Optimal Choice and Allocation of FACTS Devices using Genetic Algorithms

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***Abstract*--This paper concerns the optimal choice and allocation of FACTS (Flexible AC Transmission Systems) devices in multi-machine power system using genetic algorithm. The objective is to achieve the power system economic generation and dispatch. Using the proposed method, the locations of the FACTS devices, their types and rated values are optimized simultaneously. Different kinds of FACTS devices are simulated in this study: UPFC, TCSC, TCPST, and SVC. Furthermore, their investment costs are also considered. Simulation results validate the efficiency of this new approach in minimizing the overall system cost function, which includes generation cost and the investment costs of the FACTS devices. The proposed algorithm is an effective and practical method for the choice and allocation of FACTS devices in large power systems.**

***Index Terms*--Allocation, Cost function, Economic generation, FACTS, Genetic algorithms, Investment, Optimal power flow.**

I. INTRODUCTION

IN recent years, with the deregulation of the electricity market, the traditional concepts and practices of power systems have been changed. Better utilization of the existing power system to increase capacities by installing FACTS devices becomes imperative [1, 10].

The parameter and variables of the transmission line, i.e. line impedance, terminal voltages, and voltage angle can be controlled by FACTS devices in a fast and effective way [10,12]. The benefit brought about FACTS includes improvement of system dynamic behavior and thus enhancement of system reliability. However, their main function is to control power flows [2,4] . Provided optimal locations, FACTS devices are capable of increasing the system loadability too [1]. These aspects are playing an increasing and major role in the operation and control of competitive power systems.

Many researches were made on the optimal location of FACTS devices [1-3]. However, the investment cost of FACTS and their impact on the power generation costs are not wholly considered yet.

The objective of this paper is to develop an algorithm to find and choose the best locations for the FACTS devices. Therefore, the overall cost function, which includes the generation costs of power plants and the investment costs of FACTS, is minimized.

Different kinds of FACTS devices and their different locations have different advantages. In realizing the proposed objective, the suitable types of FACTS devices, their location and their rated values must be determined simultaneously. This combinatorial analysis problem is solved using genetic algorithm [1,11].

This paper is organized as follows: Following the introduction, different FACTS models are described in section 2. Then in section 3, the genetic algorithm for the optimal location of FACTS devices is discussed in detail. The simulation results are given in section 4. Finally, brief conclusions are deduced.

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II. FACTS MODELS

A. FACTS Devices

In this paper, four typical FACTS devices have been selected: TCSC (Thyristor Controlled Series Capacitor), TCPST (Thyristor Controlled Phase Shifting Transformer), UPFC (Unified Power Flow Controller), and SVC (Static Var Compensator). Their block diagrams are shown in Fig. 1.

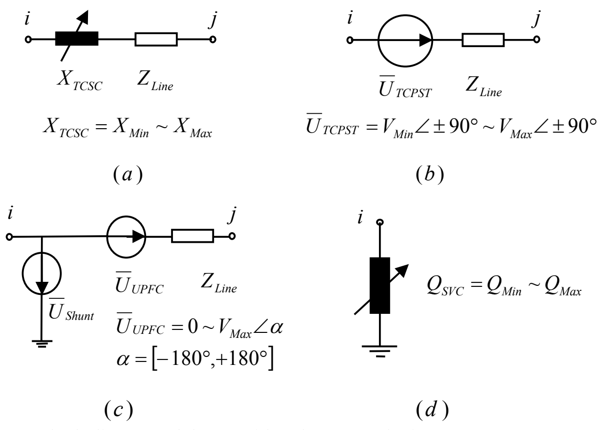

Fig. 1. Block diagram of the considered FACTS devices
(a) TCSC (b) TCPST (c) UPFC (d) SVC

As shown in Fig. 1, the reactance of the line can be changed by TCSC. TCPST varies the phase angle between the two terminal voltages and SVC can be used to control the reactive compensation. The UPFC is the most powerful and versatile FACTS device due to the facts that the line impedance, terminal voltages, and the voltage angle can be controlled by it as well.

The power flow P_{ij} through the transmission line *i-j* is a function of line impedance X_{ij}, the voltage magnitude V_j , V_j and the phase angle between the sending and receiving end voltages $\delta_i - \delta_j$.

$$P_{ij} = \frac{V_i V_j}{X_{ij}}\left(\delta_i - \delta_j\right) \tag{1}$$

The above-mentioned FACTS devices can be applied to control the power flow by changing the parameters of power systems so that the power flow can be optimized. Moreover, in a multi-machine network according to the different utilization of generation units in case of FACTS, the generation costs can also be reduced.

B. Mathematical Models of FACTS Devices

The mathematical models of the FACTS devices are developed mainly to perform the steady-state research. Therefore the TCSC is modeled to modify the reactance of the transmission directly. The SVC, TCPST and UPFC are modeled using the power injection method [4]. Furthermore, for the TCSC, TCPST and UPFC, their mathematical model is integrated into the model of the transmission line. Whereas the SVC model is only incorporated into the sending end as a shunt element of the transmission line.

The mathematical models of FACTS, as shown in Fig. 2, are implemented in Matpower 2.0 [9].

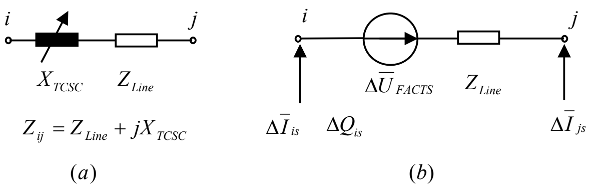

Fig. 2. Mathematical models of the FACTS devices.
(a) TCSC. (b) TCPST, UPFC and SVC.

TCSC

The TCSC can serve as the capacitive or inductive compensation respectively by modifying the reactance of the transmission line. In this simulation, the reactance of the transmission line is adjusted by TCSC directly. The rated value of TCSC is a function of the reactance of the transmission line where the TCSC is located:

$$X_{ij} = X_{Line} + X_{TCSC}, \quad X_{TCSC} = rt\csc \cdot X_{Line} \tag{2}$$

where X_{Line} is the reactance of the transmission line and *rtcsc* is the coefficient which represents the compensation degree of TCSC.

To avoid overcompensation, the working range of the TCSC is between $-0.7X_{Line}$ and 0.2 X_{Line} [1,2].

$$rt\csc_{\min} = -0.7, \quad rt\csc_{\max} = 0.2$$

TCPST

The voltage angle between the sending and receiving end of the transmission line can be regulated by TCPST. It is modeled as a series compensation voltage $\Delta\overline{U}_{FACTS} = \Delta\overline{U}_{TCPST}$, as shown in Fig 2. (b), which is perpendicular to the bus voltage. The working range of the TCPST is between -5 degrees to +5 degrees. The injected currents at bus *i* and bus *j* can be expressed as follows:

$$\Delta\overline{I}_{is} = \frac{\Delta\overline{U}_{TCPST}}{Z_{ij}}, \quad \Delta\overline{I}_{js} = -\frac{\Delta\overline{U}_{TCPST}}{Z_{ij}} \tag{3}$$

SVC

The SVC can be operated at both inductive and capacitive compensation. It is modeled as an ideal reactive power injection at at bus *i*, as shown in Fig. 2 (b). The injected power at bus *i* is:

$$\Delta Q_{is} = Q_{SVC} \tag{4}$$

UPFC

Basically, the UPFC has two voltage source inverters (VSI) sharing a common dc storage capacitor. It is connected to the system through two coupling transformers [6,7,12]. In this simulation, the series compensation $\Delta\overline{U}_{FACTS} = \Delta\overline{U}_{UPFC}$ is employed. The injected currents at bus *i* and bus *j* can be expressed as follows:

$$\Delta\overline{I}_{is} = \frac{\Delta\overline{U}_{UPFC}}{Z_{ij}}, \quad \Delta\overline{I}_{js} = -\frac{\Delta\overline{U}_{UPFC}}{Z_{ij}} \tag{5}$$

III. Cost Functions

As mentioned above, the main objective of this paper is to find the optimal locations of FACTS devices to minimize the overall cost function consisting of generation costs and FACTS devices investment costs.

For minimizing the generation costs in power systems, algorithms are well developed and being used for unit commitment and operation. In this work, a modified version of power simulation software: Matpower 2.0 is employed [9]. For the intended research, Matpower has been extended by incorporating the mathematical models of FACTS devices. Furthermore, cost functions are incorporated for:

- Generation costs.
- Investment costs of FACTS devices.

A. Generation cost function

The generation cost function is represented by a quadratic polynomial as follows:

$$c_2(P_G) = \alpha_0 + \alpha_1 P_G + \alpha_2 {P_G}^2 \tag{6}$$

Where P_G is the output of the generator (MW), and α_0, α_1 and α_2 are constant coefficients.

B. FACTS devices cost functions

Based on the Siemens AG Database [8], the cost functions for SVC, TCSC and UPFC are developed:

The cost function for UPFC is:

$$c_{1UPFC} = 0.0003s^2 - 0.2691s + 188.22(US\$/kVar) \tag{7}$$

For TCSC:

$$c_{1TCSC} = 0.0015s^2 - 0.7130s + 153.75(US\$/kVar) \tag{8}$$

For SVC:

$$c_{1SVC} = 0.0003s^2 - 0.3051s + 127.38(US\$/kVar) \tag{9}$$

where c_{1UPFC}, c_{1TCSC} and c_{1SVC} are in *US$/kVar* and *s* is the operating range of the FACTS devices in *MVar*.

The cost function for SVC, TCSC and UPFC are shown in Fig. 3.

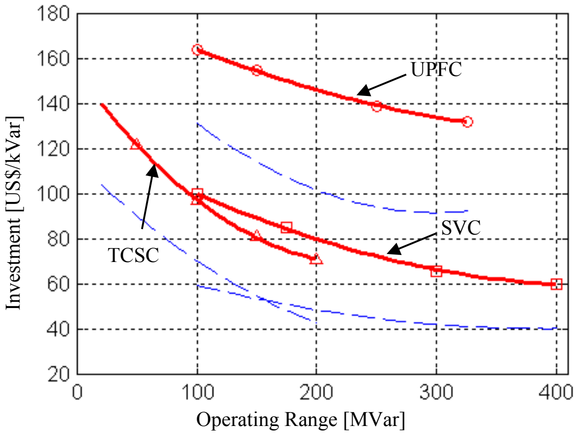

Fig. 3. Cost functions of the FACTS devices: SVC, TCSC and UPFC.
——: Upper limit: Total investment costs
------: Lower limit: Equipment costs
—o—: UPFC. —△—: TCSC. —□—: SVC.

The cost of a TCPST is more related to the operating voltage and the current rating of the circuit concerned [2,3,5]. Thus, once the TCPST is installed, the cost is fixed and the cost function can be expressed as follows [5]:

$$C_{TCPST} = d \cdot P_{\max} + IC \tag{10}$$

where d is a positive constant representing the capital cost and IC is the installation costs of the TCPST respectively. P_{max} is the thermal limit of the transmission line where TCPST is installed [5].

IV. Optimal FACTS Allocation

The formulation of the optimal allocation of FACTS devices can be expressed as follows [5]:

$$\begin{aligned} \min. \quad & c_{\text{Total}} = c_1(\mathbf{f}) + c_2(\mathbf{P_G}) \\ s.t. \quad & E(\mathbf{f},\mathbf{g}) = 0 \\ & B_1(\mathbf{f}) < \mathbf{b}_1, \;\; B_2(\mathbf{g}) < \mathbf{b}_2 \end{aligned} \tag{11}$$

where,

c_{Total}: the overall cost objective function which includes the average investment costs of FACTS devices $c_1(\mathbf{f})$ and the generation cost $c_2(\mathbf{P_G})$.

$E(\mathbf{f},\mathbf{g})$: the conventional power flow equations.

$B_1(\mathbf{f})$ and $B_2(\mathbf{g})$ are the inequality constrains for FACTS devices and the conventional power flow respectively.

f and $\mathbf{P_G}$ are vectors that represent the variables of FACTS devices and the active power outputs of the generators.

g represents the operating state of the power system.

The unit for generation cost is *US$/Hour* and for the investment costs of FACTS devices are *US$*. They must be unified into *US$/Hour*. Normally, the FACTS devices will be in-service for many years [10,12]. However, only a part of its lifetime is employed to regulate the power flow. In this paper, three years is applied to evaluate the cost function. Therefore the average value of the investment costs are calculated using the following equation:

$$c_1(\mathbf{f}) = \frac{c(\mathbf{f})}{8760 \times 3} (US\$ / Hour) \tag{12}$$

where $c(\mathbf{f})$ is the total investment costs of FACTS devices.

As mentioned above, power system parameters can be changed using FACTS devices. These different parameters derive different results on the objective function (11). Also, the variation of FACTS locations and FACTS types has also influences on the objective function. Therefore, using the conventional optimization methods is not easy to find the optimal location of FACTS devices, their types and their rated values simultaneously. To solve this problem, the genetic algorithms method is employed.

V. Genetic Algorithms

Based on the mechanisms of natural selection and genetics, GAs (genetic algorithms) are global search techniques. They can search several possible solutions simultaneously and they do not require any prior knowledge or special properties of the objective function [1,11]. Moreover, they always produce high quality solutions and, therefore, they are excellent methods for searching optimal solution in a complex problem.

The GAs start with random generation of initial population and then the selection, crossover and mutation are proceeded until the best population is found. Particularly, GAs are practical algorithm and easy to be implemented in the power system analysis.

A. Encoding

The objective is to find the optimal locations for the FACTS devices within the equality and inequality constrains. Therefore, the configuration of FACTS devices is encoded by three parameters: the location, type and its rated value [1]. Each individual is represented by n_{FACTS} number of strings, where n_{FACTS} is the number of FACTS devices needed to be analyzed in the power system, as shown in Fig. 4.

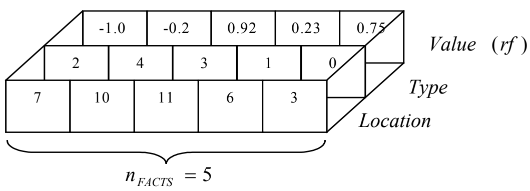

Fig. 4. Individual configuration of FACTS devices.

The first value of each string corresponds to the location information. It is the number of the transmission line where the FACTS is to be located. Each string has a different value of location [1]. In other words, it must be ensured that on one transmission line there is only one FACTS device. Moreover, SVC is installed only at one node of the transmission line and the sending node is selected in this simulation.

The second value represents the types of FACTS devices [1]. The values assigned to FACTS devices are: 1 for TCSC; 2 for TCPST; 3 for UPFC, 4 for SVC and 0 for no FACTS situation. Particularly, if there is no FACTS device needed on the transmission line, the value 0 will be employed.

The last value *rf* represents the rated value of each FACTS device. This value varies continually between –1 and +1. The real value of each FACTS device is then converted according to the different FACTS model under the following criterion:

TCSC: TCSC has a working range between $-0.7X_{Line}$ and $0.2X_{Line}$ [2,3], where X_{Line} is the reactance of the transmission line where the TCSC installed. Therefore *rf* is converted into the real compensation degree *rtcsc* using the following equation:

$$rt\,\mathrm{csc} = rf \times 0.45 - 0.25 \qquad (13)$$

UPFC: The inserted voltage of UPFC U_{UPFC} has a maximum magnitude of $0.1V_m$, where V_m is the rated voltage of the transmission line where the UPFC is installed. The angle of U_{UPFC} can be varied from $-180°$ to $180°$. Therefore *rf* is converted into the working angle *rupfc* using the following equation:

$$rupfc = rf \times 180 \ (degrees) \qquad (14)$$

TCPST: The working range of TCPST is between $-5°$ and $5°$. Then *rf* is converted into the real phase shift value *rtcpst* using the following equation:

$$rtcpst = rf \times 5 \ (degrees) \qquad (15)$$

SVC: The working range of SVC is between –100MVar and 100MVar. Then *rf* is converted into the real compensation value using:

$$rsvc = rf \times 100 \ (MVar) \qquad (16)$$

B. *Initial Population*

The initial population is generated from the following parameters [1]:

- n_{FACTS}: the number of FACTS devices to be located.
- n_{Type}: FACTS types.
- $n_{Location}$: the possible locations for FACTS devices.
- n_{Ind}: the number of individuals of the population.

First, as shown in Fig. 5, a set of n_{FACTS} numbers of strings are produced. For each string, the first value is randomly chosen from the possible locations $n_{Location}$.

The second value, which represents the types of FACTS devices, is obtained by randomly drawing numbers among the selected devices [1]. Particularly, after the optimization, if there is no FACTS device necessary for this transmission line, the second value will be set zero.

The third value of each string, which contains the rated values of the FACTS devices, are randomly selected between –1 and +1.

To obtain the entire initial population, the above operations are repeated n_{Ind} times [1]. Fig. 5 shows the calculation of the entire population.

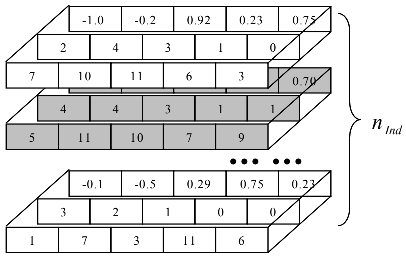

Fig. 5. Calculation of the entire population.

C. *Fitness Calculation*

After encoding, the objective function (fitness) will be evaluated for each individual of the population. The fitness is a measure of quality, which is used to compare different solutions [1, 11]. In this work, the fitness is defined as follows:

$$Fitness = m - c_{Total} \qquad (17)$$

Because the GAs can only find the maximum positive value of the objective function, a large positive constant *m* is selected to convert the objective function into a maximum one.

Then reproduction, crossover and mutation are applied successively to generate the offspring.

D. *Reproduction*

Reproduction is a process where the individual is selected to move to a new generation according to their fitness. The biased roulette wheel selection [1] is employed. The probability of an individual's reproduction is proportional to its part on the biased roulette wheel [11].

E. *Crossover*

The main objective of crossover is to reorganize the information of two different individuals and produce a new one [1,11].

A two-points crossover [1] is applied and the probability *pc* of the crossover is selected as 0.95. First, two crossing points are selected uniformly at random along the individuals. Elements outside these two points are kept to be part of the offspring. Then, from the first position of crossover to the second one, elements of the three strings of both parents are exchanged [1,11].

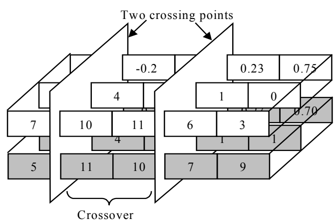

Fig. 6. Two points crossover.

F. Mutation

Mutation is used to introduce some sort of artificial diversification in the population to avoid premature convergence to local optimum [4,11]. Non-uniform mutation, which has proved to be successful in a number of studies [11], is employed in this paper.

For a given parent $X = x_1\, x_2 \ldots x_k \ldots x_l$, if the gene x_k is selected for mutation and the range of x_k is $\left[U_{Min}^k, U_{Max}^k\right]$, then the result $x_k^{'}$ is:

$$x_k^{'} = \begin{cases} x_k + \Delta\left(t, U_{Max}^k - x_k\right) & if \quad random\,(0,1) = 0 \\ x_k - \Delta\left(t, x_k - U_{Min}^k\right) & if \quad random\,(0,1) = 1 \end{cases} \tag{18}$$

where

$$\Delta(t, y) = y \cdot \left(1 - r^{\left(1 - \frac{t}{T}\right) \cdot b}\right) \tag{19}$$

$\Delta(t, y)$ (y represents $x_k - U_{Min}^k$ and $U_{Min}^k - x_k$) returns a value in the range $[0,y]$. Its probability being close to 0 and increases as t increases (t is generation number). This property enables the operator to search the space uniformly initially (when t is small), and very locally at later stages [11]. In (19), r is a random value in the range of $[0,1]$ and b is a parameter determining the degree of non-uniformity. In this simulation, b=2 is applied.

The above-mentioned operations of selection, crossover and mutation are repeated until the best individual is found.

The proposed optimization strategy is summarized in Fig. 7. In order to ensure that there is only one FACTS device on each transmission line, the process of 'Arrangement of the FACTS locations' is necessary.

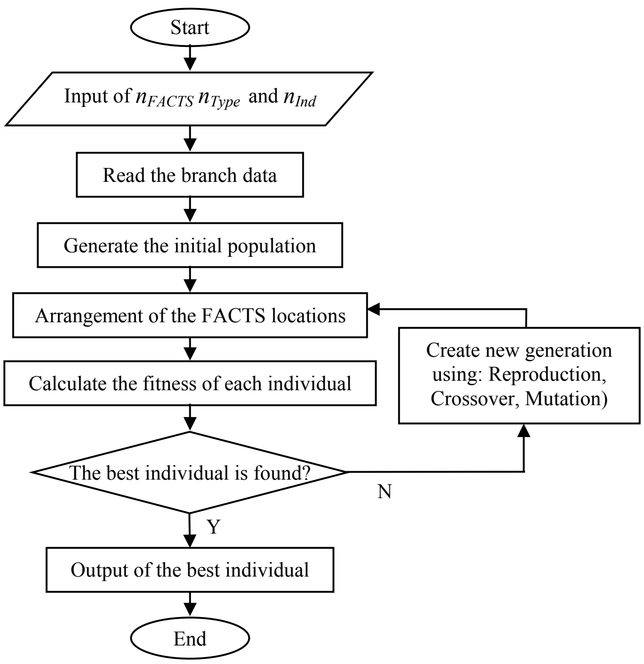

Fig. 7. Flow chart of the GA optimization.

VI. Case Study

In order to verify the effectiveness of the proposed method, the modified IEEE 14-bus test system (shown in Fig. 8) is used. The generation cost function is shown in Appendix A. Different operating conditions are simulated for the determination of the optimal FACTS locations.

The initial value of n_{FACTS}, which indicates the number of FACTS devices to be simulated, is defined as five. The total number of generation is 200 and there are 20 individuals in each generation.

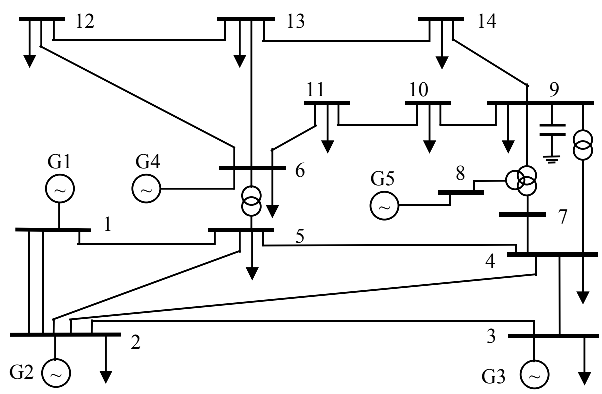

Fig. 8. IEEE 14-bus test system

A. Case 1. Voltage congestion at bus 2

In this case, all generators are in service and there is only voltage congestion at bus 2. The simulation result is shown in Fig. 9. After the optimization, for the considered power system only a SVC at bus 5 is necessary.

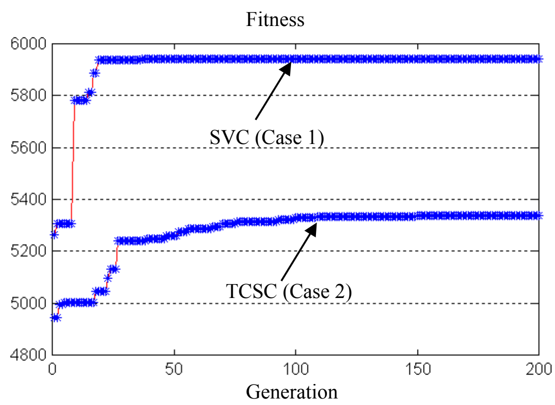

Fig. 9. Simulation results for case 1 and case 2

B. Case 2. Active power flow congestion on the transmission line between bus 1 and 5.

In this case, only generator 1 and 3 are in service and there is a congestion at the transmission line between bus 1 and bus 5. The simulation result is shown in Fig. 9. After the optimization, a TCSC needs to be installed between bus 2 and bus 5.

Simulation results show that for the considered power system, the TCSC and SVC are the most efficient. Moreover, SVC is only for the voltage congestion favorable. Even though UPFC is the most powerful FACTS device, it has not been applied due to its extremely high investment cost.

C. Practical application

Simultaneous optimization of the locations of the FACTS devices, their types and rated values is a very complicated optimization problem in large power systems. The proposed algorithm is suitable to search several possible solutions simultaneously. It always produces high quality solutions and it is faster than the traditional optimization methods in large power system researches. Furthermore, this algorithm is practical and easy to be implemented into the large power system analysis.

VII. Conclusions

In this paper, a genetic algorithm based approach is proposed to determine the suitable type of FACTS devices and its optimal location in power systems. The TCSC, UPFC, TCPST and SVC are simulated. The overall system cost function, which includes generation cost and the investment costs of the FACTS devices is employed to evaluate the power system performance.

Simulation results validate the efficiency of this new approach in minimizing the overall system cost function. Furthermore, the locations of the FACTS devices, their types and rated values are optimized simultaneously. The proposed algorithm is an effective and practical method for the allocation of FACTS devices in large power systems.

VIII. Appendices

A. Generation Cost Function

TABLE I
GENERATION COST FUNCTION

Coefficient / Generator	α_0	α_1	α_2
G1	100	60	0.06
G2	100	50	0.05
G3	500	300	1.00
G4	100	15	0.02
G5	100	45	0.03

B. Simulation Results

	Line No.	From Bus	To Bus	FACTS Type	Rated Value
Case 1	10	5	6	SVC	10.5Mvar
Case 2	5	2	5	TCSC	-22%X_{Line}

IX. References

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X. Biographies

Lijun Cai was born in 1970. He received his B.-Eng., M.-Eng. from Electrical Engineering Department, North China Electrical Power University, P. R. China in 1992 and 1997 respectively. He is now a Ph. D. candidate at the Institute of Electrical Power Systems in the University of Duisburg-Essen, Germany. His research interest is in the optimal location and multi-objective coordinated control of FACTS devices. He is a student member of IEEE.

István Erlich was born in 1953. He received his Dipl.-Ing. degree in electrical engineering from the University of Dresden, Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer-Institute IITB Dresden, respectively. During this time, he had also a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen, Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is member of IEEE and VDE.