Power System Static Voltage Stability Analysis Considering all Active and Reactive Power Controls - Singular Value Approach

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Abstract—**This paper presents a novel approach for analyzing power system static voltage stability considering all possible active and reactive power controls based on the multi-input multi-output (MIMO) transfer function. By applying the voltage magnitudes of critical buses as output and feasible controls as input signals, the MIMO transfer function of multi-machine power system can be developed. The analyses are carried out by means of the singular value decomposition (SVD). The proposed approach takes the advantages of the classical static voltage stability analysis and the modern multi-variable feedback control theory. Not only the influences of reactive power controls on the static voltage stability, which can be achieved by the classic method, the influence of active power modulation can also be analyzed. The output singular vectors provide an overview of the most critical buses that affected by the static voltage stability. The magnitudes of the input singular vectors show that which input signal has the largest effect on the most critical bus. The proposed approach is simple and easy to be implemented into large power systems.**

Index Terms—**Multi-input multi-output Transfer function, Multi-variable feedback control, Power system dynamics, Singular value analysis, Voltage stability.**

I. Introduction

THE static voltage stability problem is one of the major aspects in power system planning and secure operation. A large number of researches were made in the area of voltage stability. Most of them are based on the modal analysis of the reduced (*Q-V*) Jacobian matrix [1-6]. The bus, branch and generator participation factors on the static voltage stability can then be obtained. Moreover, the stability margin and the shortest distance to instability can also be determined [6].

The classic methods consider the active powers (both generators and loads) at all buses as constant and only the influence of the PQ-bus reactive power controls on the static voltage stability can be analyzed.

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However, the active powers have also great influences on the static voltage stability. In the deregulated electricity market, the active power exchanges in large power systems vary frequently. Especially in recent years, the wind power generations are widely developed all over the world. Due to the stochastic character of the wind energy, the active power inputs to these buses are also fluctuate. Although there are extensive literatures on voltage stability analysis, very few deal with the above mentioned issues.

In this paper, a novel approach for analyzing the static voltage stability is proposed. This method takes the advantages of classical static voltage stability analysis and modern multi-variable feedback control theory. By analyzing the MIMO transfer function, the most critical buses affected by the static voltage stability can be selected out. At the same time, all feasible controls that could influence the static voltage stability can be analyzed. Therefore, suitable controls for the static voltage stability can also be determined.

This paper is organized as follows: Following the introduction, the classic static voltage stability analysis is described in section II. Then in section III, the proposed MIMO modeling method is introduced. The singular value analysis for static voltage stability assessment is proposed in section IV. Then, the simulation results are given in section V. Finally, brief conclusions are deduced.

II. Classical Static Voltage Stability Analysis and Control

A. Static voltage stability analysis

The static voltage stability analysis is based on the modal analysis of the power flow Jacobian matrix, as shown in (1) [6]:

$$\begin{bmatrix} \Delta\mathbf{P}_{PQ,PV} \\ \Delta\mathbf{Q}_{PQ} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{P\theta} & \mathbf{J}_{PV} \\ \mathbf{J}_{Q\theta} & \mathbf{J}_{QV} \end{bmatrix} \cdot \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\mathbf{V}_{PQ} \end{bmatrix} \tag{1}$$

where

$\Delta\mathbf{P}_{PQ,PV}$: is the incremental changes in active power of PQ and PV buses

$\Delta\mathbf{Q}_{PQ}$: is the incremental changes in reactive power of PQ buses

$\Delta\boldsymbol{\theta}$: is a vector that contains the incremental changes in bus voltage angle

$\Delta\mathbf{V}$: is a vector that contains the incremental changes in bus voltage magnitudes

The elements of the Jacobian matrix represent the sensitivities between power flow bus voltage changes [6]. According to the classical static voltage stability analysis, power system voltage stability is largely affected by the reactive power [6]. Therefore, keeping active power as constant at each operating point, the *Q*-*V* analysis can be carried out. Assuming $\Delta\mathbf{P}_{PQ,PV} = 0$, it follows from (1) that [4,6]:

$$\Delta\mathbf{Q}_{PQ} = \left[\mathbf{J}_{QV} - \mathbf{J}_{Q\theta}\cdot\mathbf{J}_{P\theta}^{-1}\cdot\mathbf{J}_{PV}\right]\cdot\Delta\mathbf{V}_{PQ} = \mathbf{J}_{R}\cdot\Delta\mathbf{V}_{PQ} \quad (2)$$

and

$$\Delta\mathbf{V}_{PQ} = \mathbf{J}_{R}^{-1}\cdot\Delta\mathbf{Q}_{PQ} \quad (3)$$

Based on the $\mathbf{J}_R^{-1}$, which is the reduced *V*-*Q* Jacobian matrix, the *Q*-*V* modal analysis can be carried out. Therefore, the bus, branch and generator participation factors on the static voltage stability can also be obtained. Moreover, the stability margin and the shortest distance to instability will be determined [4-6].

As discussed in [2,3,12], since $\mathbf{J}_R^{-1}$ is a square matrix, the singular value analysis of $\mathbf{J}_R^{-1}$ provides the same result as the classical static voltage stability analysis. As mentioned above, the classical method can only analyze influences of PQ-bus reactive power controls on the static voltage stability.

B. Voltage stability control

In order to prevent voltage collapse, different measures can be applied [5,6,10]. The reactive power compensation, under voltage load shedding and the control of transformer tap-changers are the most important controls for enhancing the static voltage stability [5,6,9]. Furthermore, with the development of power electronics, FACTS (Flexible AC Transmission Systems) devices, i.e. SVC (Static Var Compensation) and STATCOM (Static Synchronous Compensator), are recognized as important and effective methods for voltage stability control [14,16,17].

C. Restrictions of the classic static voltage stability analysis method

The classic method provides clear indices of the static voltage stability. However, there are still some restrictions by applying the classic method:

1. The classic method based on the modal analysis and therefore, the matrix must be square. In other words, the number of the inputs (reactive power changes) must equal to the number of the outputs (voltage magnitude changes). In large power systems, there may only a few critical buses that are associated with static voltage stability. Moreover, there are plenty of possible controls that may have influence on the critical buses. It is difficult to use the classic method to analyze such non-square MIMO systems.
2. The classical method can only analyze influences of PQ-bus reactive power control on the static voltage stability. In practice, the active power changes have also great influences on the static voltage stability [5]. Therefore, a new approach which can analyze both active and reactive power on the static voltage stability is necessary.
3. In the classical static voltage stability analysis, the influences of generator controls cannot be considered because they are considered as PV buses and their voltage magnitudes are constant. In real power systems, the voltages of the PV buses are always controlled by the reactive power modulations (AVR-automatic voltage regulation) and these controls may have also great effects on the static voltage stability. Therefore, it is necessary to develop a new method which can analyze the influence of AVRs on the power system static voltage stability.

III. Static Voltage Stability – Modeling and Analysis using the Novel MIMO Approach

In this paper, power system static voltage stability is analyzed based on the MIMO transfer function which is widely used in control engineering. In this analysis, detailed dynamic power system, where the generators, governors, static exciters, power system stabilizers (PSS) and nonlinear voltage and frequency dependent loads are included, is modeled. In general, this mathematical model considers all possible issues that could affect static voltage stability.

Firstly, controls that could affect static voltage stability must be selected as inputs to the MIMO system. Usually, they are active power and reactive power modulations at generator and load buses. Other control variables, such as the SVC and STATCOM control signals, can also be included in the input variables. Since the tap-changer control will change the power system state, the influence of the tap-changer should be considered at the new operating point.

Secondly, similar to the classical method, the incremental changes of bus voltage magnitudes are considered as the output variables. In practice, only the most critical buses that affected by the static voltage stability are selected as output variables. Therefore, the numbers of output variables can be constrained to a small range and large power systems can also be analyzed using the proposed method.

The above mentioned formulation process of the MIMO system transfer function matrix is described in the following section:

A. *Formulation of the MIMO transfer function matrix $\mathbf{J}_V(s)$*

By linearizing around the operating point, linearized power system model can be expressed as follows:

$$\begin{aligned}\Delta\dot{\mathbf{x}} &= \mathbf{A}\cdot\Delta\mathbf{x}+\mathbf{B}\cdot\Delta\mathbf{u}\\ \Delta\mathbf{y} &= \mathbf{C}\cdot\Delta\mathbf{x}+\mathbf{D}\cdot\Delta\mathbf{u}\end{aligned} \quad (4)$$

where

- $\Delta\mathbf{x}$ is the state vector of the system
- $\Delta\mathbf{y}$ is the output vector containing bus voltages
- $\Delta\mathbf{u}$ is the input vector containing load and generation modulations
- $\mathbf{A}$ is the state matrix
- $\mathbf{B}$ is the control matrix
- $\mathbf{C}$ is the output matrix
- $\mathbf{D}$ is the feedforward matrix

Therefore, the transfer function matrix $\mathbf{J}_V(s)$ can be expressed as:

$$\mathbf{J}_V(s)=\frac{\Delta\mathbf{y}}{\Delta\mathbf{x}}=\mathbf{C}\cdot(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D} \quad (5)$$

Since the number of inputs and outputs can be restricted even in large power systems, the dimension of the transfer function matrix $\mathbf{J}_V(s)$ is usually small. However, the order of each transfer function in $\mathbf{J}_V(s)$ may be large. In order to get a relative low order MIMO transfer function, the order of the state equation (3) could be reduced by means of model reduction before calculation of $\mathbf{J}_V(s)$.

In this research, besides the SVC control signal, the active and reactive power modulations of all generators and loads are considered as input variables for demonstration of the proposed method. Also the set of output signals is extended to all bus voltage magnitude changes. The corresponding transfer function matrix $\mathbf{J}_V(s)$ is shown in Fig. 1.

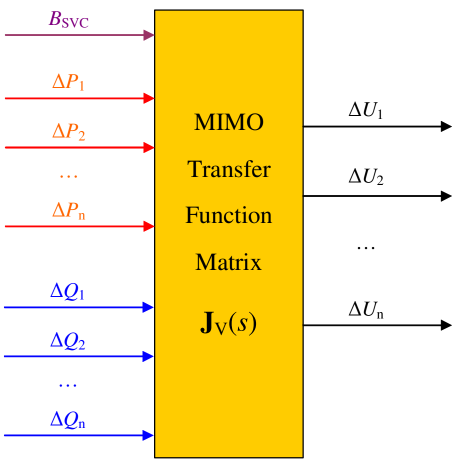

Fig. 1 Static voltage stability modeling

Following (5), the detailed MIMO transfer function matrix system can then be described as:

$$\begin{aligned}\Delta\mathbf{y}&=\begin{bmatrix}\Delta U_1 & \Delta U_2 & \dots & \Delta U_n\end{bmatrix}^T\\ &=\mathbf{J}_V(s)\cdot\left[\begin{array}{cccc|cccc}\Delta P_1 & \Delta P_2 & \dots & \Delta P_n & \Delta Q_1 & \Delta Q_2 & \dots & \Delta Q_n\end{array}\right]^T\\ &=\mathbf{J}_V(s)\cdot\Delta\mathbf{u}\end{aligned} \quad (6)$$

where

$$\mathbf{J}_V(s)=\left[\begin{array}{ccc|ccc} g_{P1\_U1}(s) & \dots & g_{Pn\_U1}(s) & g_{Q1\_U1}(s) & \dots & g_{Qn\_U1}(s)\\ g_{P1\_U2}(s) & \dots & g_{Pn\_U2}(s) & g_{Q1\_U2}(s) & \dots & g_{Qn\_U2}(s)\\ & \dots & & & \dots & \\ g_{P1\_Un}(s) & \dots & g_{Pn\_Un}(s) & g_{Q1\_Un}(s) & \dots & g_{Qn\_Un}(s)\end{array}\right] \quad (7)$$

is a $n_{Output}\times n_{Input}$ transfer function matrix.

Since the number of inputs n_{Input} and outputs n_{Output} of $\mathbf{J}_V(s)$ are always different, modal analysis can not be used. In this research, the singular value approach is applied.

IV. Singular Value Approach

A. *Singular value decomposition (SVD)*

For analyzing the MIMO transfer function matrix, singular value decomposition (SVD) of $\mathbf{J}_V(s)$ can be carried out at every frequency [3,12,13,18,19]:

$$\mathbf{J}_V(s)=\mathbf{U}(s)\Sigma(s)\mathbf{V}^T(s) \quad (8)$$

where

$\mathbf{\Sigma}$ is a $n_{Output}\times n_{Input}$ matrix with $k = \min\{n_{Output}, n_{Input}\}$ non-negative singular values, σ_i, arranged in descending order along its main diagonal; the other entries are zero.

The singular values are the positive square roots of the eigenvalues of $\mathbf{J}_V^T(s)\times\mathbf{J}_V(s)$, where $\mathbf{J}_V^T(s)$ is the complex conjugate transpose of $\mathbf{J}_V(s)$ [13].

$$\sigma_i(\mathbf{J}_V(s))=\sqrt{\lambda_i(\mathbf{J}_V^T(s)\times\mathbf{J}_V(s))} \quad (9)$$

$\mathbf{U}(s)$ is a $n_{Output}\times n_{Output}$ unitary matrix of output singular vectors, $\mathbf{u}_i(s)$.

$\mathbf{V}(s)$ is a $n_{Input}\times n_{Input}$ unitary matrix of input singular vectors, $\mathbf{v}_i(s)$.

B. Input and output singular vectors

The column vectors of $\mathbf{U}(s)$, denoted $\mathbf{u}_i(s)$, represent the output directions of the system. They are orthogonal and of unit length (orthonormal) [13]:

$$\|\mathbf{u}_i\|_2 = \sqrt{|\mathbf{u}_{i1}|^2 + |\mathbf{u}_{i2}|^2 + \ \dots \ + |\mathbf{u}_{in\_output}|^2} = 1 \quad (10)$$

$$\mathbf{u}_i^H \mathbf{u}_i = 1, \ \ \mathbf{u}_i^H \mathbf{u}_i = 0, \ \ i \neq j \quad (11)$$

Likewise, the column vectors of $\mathbf{V}(s)$, denoted $\mathbf{v}_i(s)$, are orthogonal and of unit length, represent the input directions of the system. These input and output directions are related through the singular values [13]:

Since $\mathbf{V}^H(s)\mathbf{V}(s)=\mathbf{I}$, then (11) can be rewritten as $\mathbf{J}_v(s)\mathbf{V}(s)=\mathbf{U}(s)\mathbf{\Sigma}(s)$, which for column i becomes:

$$\mathbf{J}_v(s)\mathbf{v}_i(s) = \sigma_i \mathbf{u}_i(s) \quad (12)$$

where $\mathbf{u}_i(s)$ and $\mathbf{v}_i(s)$ are vectors and σ_i is a scalar. This means that if an input in the direction $\mathbf{v}_i(s)$, then the output is in the direction $\mathbf{u}_i(s)$. Furthermore, since $\|\mathbf{u}_i(s)\|_2 = 1$ and $\|\mathbf{v}_i(s)\|_2 = 1$, the i^{th} singular value σ_i gives directly the gain of the matrix $\mathbf{J}_v(s)$ in this direction [13]:

$$\sigma_i(\mathbf{J}_v(s)) = \|\mathbf{J}_v(s)\mathbf{v}_i(s)\|_2 = \frac{\|\mathbf{J}_v(s)\mathbf{v}_i(s)\|_2}{\|\mathbf{v}_i(s)\|_2} \quad (13)$$

C. Maximum and minimum singular values

The singular values are sometimes called the principal gains. The largest gain for any input direction is equal to the maximum singular value [13].

$$\bar{\sigma}(\mathbf{J}_v(s)) \equiv \sigma_1(\mathbf{J}_v(s)) = \max_{\mathbf{d}\neq 0} \frac{\|\mathbf{J}_v(s)\mathbf{d}\|_2}{\|\mathbf{d}\|_2} = \frac{\|\mathbf{J}_v(s)\mathbf{v}_1(s)\|_2}{\|\mathbf{v}_1(s)\|_2} \quad (14)$$

and that the smallest gain for any input direction is equal to the minimum singular value [13]:

$$\underline{\sigma}(\mathbf{J}_v(s)) \equiv \sigma_k(\mathbf{J}_v(s)) = \min_{\mathbf{d}\neq 0} \frac{\|\mathbf{J}_v(s)\mathbf{d}\|_2}{\|\mathbf{d}\|_2} = \frac{\|\mathbf{J}_v(s)\mathbf{v}_k(s)\|_2}{\|\mathbf{v}_k(s)\|_2} \quad (15)$$

where $k = \min \{ n_{Output}, n_{Input} \}$.

Therefore, for any vector $\mathbf{d}$:

$$\underline{\sigma}(\mathbf{J}_v(s)) \leq \frac{\|\mathbf{J}_v(s)\mathbf{d}\|_2}{\|\mathbf{d}\|_2} \leq \bar{\sigma}(\mathbf{J}_v(s)) \quad (16)$$

Define $\mathbf{u}_1(s) = \bar{\mathbf{u}}(s)$, $\mathbf{v}_1(s) = \bar{\mathbf{v}}(s)$, $\mathbf{u}_k(s) = \underline{\mathbf{u}}(s)$ and $\mathbf{v}_k(s) = \underline{\mathbf{v}}(s)$, it follows that [13] :

$$\mathbf{J}_v(s)\bar{\mathbf{v}}(s) = \bar{\sigma}\bar{\mathbf{u}}(s), \ \mathbf{J}_v(s)\underline{\mathbf{v}}(s) = \underline{\sigma}\underline{\mathbf{u}}(s) \quad (17)$$

$\mathbf{u}_1(s) = \bar{\mathbf{u}}(s)$ and $\mathbf{v}_1(s) = \bar{\mathbf{v}}(s)$ are always defined as the maximum output and input singular vectors. The vector $\bar{\mathbf{v}}(s)$ corresponds to the input direction with largest amplification, and $\bar{\mathbf{u}}(s)$ is the corresponding output direction in which the inputs are most effective [13]. Control input can be selected based on the analysis of $\bar{\mathbf{v}}(s)$.

D. Interpretation of the singular value analysis in power system static voltage stability analysis

For dynamic analysis, the singular values and their associated directions vary with frequencies [18,19]. In power system static voltage stability analysis, only the steady state (s=0) will be considered.

By analyzing the maximum singular values and their related input and output singular vectors, the relationship between input and output variables can be obtained. For the static voltage stability analysis, these can be interpreted as follows:

1. The maximal output singular vector shows at which bus the voltage magnitude is the most critical.
2. The maximal input singular vector indicates which input has the largest influence on the corresponding output.

Therefore, by means of the singular value approach, the static voltage stability analysis can be carried out.

Some advantages of the singular value analysis over the eigenvalue decomposition for analyzing gains and directionality of multi variable systems are [13]:

1. The singular values give better information about the gains of the transfer function matrix.
2. The directions (input and output) obtained from the singular value analysis are orthogonal.
3. The singular value analysis can be applied to the non-square system directly.

V. Simulation Results

A. Power system model

The typical 4-machine two-area power system model [6,11] is used for demonstration of the proposed method, as shown in Fig. 2. The dynamic model consists of generators with 6[th] order model, governors, static exciters, power system stabilizers (PSS). Induction motor loads are modeled at Bus 4 (deep bar) and Bus 9 (double cage). For the reactive compensation, a SVC (Static Var Compensator) is applied on Bus 11 (mid-point of the tie-line).

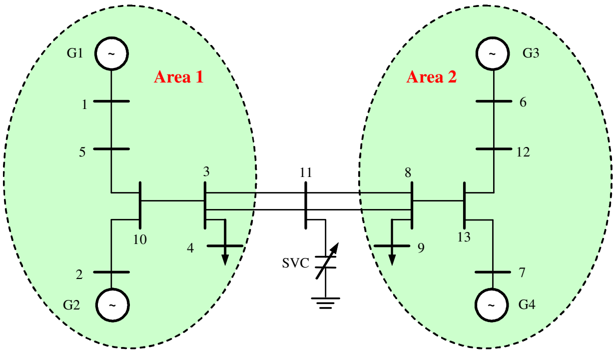

Fig. 2 4-machine two-area power system model

The purpose of SVC is to maintain the bus voltage by means of reducing the effect of power system disturbances. It is a fast acting device and always be used when mechanically switched capacitors are too slow to prevent voltage collapse [11]. The SVC dynamic model is shown in Fig. 3 [11].

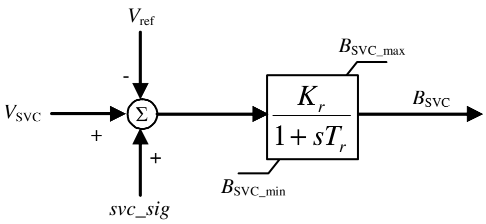

Fig. 3 Dynamic model of SVC

where

V_{ref}:	is the reference voltage magnitude of SVC
V_{SVC}:	is the terminal voltage magnitude of SVC
svc_sig:	is supplementary control signal into the reference input of SVC
K_r:	is the regulator gain
T_r:	is the regulator time constant
B_{SVC_min}:	is the minimum susceptance of SVC
B_{SVC_max}:	is the maximum susceptance of SVC
B_{SVC}:	is the controlled susceptance of SVC

As mentioned in [11], the three possible locations for SVC are Bus 3, 8 and 11. In this research, the static behaviour of SVC static reactive compensation control is mainly considered. Therefore, the location of SVC is determined at mid-point of the tie-line 3-8 to provide reactive support under high power transfer [11,15].

B. Classical static voltage stability analysis – singular value approach

Firstly, the classical static voltage stability analysis is carried out. Since the reduced Jacobian matrix $\mathbf{J}_R^{-1}$, as shown in (8), is a square matrix, both modal analysis and singular value analysis can be applied and their results are the same. In this research, the classical static voltage stability analysis is carried out using the modal analysis approach.

The simulation result is shown in Fig. 4. The maximum eigenvalue of $\mathbf{J}_R^{-1}$ is 0.14. The corresponding maximum right eigenvector is 0.59 and it associates with Bus 11. This means that disturbances on reactive loads will cause the largest variation on the voltage magnitude at Bus 11.

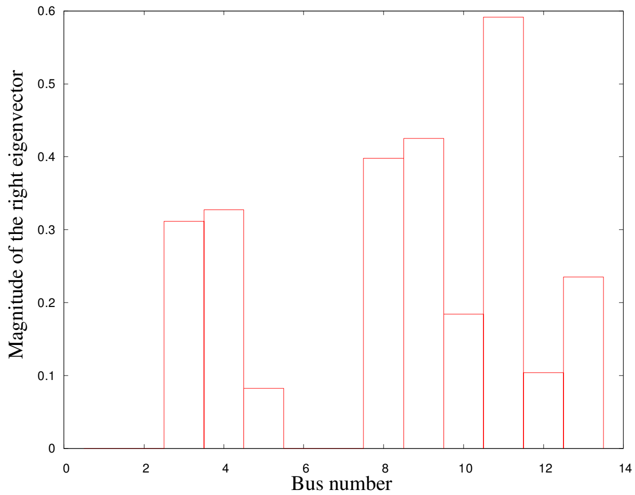

Fig. 4 Right eigenvector plot for the classical static voltage stability analysis

C. Singular value analysis of the transfer function matrix $J_V(s)$ (for steady state: s=0)

Then, the static behavior of the MIMO transfer function matrix $\mathbf{J}_V(s)$ is analyzed for the steady state (s=0). The simulation result is shown in Fig. 5.

For the classical static voltage stability assessment discussed in the above section, the influences of generators G1~G4 (Buses 1, 2, 6, 7) cannot be analyzed because they are considered as PV buses and their voltage magnitude will be constant. In real power systems, the voltages of the PV buses are always controlled by the AVR and the PV-bus voltage controls are also considered in this research. Therefore, the output singular vectors associated with PV-buses are non-zero values in the plot shown in Fig. 5.

In comparison with the simulation result of static voltage stability analysis in Fig. 4, it can be realized that the results are similar:

1. The most critical bus in both analyses is Bus 11.
2. Instead of buses 8 and 9 in the classical static voltage stability analysis, buses 3 and 4 come to be the second

critical buses.

3. The other buses have relative same result with regard to the static voltage stability.

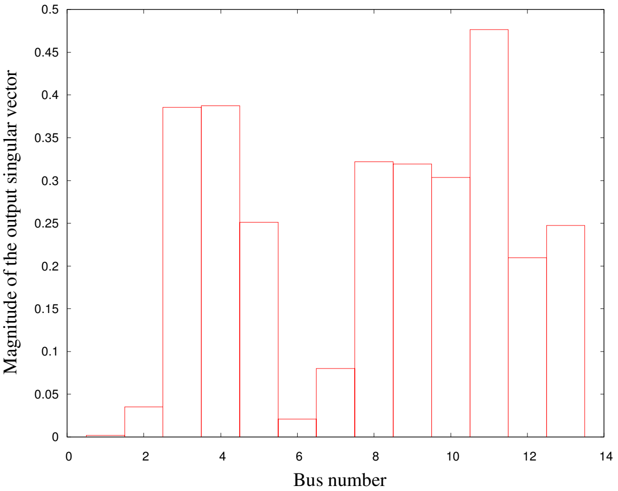

Fig. 5 Output singular vector of the transfer function matrix $\mathbf{J}_V(s)$ when s=0

The input singular vectors associated with static mode are given in Fig. 6. It shows that the active and reactive power controls of the generators (Input number 2-9) have the largest influences on the static voltage stability. Also, in comparison with the load modulations (Input number 10-27), the SVC control (Input number 1) have larger influence. The input numbers and their corresponding input variables are given in Table I and II.

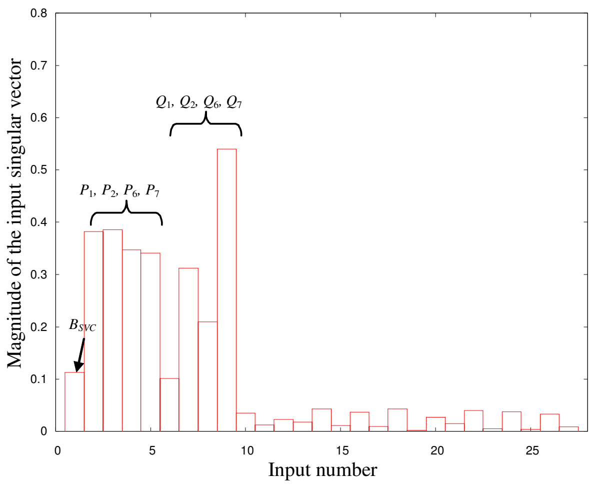

Fig. 6 Input singular vector associated with static mode s=0

TABLE I
THE INPUT NUMBERS AND THEIR CORRESPONDING INPUT VARIABLES FOR THE SVC AND GENERATION MODULATIONS

Input Number	1	2	3	4	5	6	7	8	9
	SVC	**Active Generation Modulation**				**Reactive Generation Modulation**			
Input Variable	B_{svc}	P_1	P_2	P_6	P_7	Q_1	Q_2	Q_6	Q_7

TABLE II
THE INPUT NUMBERS AND THEIR CORRESPONDING INPUT VARIABLES FOR THE ACTIVE AND REACTIVE LOAD MODULATIONS

Input Number	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
Input Variable	P_4	Q_4	P_9	Q_9	P_{11}	Q_{11}	P_3	Q_3	P_5	Q_5

Input Number	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
Input Variable	P_8	Q_8	P_{10}	Q_{10}	P_{12}	Q_{12}	P_{13}	Q_{13}

Among the suitable static voltage stability controls, the control of input number 9 (Reactive power control of generator 9) have the largest influence on the static voltage stability of Bus 11. However, always the local control signals will be selected in practice. Therefore, the SVC control is the best one.

VI. CONCLUSION

This paper presents the application of singular value analysis to a properly defined MIMO model for power system static voltage stability analysis. The singular value analysis of the MIMO transfer function matrix is carried out for the steady state (s=0). The elements of the input singular vector indicate the impact of the input variables on the outputs. Therefore input singular vectors can be used for selecting the most suitable control signal for the improvement of steady state voltage stability. From the output singular vector, the influences of static voltage stability on the selected buses can be evaluated. Moreover, the output singular vectors provide an overview of the most critical buses that affected by the static voltage stability.

In this paper, not only the active and reactive power control of the load buses, but also the controls of the generator powers are considered as the inputs to the MIMO model. Therefore even in large power systems, an overview of the feasible controls is possible. Furthermore, since the inputs and outputs can always be restricted to a small range, the proposed approach is simple and easy to be implemented in large power system static voltage stability analyses.

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VIII. Biographies

Li-Jun Cai (1970) received his B.-Eng. and M.-Eng. degrees from the Electrical Engineering Department, North China Electrical Power University, China, in 1992 and 1997, respectively, and the Ph.D. degree in electrical engineering from the University of Duisburg-Essen, Essen, Germany, in 2004. From 2004 to 2006, he worked as a post-doctorial fellow in the University of Duisburg-Essen, Germany. Presently, he is with the Vattenfall Europe Transmission GmbH, Berlin, Germany.

His research interests are in the power system planning, optimization, multi-objective coordinated control of FACTS devices and power system dynamic stability analyses and control.

István Erlich (1953) received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen/Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is a member of VDE and senior member of IEEE.Dr. Erlich is a member of VDE and Senior Member of IEEE.