

# Enhanced Reduced Order Model of Wind Turbines with DFIG for Power System Stability Studies

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**Abstract** This paper presents an extension to the stability model of the doubly fed induction generator (DFIG) used for wind turbines. With the proposed extension it is possible to consider the alternating components of the rotor current and thus the variation of the converter DC-link voltage. With realistic DC-voltage responses, a more accurate modeling of crowbar switching will now be possible. The paper also presents suitable models for rotor and line side converters, as well as for the DC-link. Moreover, the paper provides simulation results not only for model verification but also for demonstrating the behavior of the DFIG and the corresponding control system during fault.

**Index Terms**—Wind power, control system, power system stability, doubly-fed induction generator, crowbar

## I. NOMENCLATURE

|   |                                   |
|---|-----------------------------------|
| $\underline{i}, \underline{u}$  | complex current and voltage       |
| $l, x, r$   | inductance, reactance, resistance |
| $\underline{\psi}$  | complex flux-linkages             |
| $\omega, s$   | angular speed, slip               |
| $t, T$  | Torque, time constant             |
| $\Theta_m$  | Inertia of complete rotor shaft   |
| <u>subscripts</u>   |                                   |
| S, R  | Stator, rotor                     |
| d, q  | Direct, quadrature axis component |
| h, $\sigma$   | Main field, leakage               |
| Sign convention: consumed active power and inductive reactive power are considered positive |                                   |

## II. INTRODUCTION

THIS paper deals with the modeling of wind turbines equipped with doubly-fed induction generator (DFIG) in combination with static converter (dc-voltage link) and rotor side protection unit (crowbar) with a damping resistance connected in series. The basic structure is shown in Fig. 1. This variable-speed generator system permits the adjustment of rotor speed to match the optimum operating point depending on wind speed. This most commonly used system for wind power generation is characterized by high efficiency for the whole generator-converter-system.

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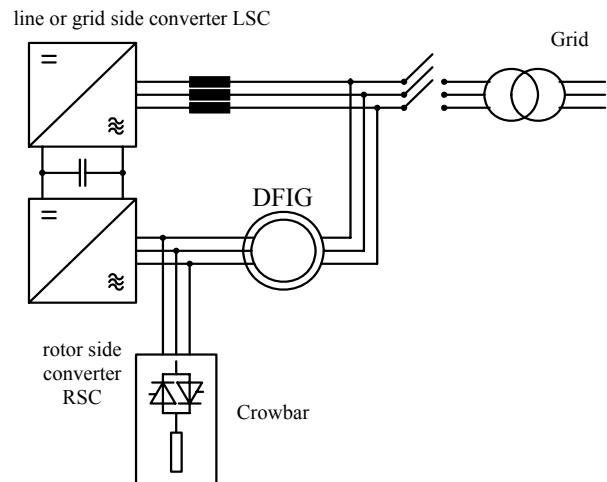


Fig. 1. Main components of the DFIG system

With the increasing share of wind turbines in electric power generation the dynamic behavior of the power system is impacted considerably. Therefore, many utilities have issued Grid Codes to define the basic requirement concerning wind turbine behavior during grid faults [1],[2]. The focus in this relation is directed at the fault ride through (FRT) capability and the voltage support to be provided by the wind turbines during fault. To prove conformity with grid requirements and to adapt control strategies to a particular grid, power system planners and grid operators need software tools and models representing wind turbines with sufficient accuracy. The challenge is to represent a large number of wind turbines in addition to conventional power plants connected to a large grid.

In [3] a reduced order model (ROM) for a DFIG is presented. This model allows the simulation of the operating performance of a large number of wind turbines (wind parks) connected to the power system. The presented model is based on neglecting the transformer terms in the stator and the grid on which the DFIG is operating. However, as a consequence of this simplification, the DC-components of stator current and stator flux are not contained in the corresponding solutions. On the other hand the stator DC- component has a fundamental influence on the behavior of the system. They result in alternating active power components on the rotor side which can lead to a fast rising voltage in the converter DC-link. When DC-link voltage exceeds a predefined limit, the crowbar has to be switched on for protection purposes. However, with the crowbar active, the DFIG operates as a

conventional slip-ring induction machine and can not be controlled by the rotor side converter. In this period the DFIG become a reactive power consumer.

One of the main objectives of this paper is to present a model extension to [3] which accounts for the neglected stator DC-components without increasing the modeling and simulation effort considerably. Thus the converter DC-circuit can be considered to represent the triggering of the crowbar firing more realistically. Simulation results at the end of this paper will show the dynamic performance of a typical wind energy DFIG system's response to a 3-phase grid fault, calculated using the improved model.

### III. GENERATOR (DFIG): BASIC MODEL

The reduced order model of the DFIG is summarized below. More detailed description is given in [3].

The fundamental system of equations for the DFIG is given by:

Voltage equations:

$$\underline{u}_S = r_S \dot{i}_S + \frac{d\underline{\psi}_S}{dt} + j\omega_S \underline{\psi}_S \quad (1)$$

$$\underline{u}_R = r_R \dot{i}_R + \frac{d\underline{\psi}_R}{dt} + j(\omega_S - \omega_R) \cdot \underline{\psi}_R \quad (2)$$

Flux equations:

$$\underline{\psi}_S = (l_h + l_{\sigma S}) \dot{i}_S + l_h \cdot \dot{i}_R \quad (3)$$

$$\underline{\psi}_R = l_h \dot{i}_S + (l_h + l_{\sigma R}) \dot{i}_R \quad (4)$$

Equation of motion

$$\frac{d\omega_R}{dt} = \frac{1}{\theta_m} (t_m + \text{Im}[\underline{\psi}_S^* \dot{i}_S]) \quad (5)$$

The orthogonal dq-reference frame used in this description is rotating at the speed  $\omega_S$ .

After some algebraic manipulation one obtains the complex state equation for the electrical stator and rotor circuits,

$$\frac{d\underline{\psi}_S}{dt} = \left( -\frac{r_S(l_h + l_{\sigma R})}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} - j\omega_S \right) \underline{\psi}_S + \frac{l_h r_S}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} \underline{\psi}_R + \underline{u}_S \quad (6)$$

$$\frac{d\underline{\psi}_R}{dt} = \frac{r_R l_h}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} \underline{\psi}_S - \left( \frac{r_R(l_h + l_{\sigma S})}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} + j(\omega_S - \omega_R) \right) \underline{\psi}_R + \underline{u}_R \quad (7)$$

which, together with the equation of motion (5), form the full order model (FOM) that can be used for instantaneous value, dynamic time domain simulations. Often the stator voltage is considered constant or interpreted as an independent input variable. That allows using the FOM for the study of a single machine operating on an infinite bus. However, the stator voltage can vary according to the type of interaction between DFIG and the grid. Basically FOM requires differential equations for the whole network due to the fact that the grid is directly connected to the stator circuit. Using the differential equations of the grid, the stator voltage in (6) can be eliminated. If the FOM is used for time domain simulation small integration step sizes are required due to the small time constants involved. The small integration time step, in

addition to the large number of differential equations especially for the grid, results in considerable simulation efforts when studying large systems. These disadvantages limit the application of the FOM to small grids or even to a single machine, infinite bus systems.

A reduced order model (ROM) can be derived by neglecting the derivative term in the stator equation, i.e. by setting  $\frac{d\underline{\psi}_S}{dt} = 0$ .

In the strict sense, this approximation is only warranted in a reference frame rotating at a speed close to the synchronous speed  $\omega_0 = 1.0 p.u.$ . The transformation of (6) and (7) into a synchronously rotating reference frame results in the transition  $\omega_S \rightarrow \omega_0$ . The ROM of the DFIG derived in [3] is described by an algebraic equivalent circuit shown in Fig. 2 where the transient impedance  $\underline{z}'$  is defined as:

$$\underline{z}' = r_S + jx' = r_S + j\omega_0 \left( l_h + l_{\sigma S} - \frac{l_h^2}{l_h + l_{\sigma R}} \right) \quad (8)$$

The Thevenin voltage source  $\underline{u}'$  behind the impedance  $\underline{z}'$  is a function of the state variable rotor flux

$$\underline{u}' = j\omega_0 k_R \underline{\psi}_R \quad (9)$$

With  $k_R = \frac{l_h}{l_h + l_{\sigma R}}$

To consider the time behavior of the rotor flux the following differential equation has to be solved:

$$\frac{d\underline{\psi}_R}{dt} = \left( -\frac{r_R}{l_h + l_{\sigma R}} + j(\omega_0 - \omega_R) \right) \underline{\psi}_R + k_R r_R \dot{i}_S + \underline{u}_R \quad (10)$$

Furthermore, the model has to be extended by the equation of motion that is also adapted according to the model reduction introduced.

$$\frac{d\omega_R}{dt} = \frac{1}{\theta_m} (t_m + \text{Im}[k_R \underline{\psi}_R^* \dot{i}_S]) \quad (11)$$

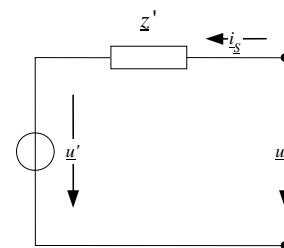


Fig. 2. Equivalent circuit of the reduced order DFIG model

In the ROM the stator flux is no more a state variable and therefore, can change instantaneously. Concerning the rotor flux behavior, the approximation introduced is clearly justified when the crowbar is not switched on (see upper part of Fig. 3). However, with the crowbar active, the rotor flux contains considerable alternating components not covered by the ROM.

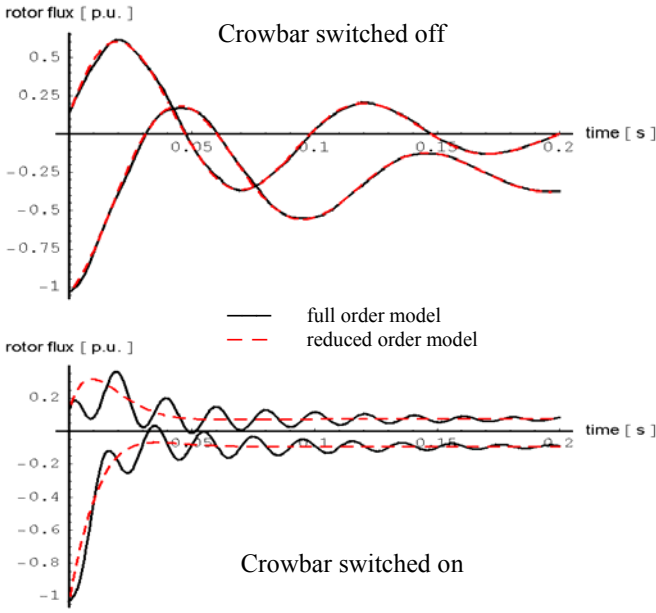


Fig. 3. Rotor flux-components  $\psi_{Rd}, \psi_{Rq}$ , caused by a three-phase voltage sag to 15 %

#### IV. ENHANCED GENERATOR (DFIG) MODEL

In a practical situation, DC-components can appear on both stator and rotor sides, which can only be captured by the FOM. They generate an alternating active power resulting in fast rise of the DC-link voltage and subsequently can lead to crowbar switching. Because the DC-components are not part of the ROM solution, this model (ROM) doesn't provide correct information for initializing crowbar control. For this purpose an enhanced model is proposed in this paper which still allows the use of the ROM, but activates an additional model part to add the DC-components to the simulation results as necessary.

The idea is based on the assumption that the stator flux calculated using the ROM is just the slow component of  $\underline{\psi}_s$ .

From (6) follows with  $\frac{d\underline{\psi}_s}{dt} = 0$

$$0 = \left( -\frac{r_s(l_h + l_{\sigma R})}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} - j\omega_0 \right) \underline{\psi}_s^{ROM} + \frac{l_h r_s}{l_h(l_{\sigma S} + l_{\sigma R}) + l_{\sigma S}l_{\sigma R}} \underline{\psi}_R^{ROM} + \underline{u}_s^{ROM} \quad (12)$$

where the superscript "ROM" signifies the slow ROM solution. In synchronous reference frame subtracting (18) from (6) results in

$$\frac{d\tilde{\psi}_s}{dt} = \left( -\frac{(r_s + r_N)(l_h + l_{\sigma R})}{lh(l_{\sigma S} + l_N + l_{\sigma R}) + (l_{\sigma S} + l_N)l_{\sigma R}} - j\omega_0 \right) (\tilde{\psi}_s - \underline{\psi}_s^{ROM}) \quad (13)$$

under the following assumptions:

- 1)  $\underline{\psi}_R \approx \underline{\psi}_R^{ROM}$  This is nearly fulfilled when the crowbar is switched off (see Fig.3). With crowbar this assumption is less tenable but still acceptable.
- 2)  $\underline{u}_s = \underline{u}_s^{ROM}$  This presupposes that the stator terminal is extended up to the Thevenin equivalent voltage of the grid

$\underline{u}_{SN}$  in accordance with Fig. 4. In this case the stator parameters must be also modified as  $r_s \rightarrow r_s + r_N$  and  $l_{\sigma S} \rightarrow l_{\sigma S} + l_N$ . However, it should be emphasized that it is not necessary to know  $\underline{u}_{SN}$  specifically, but the assumption must be, in principle, possible. The corresponding equivalent grid impedance is easily calculated from the well known short circuit capacity  $S_k^*$ . It is obvious that the suggested extension of the stator circuits to a virtual voltage source corresponds with the assumptions used for the standard short-circuit current calculation. However, the approach presupposes a constant equivalent grid impedance. Therefore, the simulation is restricted to cases where the impedance is not significantly effected by the grid fault.

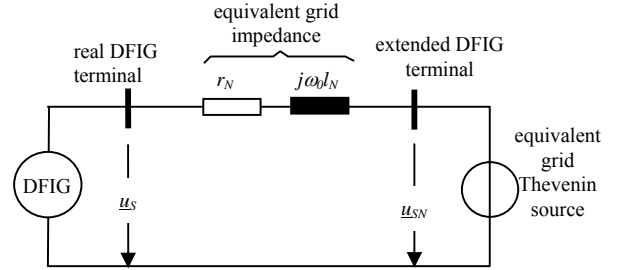


Fig. 4 Extension of the DFIG terminal

In (13) the stator flux is given as  $\tilde{\psi}_s$  to indicate the approximation used as well as the extension of the stator through the grid equivalent. Equation (13) has only the input variable  $\underline{\psi}_s^{ROM}$  that is calculated by

$$\underline{\psi}_s^{ROM} = (x' + \omega_0 l_N) \underline{i}_s^{ROM} + k_R \underline{\psi}_R^{ROM} \quad (14)$$

and so (13) can be solved parallel to the ROM equation system as shown in Fig. 5.

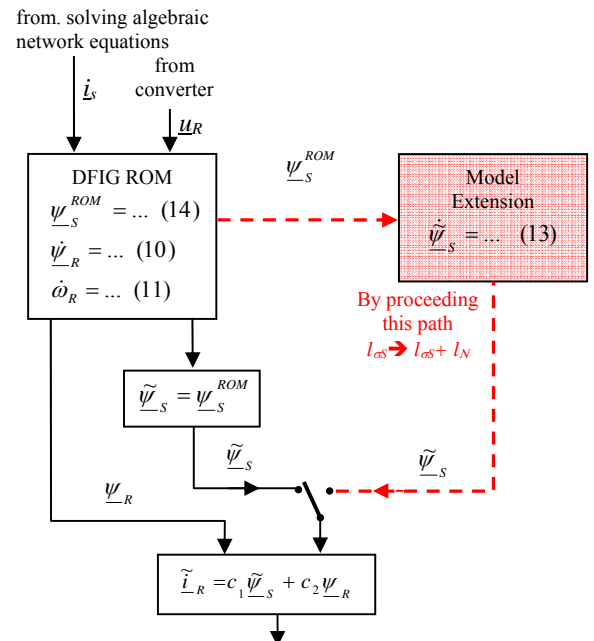


Fig. 5. Simulation algorithm using ROM extension

The coefficients  $c_1$  and  $c_2$  follow from (3) and (4):

$$c_1 = -\frac{l_h}{l_h(l_{cs} + l_N + l_{cr}) + (l_{cs} + l_N)l_{cr}} \quad (15)$$

$$c_2 = \frac{l_{cs} + l_h}{l_h(l_{cs} + l_N + l_{cr}) + (l_{cs} + l_N)l_{cr}} \quad (16)$$

Notice that the network equivalent inductance  $l_N$  is zero when only the standard ROM model is used. The model extension has to be considered just following grid faults and can be disregarded when the difference between  $\tilde{\psi}_s$  and  $\psi_s^{ROM}$  is negligible.

The ROM with the proposed extension (ROM/E) allows the calculation of the rotor currents including the corresponding alternating components. Fig. 6 demonstrates the accuracy.

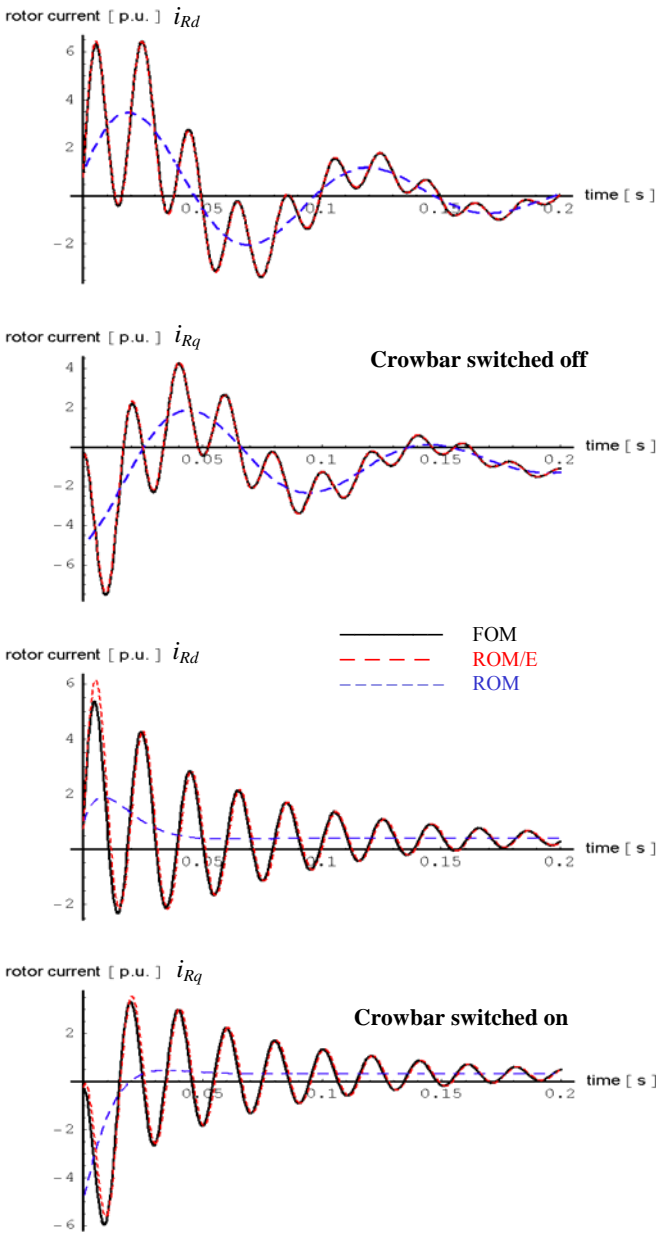


Fig. 6. Rotor currents following a three-phase voltage sag from rated voltage down to 15 %.

The ROM/E model provides very accurate results when the crowbar is not included. Actually, no difference between FOM and ROM/E results is observable in Fig. 6. With crowbar the difference between FOM and ROM/E is already visible but still acceptable from practical point of view. The impairment of model accuracy is evident, when one considers the fact that the rotor resistance increases when the crowbar is switched on. Thus the characteristic speed difference between the fast stator and the slow rotor flux is reduced, which is the physical basis for model reduction and so for the derivation of the ROM.

The ROM/E can be interpreted as an approximation of the full order DFIG model, but without using differential equations for the whole network. The integration step size required for ROM/E is usually smaller than that using the ROM only. However, the model extension is necessary only temporarily and therefore the integration step size can be increased when the model is switched back from ROM/E to ROM.

## V. MODEL OF THE ROTOR SIDE CONVERTER AND DC-LINK

A realistic simulation of the crowbar action requires modeling of the converter DC-circuit because the triggering signal is typically derived from the DC-voltage. The DC-circuit contains a capacitor which is charged/discharged by the rotor and grid side converter currents, respectively. However, the capacitor is usually not big enough for smoothing the DC-voltage variations caused by the alternating rotor current. Therefore, accurate rotor currents are the pre-requisite for modeling of crowbar switching. In some applications the converter DC-link is extended by a chopper to keep the DC-voltage within limits thereby reducing the number of the crowbar actions or even under circumstances forestalling crowbar action altogether.

The time behavior of the converter DC-voltage can be described by the following equation:

$$\frac{du_{DC}}{dt} = \frac{\Delta p_{DC}}{u_{DC} \cdot c_{DC}} \quad (17)$$

where

$$\Delta p_{DC} = p_{RSC} + p_{LSC} + p_{Chopper} - p_{RSC\_losses} - p_{LSC\_losses} \quad (18)$$

with

$$p_{RSC} = -(u_{Rd} i_{Rd} + u_{Rq} i_{Rq})$$

$$p_{LSC} = u_{LSC,d} i_{LSC,d} + u_{LSC,q} i_{LSC,q}$$

$$p_{Chopper} = -\frac{u_{DC}^2}{r_{Chopper}} \quad \text{chopper switched on}$$

$$p_{Chopper} = 0 \quad \text{chopper switched off}$$

The chopper is active when the DC-voltage exceeds a predefined threshold. The converter losses are usually small (approximately 1% of rated output power) and therefore, negligible for the desired stability type of simulations. However, for higher accuracy requirements a polynomial approach is applicable for both RSC and LSC as well











