Mohammed Ben Musa
(genannt 'al-Khwarizmi')

Auszug aus
„The algebra of Mohammed ben Musa”
(ca. 800 n.C.)

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THE AUTHOR'S PREFACE.

IN THE NAME OF GOD, GRACIOUS AND MERCIFUL! This work was written by MOHAMMED BEN MUSA, of KHOWAREZM. He commences it thus:

Praisd be God for his bounty towards those who deserve itby their virtuous acts: in performing which, as by him prescribed to his adoring creatures, we express our thanks, and render ourselves worthy of the continuance (of his mercy), and preserve ourselves from change: acknowledging his might, bending before his power, and revering his greatness! He sent MOHAMMED (on whom may the blessing of God repose!) with the mission of a prophet, long after any messenger from above had appeared, when justice had fallen into neglect, and when the true way of life was sought for in vain. Through him he cured of blindness, and saved through him from perdition, and increased through him what before was small, and collected through him what before was scattered. Praised be God our Lord! and may his glory increase, and may all his names be hallowed — besides whom there is no God; and may his benediction rest on MOHAMMED the Prophet and on his descendants!

The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge, bearing in mind those that were to come after them, and hoping for a reward proportionate to their ability, and trusting that their endeavours would meet with acknowledgment, attention, and remembrance — content as they were even with a small degree of praise; small, if compared with the pains which they had undergone, and the difficulties which they had encountered in revealing the secrets and obscurities of science.

Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method (of study), or rendered the access (to science) easier or placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-labourers, without arrogance towards them, or taking pride in what they did themselves.

That fondness for science, by which God has distinguished the IMAN AL MAMUN, the Commander of the Faithful (besides the caliphat which He has vouchsafed unto him by lawful succession, in the robe of which He has invested him, and with the honours of which He has adorned him), that affability and condescension which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, — has encouraged me to compose a short
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work on Calculating by (the rules of) Completion and Reduction confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned — relying on tbe goodness of my intention therein, and hoping that the learned will reward it, by obtaining (for me) through their pragers the excellence of the Divine mercy: in requital of which, may the choicest blessings and the abundant bount of God be theirs! My confidence rests with God, in this as in every thing, and in Him I put my trust. He is the Lord of the Sublime Throne. May His blessing descend upon all the prophets and heavenly messengers!

[...]

I found that these three kinds; namely, roots, squares, and numbers, may be combined together, and thus three compound species arise;[1] that is, "squares snd roots equal to numbers;" "squares and numbers equal to roots ;" "roots and numbers equal to squares."

Roots and Squares are equal to Numbers;[2] for instance, "one square, and ten roots of the same, amount to thirty-nine dirhems;" that is to say, what must be the square which, when increased by ten of its own roots, amount to thirty-nine? The solution is this: you halve the number[3] of the roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this, which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for; the square itself is nine.

Squares and Numbers.are equal to Roots;[4] for instance, "a square and twenty-one in number are equal to ten roots of the same square." That is to say, what must be the amount of a square, which, when twenty-one dirhems are added to it, becomes equal to the equivalent of ten roots of that square? Solution: Halve the number of the roots; the moiety is five. Multiply this by itself; the product is twenty-five. Subtract from this the twenty-one which are connected with the square; the remainder is four. Extract its root; it is two. Subtract this from the moiety of the roots, which is five; the remainder is three. This is the root of the square which you required, and the square is nine. Or you may add the root to the moiety of the roots; the sum is seven; this is the root of the square which you sought for, and the square itself is forty-nine.

When you meet with an instance which refers you to this case, try its solution by addition, and if that do not serve, then subtraction certainly will. For in this case both addition and subtraction may be employed, which will not answer in any other of the three cases in which the number of the roots must be halved. And know, that, when in a question belonging to this case you have halved the number of the roots and multiplied the moiety by itself, if the product be less than the number of dirhems connected with the square, then

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[1] The three cases considered are,

1st. $cx^2 + bx = a$

2d: $cx^2 + a = bx$

3d: $cx^2 = bx + a$

[2] 1st case: $cx^2 + bx = a$

Example: $x^2 + 10x = 39$

$$x = \sqrt{\left(\tfrac{10}{2}\right)^2 + 39} - \tfrac{10}{2} = \sqrt{64} - 5 = 8 - 5 = 3$$

[3] i.e. the coefficient.

[4] 2d. case: $cx^2 + a = bx$

Example: $x^2 + 21 = 10x$

$$x = \frac{10}{2} \pm \sqrt{\left(\tfrac{10}{2}\right)^2 - 21} = 5 \pm \sqrt{25 - 21} = 5 \pm \sqrt{4} = 5 \pm 2$$

the instance is impossible;[5] but if the product be equal to the dirhems by themselves, then the root of the square is equal to the moiety of the roots alone, without either addition or subtraction.

In every instance where you have two squares, or more or less, reduce them to one entire square,[6] as I have explained under the first case.

Roots and Numbers are equal to Squares;[7] for instance, "three roots and four of simple numbers are equal to a square." Solution: Halve the roots; the moiety is one and a half. Multiply this by itself; the product is two and a quarter. Add this to the four; the sum is six and a quarter. Extract its root; it is two and a half. Add this to the moiety of the roots, which was one and a half; the sum is four. This is the root of the square, snd the square is sixteen.

Whenever you meet with a multiple or sub-multiple of a square, reduce it to one entire square.

These are the six cases which I mentioned in the introduction to this book. They have now been explained. I have shown that three among them do not require that the roots be halved, and I have taught how they must be resolved. As for the other three, in which halving the roots is necessary, I think it expedient, more accurately, to explain them by separate chapters, in which a figure will be giren for each case, to point out the reasons for halving.

Demonstrations of the Case: "a Square and ten Roots are equal to thirty-nine Dirhems."[8]

The figure to explain this a quadrate, the sides of which are unknown. It represents the square, the which, or the root of which, you wish to know. This is the figure A B, each side of which may be considered as one of its roots; and if you multiply one of these sides by any number, then the amount of that number may be looked upon as the number os the roots which are added to the square. Each side of the quadrate represents the root of the square; and, as in the instance, the roots were connected with the square, we may take one-fourth of ten, that is to say, two and a half, and combine it with each of the four sides of the figure. Thus with the original quadrate A B, four new parallelograms are combined, each having a side of the quadrate as its length, and the number of two and a half as its breadth; they are the parallelograms C, G, T, and K. We have now a quadrate of equal, though unknown sides; but in each of the four corners of which a square piece of two and a half multiplied by two and a half is wanting. In order to compensate for this want and to complete the quadrate, we must add (to that which we have already) four times the square of two and a half, that is, twenty-five. We know (by the statement) that the first figure, namely, the quadrate representing the square, together with the four parallelograms around it, which represent the ten roots, is equal to thirty-nine of numbers. If to this we add twenty-five,which is the equivalent of the four quadrates at the corners of the figure A B, by which the great figure D H is completed, then we know that this together makes

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[5] If in an equation, of the form $x^2 + a = bx, \left(\frac{b}{2}\right)^2 < a$, the case supposed in the equation cannot happen. If $\left(\frac{b}{2}\right)^2 = a$ then $x = \frac{b}{2}$.

[6] $cx^2 + a = bx$ is to be reduced to $x^2 + \frac{a}{c} = \frac{b}{c}x$.

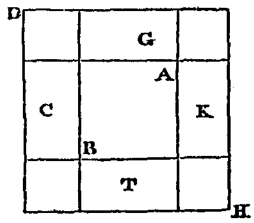
[7] 3d case: $cx^2 = bx + a$

Example: $x^2 = 3x + 4$

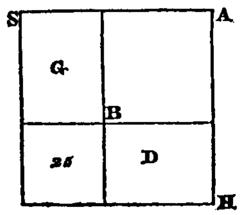
$$x^2 = \sqrt{\left(\tfrac{3}{2}\right)^2 + 4} + \tfrac{3}{2} = \sqrt{\left(1\tfrac{1}{4}\right)^2 + 4} + 1\tfrac{1}{2} = \sqrt{2\tfrac{1}{4} + 4} + 1\tfrac{1}{2} = \sqrt{6\tfrac{1}{4}} + 1\tfrac{1}{2} = 2\tfrac{1}{2} + 1\tfrac{1}{2} = 4$$

[8] Geometrical illustration of the case, $x^2 + 10x = 39$.

sixty-four. One side of this great quadrate is its root, that is, eight. If we subtract twice a fourth of ten, that is five, from eight, as from the two estremities of the side of the great quadrate D H, then the remainder of such a side will be three, and that is thee root of the square, or the side of the original figure A B. It must be observed, that we have halved the number of the roots, and added the product of the moiety multiplied by itself to the number thirty-nine, in order to complete the great figure in its four corners; because the fourth of any number multiplied by itself, and then by four, is equal to tbe product of the moiety of that number multiplied by itself.[9] Accordingly, we multiplied only the moiety of the roots by itself, instead of multiplying ist fourth by itself, and then by four. This is the figure:



The same may also be explained by another figure. We proceed from the quadrate AB, which represents the square. It is our next business to add to it the ten roots of the same. We halve for this purpose the ten, so that it becomes five, and construct two quadrangles on two sides of the quadrate A B, namely, G and D, the length of each of them being five, as the moiety of the ten roots, whilst the breadth of each is equal to a side of the quadrate A B. Then a quadrate remains opposite the corner of the quadrate A B. This is equal to five multiplied by five: this five being half of the number of the roots which ve have added to each of tbe two sides of the first quadrate. Thus we know that the first quadrate, which is the square, and the two quadrangles on its sides, which are the ten roots, make together thirty-nine. In order to complete the great quadrate, there wants only a square of five multiplied by five, or twenty-five. This we add to thirty-nine, in order to complete the great square S H. The sum is sixty-four. We extract its root, eight, which is one of the sides of the great quadrangle. By subtracting from this the same quantity whichwe have before added, namely five, ve obtain three as the remainder. This is the side of the quadrangle A B, which represents the square; it is the root of this square, and the square itself is nine. This is the figure:—



Demonstration of the Case: "a Square and twenty-one Dirhems are equal to ten Roots."[10]

We represent the square by a quadrate A D, the length of whose side we do not know. To this we join a parallelogram, the breadth of which is equal to one of the sides of the quadrate A D, such as the side H N. This paralellogram is H B. The length of the two

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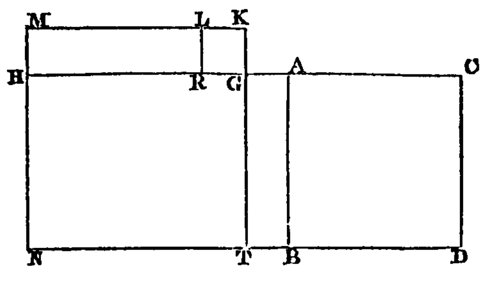
9 $4 \times \left(\frac{b}{4}\right)^2 = \left(\frac{b}{2}\right)^2$

10 Geometrical illustration of the case, $x^2 + 21 = 10x$.

figures together is equal to the line H C. We know that its length is ten of numbers; for every quadrate has equal sides and angles, and one of its sides multiplied by a unit is the root of the quadrate, or multiplied by two it is twice the root of the same. As it is stated, therefore, that a square and twenty-one of numbers are equal to ten roots, we may conclude that the length of the line H C is equal to ten of numbers, since the line C D represents the root of the square. We now divide the line C H into two equal parts at the point G: the line G C is then equal to H G. It is also evident that the line G T is equal to the line C D. At present we add to the line G T, in the same direction, a piece equal to the difference between C G and G T, in order to complete the square. Then the line T K becomes equal to K M, and we have a new quadrate of equal sides and angles, namely, the quadrate M T. We know that the line T K is five; this is consequently the length also of the other sides: the quadrate itself is twenty-five, this being the product of the multiplication of half the number of the roots by themselves, for five times five is twenty-five. We have perceived that the quadrangle E B represents the twenty-one of numbers which were added to the quadrate. We have then cut off a piece from the quadrangle H B by the line K T (which is one of the sides of the quadrate M T), so that only the part T A remains. At present we take from the line K M the piece K L, which is equal to G K; it then appears that the line T G is equal to M L; moreover, the line K L, which has been cut off from K M, is equal to K G; consequently, the quadrangle M R is equal to T A. Thus it is evident that the quadrangle H T, augmented by the quadrangle M R, is equal to the quadrangle H B, which represents the twenty-one. The whole quadrate M T was found to be equal to twenty-five. If we now subtract from this quadrate, M T, the quadrangles H T and M R, which are equal to twenty-one, there remains a small quadrate K R, which represents the difference between twenty-five and twenty-one. This is four; and its root, represented by the line R G, which is equal to G A, is two. If you subtract this number two from the line C G, which is the moiety of the roots, then the remainder is the line A C; that is to say, three, which is the root of the original square. But if you add the number two to the line C G, which is the moiety of the number of the roots, then the sum is seven, represented by the line C R, which is the root to n larger square. However, if you add twenty-one to this square, then the sum will likewise be equal to ten roots of the same square. Here is the figure:—

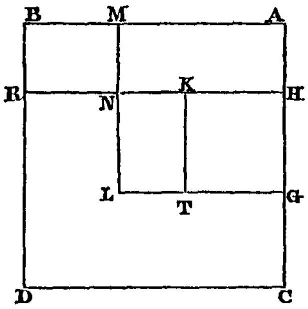


Demonstration of the Case: "three Roots and four of Simple Numbers are equal to a Square."[11]

Let the square be represented by a quadrangle, the sides of which are unknown to us, though they are equal among themselves, as also the angles. This is the quadrate A D, which comprises the three roots and the four of numbers mentioned in this instance. In every quadrate one of its sides, multiplied by a unit, is its root. We now cut of the quadrangle H D from the quadrate A D, and take one of its sides H C for three, which is the number of the roots. The same is equal to R D. It follows, then, that the quadrangle H

[11] Geometrical illustration of the 3d case, $x^2 = 3x + 4$.

B represents the four of numbers which are added to the roots. Now we halve the side C H, which is equal to three roots, at the point G; from this division we construct the square H T, which is the produot of half the roots (or one and a half) multiplied by themselves, that is to say, two and a quarter. We add then to the line G T a piece equal; to the line A H, namely, the piece T L; accordingly the line G L becomes equal to A G, and the line K N equal to T L. Thus a new quadrangle, with equal sides and angles, arises, namely, the quadrangle G M; and we find that the line A G is equal to M L, and the same line A G is equal to G L. By these means the line C G remains equal to N R, and the line M N equal to T L, and from the quadrangle H B a piece equal to the quadrangle K L is cut off.
But we know that the quadrangle A R represents the four of numbers which are added to the three roots. The quadrangle A N and the quadrangle K L are together equal to the quadrangle A R, which represents the four of numbers.
We have seen, also, that the quadrangle G M comprises the product of the moiety of the roots, or of one and a half, multiplied by itself; that is to say two and a quarter, together with the four of numbers, which are represented by the quadrangles A N and K L. There remains now from the side of the great original quadrate A D, which represents the whole square, only the moiety of the roots, that is to say, one and a half, namely, the line G C. If we add this to the line A G, which is the root of the quadrate G M, being equal to two and a half; then this, together with C G, or the moiety of the three roots, namely, one and a half, makes four, which is the line A C, or the root to a square, which is represented by the quadrate A D. Here follows the figure. This it was which we were desirous to explain.



We have observed that every question which requires equation or reduction for its solution, will refer you to one of the six cases which I have proposed in this book. I have now also explained their arguments. Bear them, therefore, in mind.
[...]
If the instance be: "To find a square, of which if one-third be added to three dirhems, nnd the sum be subtracted from the square, the remainder multiplied by itself restores the square;" then the computation is this: If you subtract one-third and three dirhems from the square there remain two-thirds of it less three dirhems. This is the root. Multiply therefore two-thirds of thing less three dirhems by itself. You say twothirds by two-thirds is four ninths of a square; and less two-thirds by three dirhems is two roots: and again, two-thirds by three dirhems is two roots; and less three dirhems by less three dirhems is nine dirhems. You have, therefore, four-ninths of a square and nine dirhems less four roots, which are equal to one root. Add the four roots to the one root, then you have five roots, which are equal to four-ninths of a square and nine dirhems. Complete now your square; that is, multiply the four-ninths of a square by two and a fourth, which gives one square; multiply likewise the nine dirhems by two and a quarter; this gives twenty and a quarter; finally, multiply the five roots by two and a quarter; this gives eleven roots and a quarter. You have, therefore, a square and twenty dirhems and a quarter, equal to eleven roots and a quarter. Reduce this according to what I taught you about halving the roots.