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Mathematical Learning as a Social and Interactive Work —Variations of Teacher's Types of Interaction: Different Types of 'MATOME'

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1. Background

The behaviour and the forms communication that an external observer can recognize when watching the mathematical discourse in a mathematics classroom strongly depends on essential underlying cultural and social constraints that regulate the norms, the social interaction and the different expectations that the participants in a certain culture of mathematical interaction have to respect and to meet. In most cases these social regulations and expectations are not explicitly noticed by the participants, nevertheless they are strongly and efficiently active, and in most cases the participants cannot simply escape from these social constraints. Further, these social regulations in an actual communication process function in a way that the different participants and their specific social expectations and tasks they have to fulfill start to influence the activity of the social norms and expectation of other person in a reciprocal manner, the regulations or reciprocally reinforced and they this way stabilize to a firm social pattern of behaviour. (cf. Bauersfeld 1988, Voigt 1985, Steinbring 1989)

In this paper, we will focus on what is socially expected and also accepted in Japanese lessons: 'MATOME', which is one of the most important Japanese teaching concepts and is regarded as an indispensable factor for a successful lesson in Japan. (cf. Shimizu 2007, p.111)

What is happening in the interaction of 'MATOME' in Japanese mathematics lessons? What kinds of 'MATOME' are there? How do they work and influence for students' mathematical learning?

To answer these research questions, we will examine a mathematics lesson conducted by an experienced mathematics teacher in our lesson study based on mathematics education research.

2. MATOME — new constructed mathematical summary of the experts

MATOME is one of the most important teaching concepts, one can say a key issue, of the Japanese traditional mathematics lesson study and lesson preparation and conduction. 'Usually at the end of the lesson, and sometimes during the lesson, the teacher presents a brief lecture on the main point(s) of the lesson.'(Stigler & Hiebert 1999, p.80) MATOME has to be done in an expert manner by the teacher, he/she is expected to perform a good, a perfect MATOME. Also students — unconsciously — react in social interaction in the classroom with the teacher's starting MATOME; the students stop other activities (mathematical, social, etc.), they begin to strictly concentrate on the teacher in front of the class at the blackboard and the students start writing into their notebooks what the teacher explains and what he/she is displaying on the blackboard. In their school life the students have seen and learned this expected social form of MATOME (cf. 'metalearning' Mellin-Olsen 1987; 'cultural scripts' Stigler &

Hiebert 1999) and they know how to correctly participate in this social interplay. The changed behaviour of the students is expected by the teacher, and vice versa the teacher reacts to students behaviour in the expected way of giving a good performance of MATOME.

The start of the communication process of MATOME is based on the teachers asking the students for some hints, for some ideas, he/she collects short remarks, descriptions and catchwords of students for his/her MATOME. Sometimes the teacher says: "The student A gave a good answer *for me*. My success of the lesson is all thanks to him/her.", a way of urging students contributions so that the teacher gets a sufficient base to start and to offer a good, a perfect MATOME.

An important aspect that has to be seen is, that the MATOME is not in the first line a summary of the students own ideas, it is not the attempt of a MATOME to further explain and to unscramble those students' contributions that have been very difficult to understand, and that have been presented in an unclear and in a very student specific manner. The MATOME is a rather new constructed mathematical summary of the expert, the teacher, that uses some hints — and only those that are really useful — to elaborate the perfect teacher's MATOME.

3. Data and Process of the Analysis

The data which we will analyse is from the 5th lesson from our lesson study *based on mathematics education research* which was conducted over a four-week period with 8 lessons given to a 7th grade class in Joetsu City (41 students; 22 male, 19 female) on May 13, 20, 22, 27, 28, 29, June 3 and 4, 2008. The lesson study based on mathematics education research was conducted by the authers, three incumbent postgraduates and one postgraduate. The mathematics teacher is one of the incumbent postgraduates who has more than fifteen years' experience in junior high schools. The topics were multiplication and division of negative numbers. The 5th lesson aimed to help students understand multiplication and division of positive and negative numbers and to have them explain answers and reasons of (1)$(+12) \div (+4)$ (2)$(-12) \div (+4)$ (3)$(+12) \div (-4)$ (4)$(-12) \div (-4)$.

The methodology is compatible with the idea of Action Research with including a cycle of 'Examination of the Lesson Plans', 'Implementing and Recording Lessons' and 'Reflection' where the practice was examined according to various and beneficial theories of mathematics education to concrete issues in each of the practical contexts. (cf. Iwasaki 2005) It could be characterized as an alternative or an extension of Japanese traditional lesson study. The theories mainly referred to are as follows: Mathematics as a science of patterns and structures (E. Ch. Wittmann), Epistemological Triangle (H. Steinbring), Relationship between interaction and mathematics learning (H. Bauersfeld, G. Krummheuer, J. Voigt), Transfer (from a teacher to students) of the responsibility to explain (G. Broussesau, N. Balacheff), View of mathematical Activity (G. Polya) and so on.

The lessons were recorded with 4 video cameras in total. We used one for recording the teacher from the side of the classroom and the others for recording the students. Based on a very careful, detailed and video-based analysis of the lesson by using a thorough translation of the Japanese transcripts into English, we reconstructed and identified different communication patterns of MATOME. In the following, these identified different types are described as they have been found in the discussion and analysis of the lesson.

4. Different Types of MATOME

Type I : MATOME in its traditional and classical form

The teacher conducts this type of MATOME in a rather perfect way; he presents a good (perfect) summary of the mathematical topic dealt with in a phase of the lesson or concerning the topic of the whole lesson. '··· Japanese teachers tend to make a final and careful comment on students' work in terms of mathematical sophistication.' (Shimizu 2007, p.111)

We found an example of classical MATOME in the lesson as follows, where the teacher summarizes the student Osa's contribution/explanation of the problem (1) by using a equivalent expression: $\square \times (+4) = (+12)$ at the blackboard.

***153- Osa: Since the sign is plus, for the sign of the answer to be plus, it has to be (a positive number) ×(a positive number). That is why the sign inside □ has to be plus.**

156- T: Yes.** *(writes " because (positive) × (positive) = (positive) ")(a positive number) × (a positive number) or (plus) × (plus) is plus, so...well, Inside □, sorry, it has to be lower, though, □ has to be positive.** *(writes " □ should be positive " on the black board)*

It has to be. OK? It means that you see that the sign has to be plus.

This classical teacher's MATOME is characterized by following aspects: The teacher might eventually take up some students ideas and contributions that have made before, but he is now the dominating presenter of the correct and perfect mathematical summary, he demonstrates his expert responsibility and he shows the mathematical idea in front of the whole class at the blackboard by speaking, writing at the blackboard and by definitely explaining in a sometimes new and well organized manner the underlying mathematical idea ('···in terms of mathematical sophistication'). This dominating role of the teacher presenting a perfect MATOME corresponds the way the students are now acting and interacting in the classroom: The students are silent, they do no longer interact, they do not ask, but they concentrate individually on the teacher's explanation and they try to note everything the teacher is writing at the blackboard into their notebooks.

Type II: a short and abbreviated version of Type I

This type of teacher's MATOME sometimes looks like a short 'addition' to a student's mathematical contribution; the teacher nevertheless in his short summary clearly shows his expert responsibility; it is the teacher who guarantees the correctness of a mathematical explanation. Also here the students turn from giving attention to a student's contribution to the teacher now, who is shortly explaining. Similar aspects to a classical MATOME (Type I) are to be observed her: The teacher is in front of the class, the students paying attention directly to the teacher.

***315- T: Well, Well. Divide into four. Ishi's was the same. Divide into four. What dividing into four mean?** *(a short pause)* **where does it come from? Why dividing into four?**

***319- Iwa: Because it is divided by 4.**

***320-) T: Oh, you mean divided by 4. This part.** *(seems to point to (+4) in the equation* $(-12) \div (+4)$*, but it's not clear)* **I got it. So, he wrote it with blue here, divide by 4, divide by plus 4.** *(writes "*$\div(+4)$*" below Iwasaki's diagram on the black board)* **Four** $\cdots$ *(writes "*$\cdots$ $\cdots$ *divide into four" on the right of* $\div(+4)$ *below Iwa's diagram on the black board)*

The teacher starts here to compare the contributions made by the students Ishi and Iwa. In his summary the teacher in the end poses a new problem: "Ishi's idea was the same (as Iwa's)". Seemingly the students

do not explain the mathematical reason of this "sameness"; the teacher gives then his explanation, but obviously not in a sophisticated way, as to offer the students a real understanding of this new question of sameness of Ishi's and Iwa's explanations.

Type III: Teacher-Guided MATOME

In this type of MATOME the teacher tries to more explicitly include students' contributions into the development of the mathematical summary. He is asking in the course of giving his MATOME also in between one or more students for short answers. This type of MATOME has to be analysed and interpreted very carefully, because during such a mathematical interaction between teacher and students two types of MATOME could develop:

a) Despite the inclusion of students' short reactions, the teacher remains the dominating expert, who is responsible for the correct mathematical explanation and the optimal understanding of the mathematical problem/idea that is in question here.

b) It is not the teacher who makes the main and central explanations, but the student or some students in the course of a teacher guided MATOME are able to more or less completely present and explain their ideas of the mathematical problem in question; the teacher rather moderates the interaction and he lets the students contributions for the whole class without adding his expert position as the last correct explanation.

***193- Kubo: Well, I have done it the same way as Osa's.**

***194- T: What do you mean by Osa's?**

***195- Kubo: The lower □ thing.**

***196- T: □ thing.** *(points to the formula on the black board)* **Yes.**

***200- Kubo: Well, change ' minus 12 ÷ plus 4' to multiplication, and ' □ × plus 4 is minus12'.** *(The teacher writes " □ × (+4) = " on the blackboard.)*

***203- T: Yes.** *(writes " -12 " to the right of the equation)*

***205- Kubo: Let's see, If it is multiplication, in case that the formula is minus, well,**

***207- T: The formula becomes minus.** *(writes " (-) " below -12 in the equation)*

***209- Kubo: Since the numbers of the formula should be different sign, and □ part number is minus, and then 12, in reverse, divide 12 by 4, the number of □ is -3.**

***212- T: Different signs mean?**

***213- Kubo: It must be plus and minus.**

214- T: It should be that kind of relationships. This is plus.** *(writes "+ with a circle" below (+4) in the equation □ × (+4) = -12)* **The partner is** *(writes "- with a circle" below □ in the equation □ × (+4) = -12)* **like this, this was done Osa's way** *(points to Osa's explanation), thinking the same way. Kubo, at that time, you didn't use this way of writing?** *(points to Osa's explanation and asks looking toward Kubo)*

The teacher develops a MATOME including the question to and the answers of the student Kubo for the problem (2). In the first part, Kubo can present step by step in some parts her mathematical ideas (*193-*209: $III_{a)}$), but in the end it is clear, that the teacher as an expert adds his correct and official explanation of Kubo's ideas to the whole class. (*212-*214: $III_{b)}$)

Type IV: Student's MATOME

In this Type of MATOME one or more students present a MATOME in their own responsibility with their own descriptions and words, the teacher does not give further comments. In the course of mathematical interaction where one can observe this type of MATOME, in a certain way the social roles of teacher and student are exchanged: A student (or some students together) presents and develops his summary of a mathematical problem by giving a mathematical explanation. Sometimes this student

takes the teacher's role in front of the class at the blackboard and he explains, writes at the blackboard and the whole class concentrates on the presentation of the student. The teacher refrains from giving additional comments, so the students in the class concentrate on the student in front as the "expert who explains mathematical questions".

In problem (2) $(-12) \div (+4)$ **Ishi began to explain the ways of calculation at his seat.**

***227- Ishi: Yes. On the number line, from zero to minus 12**

***228- T: From zero to minus 12. Is it OK on this side?** *(draws a straight line on the black board and writes 0 as a scale to the right.)* **Well, Ishida** *(stops writing on the black board and urges Ishi to come to the front of the class)* **On the diagram, my sense might be different from yours.** *(moves to the rear of the class and watches Ishi) (Isi moves to the front of the class and puts scales to the number line; The scales have larger ones at five intervals)*

***241- T:** *(Ishida in still working on his number line on the black board)* **Watch carefully how Ishida draws his number line.** *(Isida puts -5, -10 and -12 and starts to explains)*

245- Is: Well, first, from this zero to -12** *(traces the number line from -12 to zero with red chalk), this -12** *(points to the part of (-12) ÷ (+4) with his right hand)* **Divides it into four with +4 and divides it into 4** *(puts larger marks to -3, -6, -9, -12 on each scales)* **When you divide it into four, one of the four was minus 3** *(saying "minus 3" , parenthesizes 0 to -3, and in between writes -3)***, the answer is minus 3.** *(takes a glance at the teacher and gets back to his desk)*

260- T: What do you think? Anyone?** *(stays in the rear)* **Is it OK with you all? OK?** *(moves to the front)* **Though nothing like this was presented a little while ago, there seems to be something new coming up.(acts like raising his hand) Anything else? (a few students raise their hands)* **How about you, Hiro?**

In the Ishi-example it is surprising, that the teacher first starts with a typical teacher guided MATOME with Ishi, but suddenly the teacher stops and invites Ishi to come to the blackboard and to explain. This change of social roles with also the positioning in the classroom is very obvious and the other students in the class now turn from the teacher to the "student-teacher" Ishi, who writes at the blackboard and explains. The teacher does not at all intervene in Ishi's mathematical explanation and he also does not add at the end of his explanation a further official teacher's comment, but the only remark that is made by the teacher is of a different kind: "Watch carefully how Ishida draws his number line.". This comment supports the student's role of a self-responsible presenter of his own mathematical ideas.

Type V: in a short way of Type IV

The complete and detailed student's MATOME can also appear in a variation of a short or abbreviated version (as a variation of MATOME IV).

***135- Osa: Well, If you think like that, well, since the sign of the answer is plus, that sign should be plus, too.** *(Teacher points to + sign and then points to □ at the answer +12 in the equation)* **It means that the sign should be plus.**

***142- T: What do you think about Osa's explanation? If you have any question and need an explanation once again, don't hesitate to ask because we won't be able to do that in the next class. Anyone?**

Osa summarizes; "Well, if you think like that, well, since the sign of the answer is plus, that sign should be plus too. It means that the sign should be plus." Here Osa is giving an mathematical type of argument, she verbalizes it mathematically ("$\cdots$ that sign should be plus."), and she gives a description in her own words (not with formal mathematical terms, for instance). She is remaining at her seat (she comes not to the front of the class). And then after her explanation, the teacher starts with the class and with Osa a teacher-guided summary (MATOME (Type I)).

5. Conclusion

Teacher's MATOME and Student's MATOME

Based on a very careful, detailed and video-based analysis of the lesson with the transcripts, we identified and reconstructed different types of MATOME, the range of which were from its traditional and classical form by the teacher to a new one by the students. They are classified into two groups: Teacher's MATOME (I,II,$III_{a)}$) and the Studen's MATOME ($III_{b)}$, IV,V). Teacher's MATOME that the intellectual responsibility for explaining mathematical ideas is in the teacher's side is different from Student's MATOME in that it is in the student side. The difference of the social roles between Type I and Type IV, which are the two poles, seems to be more clear because in Type I the teacher's authority was exercised to guarantee the correctness of mathematical explanations but in Type IV it was done in order to support the student's role of a self-responsible presenter of his/her own mathematical ideas or explanations.

The limits of MATOME

The role and the assessment of the traditional Japanese concept of MATOME for lesson study and for lesson plan has to be carefully regarded with looking at the underlying concept or model of what is seen as the required type of students' learning of mathematics in the classroom. There can be (at least) two different views:

Model 1 : Students have in the end individually to learn and understand mathematics deeply in their own way, they have to get the best and correct idea of a mathematical problem/question. The learning in the classroom, and the teacher's guidance serve to follow and to reach this aim: the individual student's mathematical understanding.

Model 2 : Students have to learn mathematics in an interactive way, in communication with other students, in negotiating mathematical ideas and understandings, in learning to express themselves in a mathematically appropriate and understandable manner, and to jointly develop a clear idea about what is a mathematical argument and what is not a mathematical argument. To reach this aim, the learning of mathematics in the classroom becomes a social and interactive task, with concentration on students' own and joint mathematical understanding and thinking; the students own ideas have to be developed and presented in the classroom in a way, that it is clear that the responsibility of the students is important, and that the teachers official and correct summaries cannot replace students' own mathematical thinking.

According to these two fundamental underlying models of "mathematical learning in the classroom" the role of MATOME has to be seen more critically. If one accepts Model 1, then MATOME in its traditional form is a positive factor in promoting students' individual mathematical thinking. If one instead argues for Model 2, one has to carefully analyse different types of MATOME, one has to carefully look at advantages and at limits and restrictions of MATOME for a mathematical learning process of students in the sense of Model 2 (as it is done in this analysis here).

MATOME in its traditional and classical form (Type I), which is characteristic of Japanese lessons and was actually realized in a perfect way by the experienced mathematics teachers who are highly trained professional, could be a positive factor in promoting students' individual mathematical thinking. But, to promote the leaning of mathematics in the classroom to social and interactive work, which is compatible with Japanese new educational aim stated by the School Education Act revised in 2007 because the law aims to foster students ability to think , judge and express themselves as well as to help their acquire basic and fundamental knowledge and skills, one has to carefully look at not only advantages described

above but also limits of ‘ MATOME’, as cultural and social restrictions, for students’ mathematical learning as social and interactive work.

The importance of the lesson study based on mathematics education research

Teaching is a complex system and MATOME is an important part of Japanese traditional mathematics teaching culture. It would be the reason why it is difficult for teachers to change MATOME in the process of Japanese traditional lesson study.

It should be noted that the teacher’s MATOME is mostly done where the students’ presentation and explanation have much to do with the teacher’s aim of the lesson with including some key words: for example, “$\cdots$ has to be plus/minus.”. Indeed at the Examination of the Lesson Plans, Osa’s argumentation described above was shared in our team as one of the typical and her own argumentation/explanation of mathematics. That fact, which must be confirmed by additional observations, is relevant to Stigler & Hiebert’s interpretation of MATOME cited in section 2. It is reasonable to suppose that Japanese teachers conduct the MATOME in type I in a very natural way and rather in believing as “a typical good teaching” in spite of the disadvantage of the teacher’s MATOME described above.

Indeed MATOME, especially in its traditional and classical form, which is a widely shared cultural expectation in Japan, would be a blind spot of Japanese mathematics teaching because it makes ‘teachers fail to see alternatives to what they are doing in the classroom, thinking, “ this is just the way things are.” ’(Stigler & Hiebert 1999, p.100) But the episode of the lesson described in this paper, which was conducted in our lesson study *based on mathematics education research*, suggests a possibility of the alternative version of MATOME: Student’s MATOME (Type IV).

In Japan Student’s MATOME (Type IV) seems to be very rare case especially in junior high schools. When you could observe a students’ explanation of mathematical ideas, you would find it was superficially done by the student and was done by the teacher in fact — so not the Type IV if you watch the interacion carefully in a way we have analysed in Type III.

‘In order to enable the teacher to act in a didactically founded way, it would seem necessary to have him develop a critical attitude to the tradition of classroom discourse in short steps’. (Voigt 1985, p.113) A further important and essential point has to be taken into consideration, i.e. that a teacher should have the opportunity to reflect his/her interaction process with other teachers (and/or researchers) in a joint group of discussion and examination as for instance in the frame of a lesson study *based on mathematics education research.* In this context MATOME in its traditional and classical form — which is considered by the teachers without any reservation as a natural and rather important type of classroom interaction — could really become the object of careful investigation.

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