Multi-User Power-line Communication

A.J. Han Vinck
Institute for Experimental mathematics
University Duisburg-Essen, Essen, Germany
Vinck@iem.uni-due.de

Bin Dai
Shanghai Jiaotong University
P.R. China
Daibin@sjtu.edu.cn

***Abstract*— We discuss the transmission over a power line channel that consists of two links. We determine the obtainable rates for time- , and frequency sharing with the "water-filling" argument. Then we show that the two link channel is equivalent to a degraded broadcast channel for which the capacity region is known. Power- and bandwidth constraints determine whether a repeater can improve performance. Another concept from multi-user Information theory, the multi access concept is useful for determining the transmission efficiency from multiple users to a single receiver.**

Keywords— Power-line; link attenuation; bus communication; time- and frequency sharing; degraded broadcast channel; multiple access

I. INTRODUCTION

One of the dominant parameters in power line communications is that of attenuation. Attenuation can vary between 10-100 dB per kilometer. Investigations to overcome the high attenuation are therefore highly important. Instead of increasing signaling power, we choose the application of repeaters. This will be realized assuming that communication participants are connected to the same main supply line (bus), see [1] and Fig. 1, where an injected signal can be observed by all connected users.

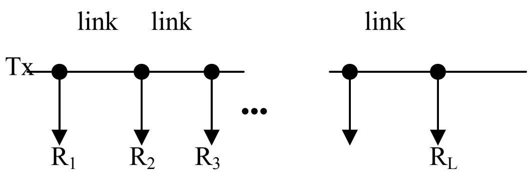

Fig. 1. Simplified bus topology.

To simplify the situation and to be able to connect the problem to known results in Information theory, we concentrate on a connection that consists of maximum two links that have the same properties. In our example, we consider the connection between Tx, R_1 and R_2.

The goal of the paper is to find the basic communication models and their communication efficiencies. To illustrate the problem, we replace every link by an AWGN channel with signal attenuation factor f and noise variance $N_0/2$. A transmitter uses PSK modulation with E_b the energy per information bit. The detection error probability for the single link connection Tx→R_1 is then given by the Gaussian Q function

$$P_1 \propto Q\left(\sqrt{\frac{E_b}{N_0} f^2}\right) .$$

For the connection Tx→R_2 we have a connection using two links: thus the attenuation factor is f^2 and the detection error probability is given by

$$P_2 \propto Q\left(\sqrt{\frac{E_b}{N_0} f^4}\right) \approx P_1^{f^2} .$$

We see a dramatic degradation in performance.

In this contribution, we first look at the communication from one transmitter to two receivers, Fig. 3a, and give the efficiency for time sharing (TS) as a reference value for further comparisons. For frequency sharing (FS), we use the water-filling argument [5], to find the optimum frequency sharing power allocation that maximizes the capacity.

We use principles from Information theory to translate the channel, consisting of two links, into a degraded broadcast channel [2] and calculate its capacity. This capacity improves on the performance for TS and FS. However, practical coding methods for broadcast channels are not known (yet) and we conclude that for the 2-link channel without repeater, time sharing is the best option.

In [1] we describe the use of a repeater and its consequences for the transmission efficiency. It is interesting to compare the repeater strategy with the broadcast results and determine the conditions for which it is beneficial to use a repeater.

We introduce a multiple access model for the communication between two transmitters and one receiver, see Fig. 4b. Information theoretical principles are used to show equivalence between the broadcast and the multiple access models.

II. COMMMUNICATION ASPECTS

We investigate the communication concepts for two links: TS, FS, repeating, Broadcast and Multiple Access (MAC). For simplicity, we assume that the attenuation factor f, $f < 1$, is the same for both links. The "background" noise is considered to be Additive White Gaussian Noise (AWGN) with power spectral density σ^2 W/Hz and average value 0. The bandwidth and transmit power are denoted by B and P, respectively.

The channel capacity $C(T\rightarrow R_1)$ for a single link and $C(T\rightarrow R_2)$ for 2 links, see also Fig. 3a, are given by

$$C(T \rightarrow R_1) = B\ln\left(1+\frac{Pf^2}{2\sigma^2 B}\right) \text{ nat/s,}$$

$$C(T \rightarrow R2) = B\ln\left(1+\frac{Pf^4}{2\sigma^2 B}\right) \text{ nat/s.}$$

All further derived transmission efficiencies are expressed in the same way.

A. Time sharing(TS)

If we use TS with parameter α from transmitter to both the receivers, then the achievable communication rate or transmission efficiency is given by

$$R_{TS}(T \rightarrow R_1) = \alpha B\ln\left(1+\frac{Pf^2}{2\sigma^2 B}\right) \text{ nat/s,}$$

$$R_{TS}(T \rightarrow R_2) = \bar{\alpha} B\ln\left(1+\frac{Pf^4}{2\sigma^2 B}\right) \text{ nat/s,}$$

$$0 \leq \alpha \leq 1; \bar{\alpha} = 1-\alpha.$$

B. Frequency sharing(FS)

For FS, we use the water-filling concept from information theory [5] to obtain the power allocation that maximizes the efficiencies, i.e. we have

$$R_{FS}(T \rightarrow R_1) = \beta B\ln\left(1+\frac{P_1 f^2}{2\beta\sigma^2 B}\right) \text{ nat/s,}$$

$$R_{FS}(T \rightarrow R_2) = \gamma B\ln\left(1+\frac{P_2 f^4}{2\gamma\sigma^2 B}\right) \text{ nat/s,}$$

where $P = P_1 + P_2$, $\gamma + \beta \leq 1$.

The "naïve" approach is to make $P_1 = \beta P$, $P_2 = \gamma P$ and $\beta + \gamma = 1$. We then have the same region as for time sharing. To obtain the efficiency region for FS using the water-filling argument, we have to find the maximum sum value as a function of β and γ [7]. The general behavior is illustrated by the dashed line in Fig. 5. Analyzing the results from A and B, we conclude that FS is always better than TS, see also [7]. Note that both efficiency regions have the same boundary points.

C. Receiver R_1 acts as repeater

If receiver R_1 can act as an intelligent repeater, a simple decoding and re-encoding strategy would give a transmission rate region

$$R_{REP}(T \rightarrow R_1) = \alpha B\ln\left(1+\frac{Pf^2}{2\sigma^2 B}\right) \text{ nat/s}$$
$$= \alpha C_1 \text{ nats/s,}$$

$$R_{REP}(T \rightarrow R_2) = \frac{\bar{\alpha}}{2} B\ln\left(1+\frac{Pf^2}{2\sigma^2 B}\right) \text{ nat/s,}$$

$$= \frac{\bar{\alpha}}{2} C(T \rightarrow R_1) \text{ nats/s,}$$

where α acts as a TS parameter. Comparing the limiting points, i.e. for $\alpha = 0$ or $\alpha = 1$, we conclude that repeating is worse than TS for

$$\frac{P}{\sigma^2 B} > \frac{1-2f^2}{f^6}.$$

We see that in a situation where power is large and bandwidth small, TS is better than repeating. For small f, repeating is preferable.

Example For $\sigma^2 = 10^{-8}$ W/Hz and $B = 10^5$ Hz, $P = 25$ W, an attenuation factor $f > 0.2$ satisfies the condition.

We have shown in [1] that the performance for the repetition strategy can be improved to

$$R^*_{REP}(T \rightarrow R_2) = \bar{\alpha}\frac{C(T \rightarrow R_1)}{2-\frac{C(T \rightarrow R_2)}{C(T \rightarrow R_1)}} \text{ nat / s.}$$

In general, this improved repetition strategy needs a complex encoding and decoding strategy. In a practical situation, we can use Reed-Solomon (R-S) codes. Suppose that the link error rate is upper bounded by t_1 and the two link error rate is upper bounded by t_2, where $½ > t_2 > t_1$. The dimensions of the R-S canonical encoding matrix that we are going to use, are given in Fig. 2. The relations between the dimensions and error rate are as follows:

$$m - (k_1 + k_2) = 2t_1 m, \quad (1a)$$

$$m - k_1 = 2t_2 m, \quad (1b)$$

$$m_2 - k_2 = 2t_1 m_2. \quad (1c)$$

The encoded information to be transmitted to user R_2 is given by $k = (k_1 + k_2)$ symbols. The communication uses three steps.

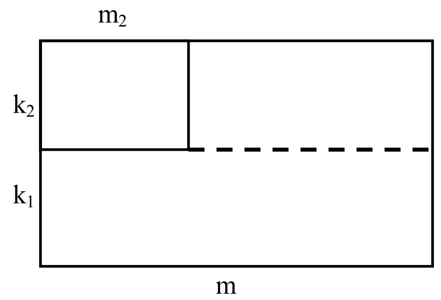

Fig. 2. Dimensions for the R-S encoding.

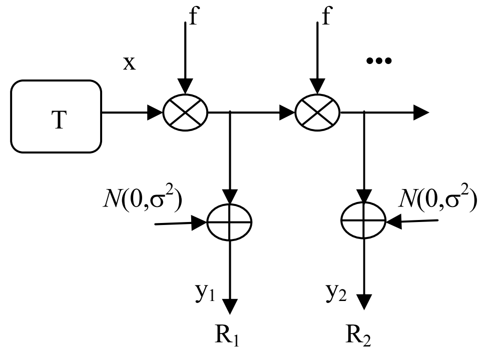

Fig. 3a. One transmitter to two receivers.

1) In the first step we encode k information symbols and transmit the m code symbols to R_1. Using (1a), R_1 can decode. Receiver R_2 receives a noisy version of the code word with a maximum of t_2m errors, which is assumed to be beyond the error correcting capability of the code with the given parameters.

2) In the second step, receiver R_1 forwards the code word, that corresponds to k_2 information symbols, of length m_2. Using the R-S encoder matrix properties, this is again an R-S code word.

3) From (1c), it follows that R_2 is able to decode the k_2 information symbols. Since R_2 can subtract the influence of the decoded information from the code word in the first step, R_2 is able to decode the remaining k_1 information symbols using property (1b).

Using (1), we can evaluate the transmission efficiency for simple repeating and R-S encoded retransmission as

$$R_{REP}(T \to R_2) = \frac{k}{2m},$$

$$R_{R-S}(T \to R_2) = \frac{k}{m + m\dfrac{2t_2 - 2t_1}{1 - 2t_1}}.$$

Since $½ > t_2 > t_1$, we can see the improvement that depends on the error rate t_2 and t_1. For t_2 close to t_1 the efficiency is close to the situation without repeater. For t_2 close to ½, the un-coded repeating efficiency appears.

Remark For error rates larger than t_2 or t_1, we have to accept decoding errors.

D. The Gaussian Broadcast channel

The two link model can be transformed into a broadcast model, i.e.

$$y_1 = x\,f + n_1,$$

$$y_2 = x\,f^2 + n_2,$$

where n_1 and n_2 represent the Gaussian noise with variance σ^2 and the input average power constraint is given by $E(X^2) \le P$, see Fig. 3b.

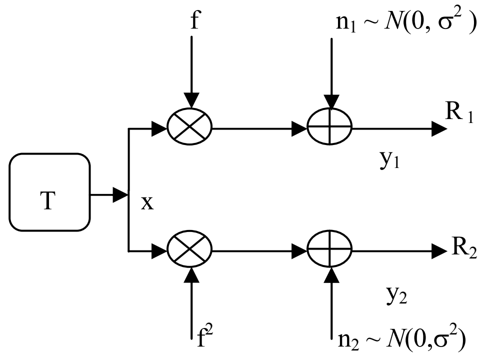

Fig. 3b. The 2-link broadcast model.

The capacity region for the Gaussian broadcast channel [2] is characterized by

$$R_{BC}(T \to R_1) = B\ln\left(1 + \frac{\gamma P f^2}{2\sigma^2 B}\right) \text{ nat/s},$$

$$R_{BS}(T \to R_2) = B\ln\left(1 + \frac{\bar{\gamma} P f^4}{2\sigma^2 B + \gamma P f^4}\right) \text{ nat/s}.$$

Note that for $(\gamma,\alpha) = (1,1)$ or $(0,0)$ we have the same result for the broadcast efficiency as for TS and FS. However, simple calculations show that the capacity region for the broadcast channel is strictly larger than the time sharing result for all other values of γ. The rate regions approach each other for $W \to \infty$. This conclusion can also be found in [4].

Example For $\sigma^2 = 10^{-8}$ W/Hz, $B = 10^5$ Hz, $P = 25$ W, $f = 0.3$, $\alpha = 1/2$ and $\gamma = 0.02$ we obtain

$$R_{TS}(T \to R_1) = R_{BC}(T \to R_1),$$

$$R_{BC}(T \to R_2) \approx 2\, R_{TS}(T \to R_2).$$

Hence, we obtain a clear improvement of the broadcast efficiency over TS. The challenging problem is to design practical coding strategies that achieve this gain in efficiency. This is still an open problem.

In [2] a general bus model with more than 2 receivers is given. This can lead to solutions for the communication problem connected to the bus situation in Fig. 1, where L links are between transmitter Tx and receiver R_L. In this general case, the broadcasting capacity is given by

$$R_{BC}(T \to R_1) = B\ln\left(1+\frac{\alpha_1 P f^2}{2\sigma^2 B}\right) \text{ nat/s},$$

$$R_{BS}(T \to R_2) = B\ln\left(1+\frac{\alpha_2 P f^4}{2\sigma^2 B+\alpha_1 P f^4}\right) \text{ nat/s},$$

$$R_{BS}(T \to R_3) = B\ln\left(1+\frac{\alpha_3 P f^6}{2\sigma^2 B+\alpha_1 P f^6+\alpha_2 P f^6}\right) \text{ nat/s},$$

• • • ,

where $\alpha_1 + \alpha_2 + \bullet\bullet\bullet + \alpha_N = 1$. Calculation of the capacity then becomes an optimization problem.

E. The Multiple Access Channel (MAC)

The two transmitter one receiver concept of Fig. 4a, can be transformed into a multiple access channel as shown in Fig. 4b. The output y at the receiver can be written as

$$y = f x_1 + f^2 x_2 + n, \tag{2}$$

where n is the additive white Gaussian noise with variance σ^2. The average power is $P = P_1 + P_2$, where $P_1 = \delta P$; $P_2 = (1-\delta)P$. The capacity region for the two access channel [6], is given by

$$R_{MAC}(T_1 \to R) = B\ln\left(1+\frac{\delta P f^2}{2\sigma^2 B}\right) \text{ nat/s},$$

$$R_{MAC}(T_2 \to R) = B\ln\left(1+\frac{\bar{\delta} P f^4}{2\sigma^2 B+\delta P f^4}\right) \text{ nat/s}.$$

To achieve the capacity, the first receiver first decodes x_2, then subtracts x_2 from y and decodes x_1.

As was observed in [6], the multiple access - and the broadcast capacity region can be shown to be equivalent when the sum of the average power in the two transmitters is the same as the sum power in the broadcast channel model. As a consequence, any point in the capacity region for the broadcast channel can also be achieved in the multiple access channel capacity region. The proof is based on the transformation of power from the access to the broadcast domain.

For TS and FS, we refer to the first part of the section. The results are the same, since for this situation, the links in the bus can be considered to be point-to-point connections.

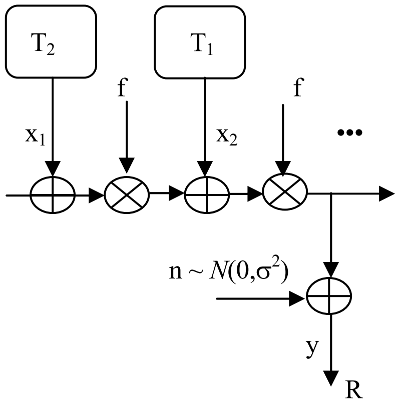

Fig. 4a. Two transmitters to one receiver.

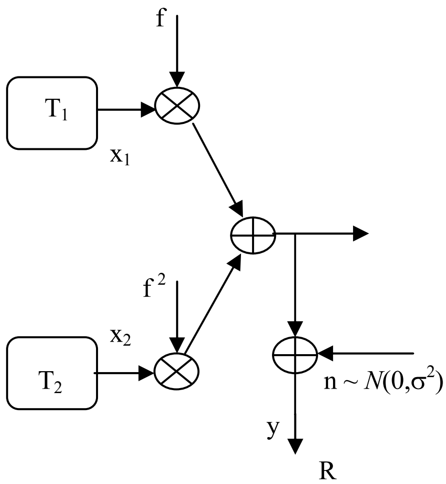

Fig. 4b. The 2-link multiple access model.

III Results and Conclusions

In Fig. 5 we present the results for the different strategies in a general way. FS and TS are the frequency- and the time sharing regions, respectively. We also indicate the performance for the degraded broadcast channel together with the repeating strategies. Of course, the different points have to be evaluated for particular link channel parameters and conditions.

We show that FS is preferable over TS with water-filling optimization. TS also out- performs repeating for specific values of the link attenuation factor f.

From the presented material, it follows that there is an interesting connection between multi-user Information theory and real communications, such as communication over multiple links.

REFERENCES

[1] Lutz Lampe and A.J. Han Vinck, "On Cooperative Coding for Narrowband PLC Networks," AEÜ, 2011

[2] P. P. Bergmans, "A simple converse for broadcast channels with additive white Gaussian noise," IEEE Trans. on Information Theory, Vol. IT-20, pp. 279-280, 1974

[3] V. Balakirsky and A.J. Han Vinck, "Potential performance of PLC systems composed of several communication links," ISPLC 2005, p.12–16, 2005

[4] R.J. McEliece and L. Swanson, "A note on the wide-band Gaussian broadcast channel," IEEE Transactions on Communications, Vol. Com 35, pp. 452-453, 1987

[5] T.M. Cover and Joy Thomas, *Elements of Information Theory*, Wiley Series in Telecommunications and Signal Processing

[6] N. Jindal, S. Vishwanath and A. Goldsmith, "On the Duality of Gaussian Multiple-Access and Broadcast Channels," IEEE Transactions on Information Theory, Vol. 50, No. 5, pp. 768-783, May 2004

[7] P. P. Bergmans and T. M. Cover, "Cooperative broadcasting," IEEE Transactions on Information Theory, vol. IT-20, pp. 317–324, May 1974

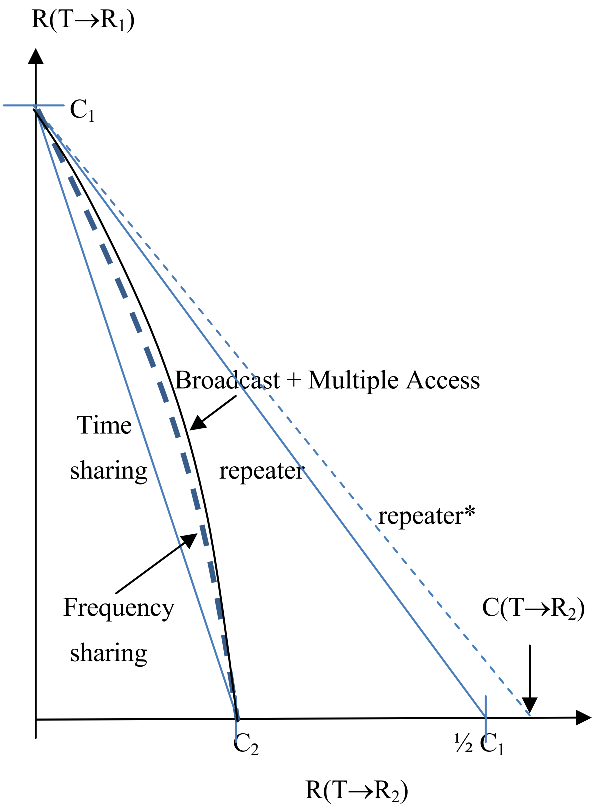

Fig. 5. Overview of efficiencies.