1} |n_{i+\mu}| + \sigma^2(N-L)\ln(2)\right) \geqq T_B\right]. \end{aligned} \tag{8}$$

Using the Chernoff bounding technique, we obtain for small values of σ^2,

$$\Pr[L_B(\mu) \geqq T_B] < \exp\left(-\frac{N}{8\sigma^2}\right), \tag{9}$$

where $T_B \to N/2$ when $\sigma^2 \to 0$. For our simulations, the value of threshold T_B is set to $N/2$. Comparing (9) and (6), we conclude that the gain in signal-to-noise ratio for the false detection rate is $10\log_{10}(N/L)$ dB. This can also be observed from the simulation results as presented in Figs. 5, 7, 9. For the detection method based on P_B, an undetected sync error occurs whenever for the correct sync position μ, $L_B(\mu) < T_B$, or $L_B(\mu - i) \geqq L_B(\mu) \geqq T_B$, $0 < |i| < N$. Obviously, the undetected sync error rate decreases when we decrease the threshold T_B. For the correct sync position,

$$\begin{aligned} L_B(\mu) &= L + \sum_{i=0}^{L-1} n_{i+\mu} \cdot c_i \\ &\quad + \sigma^2 \cdot \sum_{i=L}^{N-1} \ln\left(e^{-\rho_{i+\mu}/\sigma^2} + e^{\rho_{i+\mu}/\sigma^2}\right) \\ &> L + \sum_{i=0}^{L-1} n_{i+\mu} \cdot c_i + \sum_{i=L}^{N-1} |\rho_{i+\mu}|. \end{aligned} \tag{10}$$

For a threshold $T_B = N/2$ it can be shown that for the correct sync position

$$\Pr[L_B(\mu) < N/2] < \exp\left(-\frac{N}{8\sigma^2}\right). \tag{11}$$

The above analysis does not include the computation of $\Pr[L_B(\mu - i) \geqq L_B(\mu) | L_B(\mu) \geqq T_B]$, $0 < |i| < N$ for the method based on P_B in the correct sync position. This probablity depends on the structure of the data and the sync word. Probability $\Pr[L_B(\mu - i) \geqq L_B(\mu) | L_B(\mu) \geqq T_B]$, $0 < |i| < N$ is the dominant term in the undetected sync rate. Inspection of $L_B(\mu)$ shows that for high SNR, for $0 < |i| < N$, the smallest average value

$$\min_{0<|i|<N} E\left[L_B(\mu) - L_B(\mu - i)\right] \geqq L - 1,$$

when the sync word is a Barker sequence and μ is the correct sync position. As an example, for $\mu = 0$, and $L = 7$, the value

$$L_B(0) - L_B(-1) \to L + 2n_2 - 2n_4 + 2n_5 - 2n_6 + n_{N-1},$$

for high SNR. Since

$$\Pr[L_B(\mu - i) \geqq L_B(\mu) | L_B(\mu) \geqq T_B] < \Pr[L_B(\mu) - L_B(\mu - i) < 0], \qquad 0 < |i| < N,$$

the undetected sync rate for AWGN decreases negative exponentially with L as $\exp(-(L-1)/8\sigma^2)$. The difference in the undetected sync rate for $L = 7$ and $L = 13$ is about 3 dB, see also Fig. 10.

In the next section we give the simulation results for both detection methods.

3. Simulation Results

We simulated packet transmission with embedded sync words of different length. We considered two different simulations. The first simulation method divides transmission into blocks of length $N = 256$, where each block contains a packet (including the sync word) prefixed and suffixed by 256 idle transmissions, i.e., AWGN only. A sync error occurs if the correct sync position does not give the maximum value for P_B, see Fig. 3. The simulation results are shown for a Barker sequence of length 13 and a particular sequence of length 17 with good cross-correlation properties. We compare the normal correlation rule based on P_C with the rule based on P_B. The rule based on P_B clearly outperforms the correlation rule based on P_C.

In Fig. 4 we give the false acquisition probability for a Barker sync word of length 13 for several lengths of the packet. With increasing packet length, we have an increasing false sync probability, caused by the occurrence of the sync word within the data. We observe that the rule based on P_B is not sensitive to packet length variations.

To further investigate the performance of the synchronization rule, we simulated the transmission of packets of a fixed length and with probability P_P of a packet start after an idle transmission slot.

The results are given in Figs. 5–10. In Fig. 5 and Fig. 7, we show the number of incorrectly detected packets in the case where 10,000 packets of different length were transmitted with packet occurrence probability $P_P = 0.001$ using sync words of length 7 and 13, respectively. From Fig. 7, we see that the performance for the method based on P_C is much worse than the performance of the method based on P_B. The gain in SNR varies from 1 dB for $N = 26$ to 5 dB for $N = 91$. From (9) and (6), we expect a gain of $10 \log_{10}(7) = 8.45$ dB

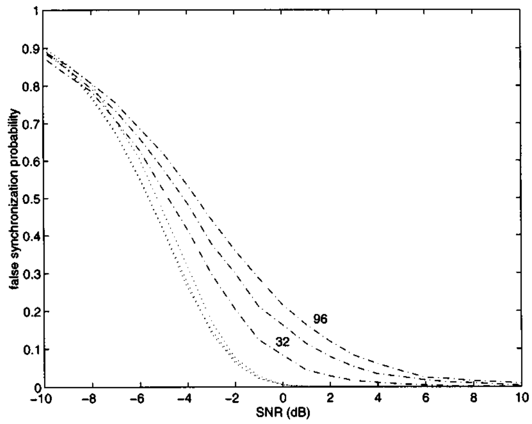

Fig. 4 False acquisition probability for packets of length 32, 64, and 96, and sync word 1111100110101 of length 13 using rule P_C (dashed lines) and P_B (dotted lines). Each packet is prefixed and suffixed by 128 idle transmissions.

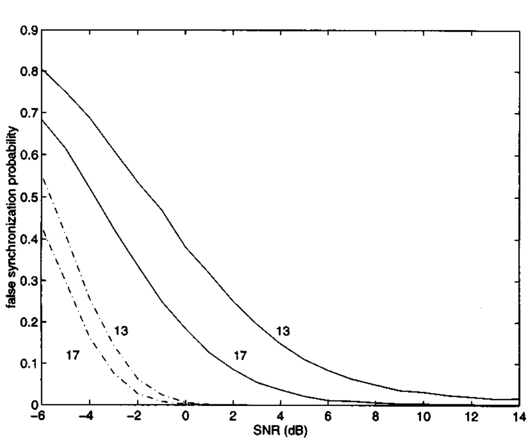

Fig. 3 False acquisition probability for packets of length $N = 256$ with sync words of length 13 and 17 using rule P_C (solid lines) and P_B (dashed lines). Each packet is prefixed and suffixed by 256 idle transmissions.

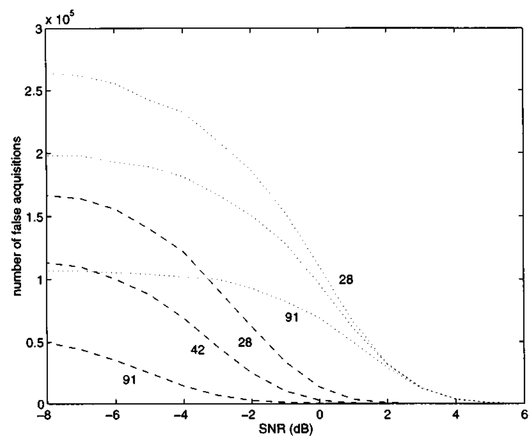

Fig. 5 Number of incorrectly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 28, 42, 91$ with sync word 1110010 of length 7 were transmitted with packet probability $P_P = 0.001$.

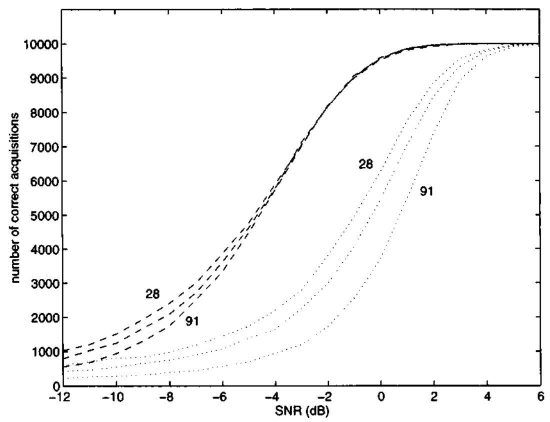

Fig. 6 Number of correctly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 28, 42, 91$ with sync word 1110010 of length 7 were transmitted with packet probability $P_P = 0.001$.

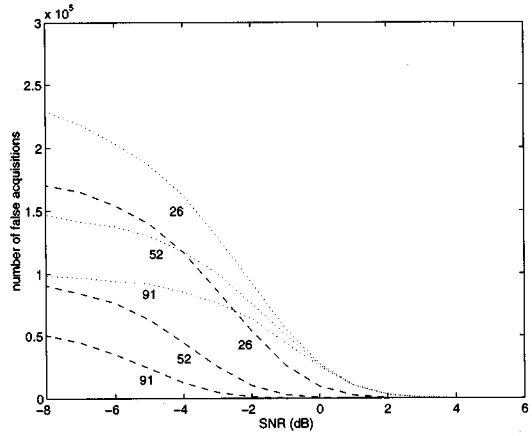

Fig. 7 Number of incorrectly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 26, 52, 91$ with sync word 1111100110101 of length 13 were transmitted with packet probability $P_P = 0.001$.

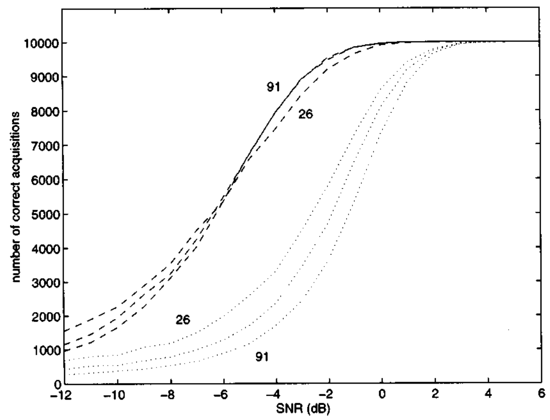

Fig. 8 Number of correctly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 26, 52, 91$ with sync word 1111100110101 of length 13 were transmitted with packet probability $P_P = 0.001$.

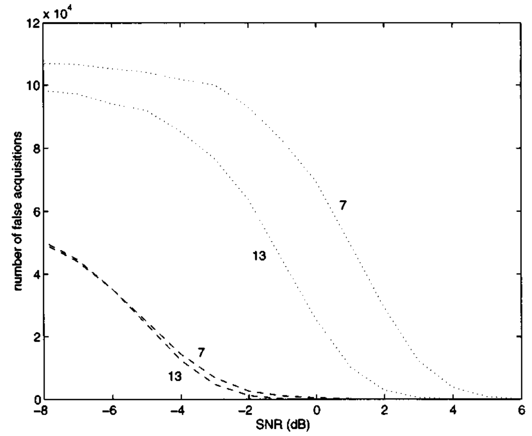

Fig. 9 Number of incorrectly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 91$ with sync word 1110010 and 1111100110101 of lengths 7 and 13 were transmitted with packet probability $P_P = 0.001$.

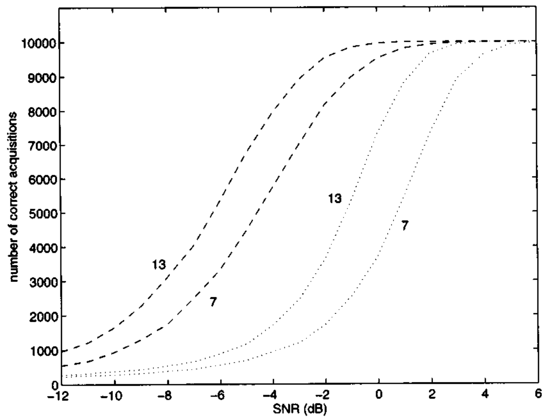

Fig. 10 Number of correctly detected packets using rule P_C (dotted lines) and P_B (dashed lines) in the case where 10,000 packets of length $N = 91$ with sync word 1110010 and 1111100110101 of lengths 7 and 13 were transmitted with packet probability $P_P = 0.001$.

for $N = 91$. We see a dependency for rule based on P_B on the block length N as predicted by (9). By increasing the block length N, the number of incorrectly detected packets is reduced.

In Fig. 9 we fix the blocklength $N = 91$ and vary the length of the sync word. The dependency of L for the rule based on P_C is in agreement with the bound as described by (6). Note that rule based on P_B is independent from the length of the sync word. The difference between the curves for $L = 7$ and $L = 13$ is about 2 dB and thus in agreement with (6).

In Fig. 6 and Fig. 8 we give the number of correctly detected packets in the case where 10,000 packets of different length were transmitted with packet occurrence probability 0.001 and sync words of length 7 and 13, respectively. For the detection method based on P_C, performance gets worse as the block length increases.

This is caused by the fact that more candidates may give a value of $L_C(\mu)$ above the threshold $L/2$ and thus exclude the correct sync position. One also clearly observes that for the method based on P_B, the block length hardly influences the performance. In Fig. 10, we see that for the undetected sync rate the dependency of L is dominant. The difference between the curves for $L = 7$ and 13 is about 2 dB.

4. Conclusion

We consider a method to allocate sync words in packet or frame transmission over the AWGN channel. We extend the method originally developed by Massey to the one-shot synchronization problem where the sync word is prefixed to the data stream and the packet is surrounded by idle transmissions. We give simulation results for several interesting sync words such as Barker sequences of length 7 and 13. The simulation results show that the extended rule makes the false sync probability almost independent of the packet length. The latter fact is of advantage for short sync words. Furthermore, it was verified by simulation that the newly derived rule significantly improves on the ordinary correlation rule.

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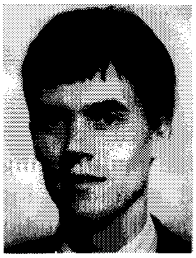
A.J. Han Vinck was born in 1949 in Breda, The Netherlands. He received the M.Sc. degree in 1974 and the Ph.D. degree in 1980 from the Eindhoven University of Technology, Eindhoven, The Netherlands. From 1985 to 1990, he was an Associate Professor at the University of Eindhoven, and became a Full Professor in 1990 at the University of Essen, Essen, Germany. From 1992 to 1994, he was Director of the Institute for Experimental Mathematics at the University of Essen. His research interests include digital communications. He organized the Veldhoven Workshop on Information Theory in 1990 and is Co-Chairman of the International Symposium on Information Theory (ISIT '97) in Ulm, Germany.



Adriaan J. van Wijngaarden was born in Amsterdam, The Netherlands, on June 23, 1968. He received the engineering (M.Sc. equivalent) degree in electrical engineering from the Eindhoven University of Technology, Eindhoven, The Netherlands, in 1992. He performed part of his study at Philips Research Laboratories, Eindhoven. Since 1992 he has been a Research Engineer of the Digital Communication Group at the Institute for Experimental Mathematics, University of Essen, Germany, where he is currently working towards the doctorate in engineering degree. During this period, he has also been a Visiting Scientist at the Communications Research Centre, Ottawa, Ont., Canada, and at the University of Electro-Communications, Tokyo, Japan. His research interests include frame synchronization, source and error control coding, and communication techniques.