

# A NEW ADAPTIVE DIFFERENTIAL EVOLUTION ALGORITHM FOR VOLTAGE STABILITY CONSTRAINED OPTIMAL POWER FLOW

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**Abstract** – This paper presents a new adaptive differential evolution algorithm namely JADE-vPS for the voltage stability constrained optimal power flow (VSCOPF) problem. The active and reactive power dispatch problems are solved in a single step. The objective is to minimize the total fuel cost while taking security and voltage stability constraints into consideration. VSCOPF is formulated as a mixed-integer nonlinear programming problem. The proposed JADE-vPS algorithm eliminates the common task of users in tuning control parameters before commencing the optimization process. Simulations were carried out on IEEE-30 bus system. Results of JADE-vPS are compared with few other differential evolution algorithms. It is shown that JADE-vPS is a convenient and powerful optimization algorithm to handle VSCOPF.

**Keywords:** *Optimal power flow, Voltage stability, Differential evolution, Control parameter adaptation*

## 1 INTRODUCTION

Optimal power flow (OPF) is an optimization tool used to schedule the control parameters of power systems in such a manner that the objective function is minimized or maximized [1]. Operating constraints of equipments, security requirement and stability limits are enforced to the solution. Optimal reactive power and voltage (Q-U) control is an OPF sub-problem which has a significant impact on economic and secure operation of power systems.

Traditionally, the feasible operation of Q-U control has been defined by voltage profile criteria ensuring that system voltage profile is maintained both in normal and post contingency conditions. In some power utilities concerned with voltage stability problems, the voltage profile criteria alone are not sufficient. Reactive power sources in the transmission system should be properly scheduled in order to protect the system against voltage instability by ensuring a sufficient voltage stability margin at all times [2]. In other words, the Q-U control problem of those utilities can be considered as the voltage stability constrained OPF problem (VSCOPF) such as proposed in [3]. This is an important task in operating modern power systems besides economic objectives, such as minimization of power generation costs, etc.

In literature several techniques, such as nonlinear programming, successive linear programming, interior point methods, have been applied to solve the OPF problem [4]. Although these methods have demonstrated efficacy in handling large scale OPF problems, they may fail to find the global optimum in many difficult problems and thereby reach a local optimum.

Major difficulties of the OPF are due to non-convex and discontinuous cost characteristics, mixed integer variables, a large number of constraints. Moreover, the solution quality of classical methods is quite sensitive to the starting points. To eliminate these problems, meta-heuristics becomes an attractive tool for solving real-world and complex OPF. These methods include genetic algorithm (GA) [5, 6], particle swarm optimization (PSO) [7, 8], differential evolution (DE) [9, 10], evolutionary programming (EP) [11], bacterial foraging optimization (BFO) [12], etc. It is a common practice of a user of these algorithms to specify an appropriate set of parameters.

To eliminate this process and enhance the search capability, these control parameters are adapted during the optimization course. Recently, an adaptive differential evolution algorithm JADE [13] as named by the authors has demonstrated significant improvement in quality of solutions. In JADE, two major parameters namely mutation factor and crossover rate are already self-adapted. The only remaining parameter; population size (*PS*) must be still specified by the user. In this paper, we explore the possibility of allowing *PS* to be automatically adapted in a very similar manner to the other two parameters. We name this improved version the JADE with variable population size (JADE-vPS). This algorithm is applied to solve the VSCOPF problem with the objective to minimize the cost of power generation while maintaining a sufficient voltage stability margin. The key advantage of the problem formulation here is that active and reactive powers are optimized simultaneously.

The rest of this paper is organized as follows. Section 2 provides the VSCOPF problem formulation. The developed JADE-vPS algorithm is presented in Section 3. A short description of the constraint handling technique is also given in this section. The test power system and simulation settings are described at the beginning of Section 4 and followed by simulation results. Then, the paper is finally concluded in Section 5.

## 2 VSCOPF PROBLEM FORMULATION

The VSCOPF problem solves for the optimal settings of control variables consisting of power output of generators and settings of reactive power controls. The objective is to minimize the total fuel cost of all generating units. Operational ranges of equipments such as transformer tap position, security limits and the system voltage stability margin are taken into consideration.

Therefore, the optimization problem involves solving a set of nonlinear equations expressed as:

$$\begin{aligned} & \text{Minimize } f(\mathbf{x}, \mathbf{u}) \\ & \text{subject to } g(\mathbf{x}, \mathbf{u}) \leq 0 \\ & \quad h(\mathbf{x}, \mathbf{u}) = 0 \end{aligned} \quad (1)$$

The vector  $\mathbf{x}$  contains control variables and  $\mathbf{u}$  contains dependent variables. Here, the objective function  $f$  is the total cost of power generation of all  $n$  generating units represented by:

$$f = \sum_{i=1}^n FC_i(P_i) \quad (2)$$

where  $FC_i$  is the fuel cost function of generator  $i$ ;  $P_i$  is the power output of generator  $i$ . The valve-point effect is derived from the opening process of multivalve steam turbines. It produces ripples in the generator heat rate characteristic. Therefore, the fuel cost function of the generator with this characteristic shows the higher degree of nonlinearity as follows:

$$FC_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \sin(f_i (P_i^{\min} - P_i)) \right| \quad (3)$$

where  $a_i$ ,  $b_i$ ,  $c_i$ ,  $e_i$  and  $f_i$  are cost coefficients of generator  $i$ , obtained by the heat-characteristic test;  $P_i^{\min}$  is the minimum generation capacity of generator  $i$ .

The set of nonlinear inequality constraints is represented by  $g(\mathbf{x}, \mathbf{u})$ . In this paper, control variables consist of active and reactive power outputs of generators and transformer tap positions as shown in (4)-(6). Dependent variables comprise bus voltage, power line flow and voltage stability margin as given in (7)-(9). All constraints are listed below.

- Prohibited operating zone(POZ) limits

Some thermal units cannot be operated in the entire capacity range due to some physical limitations. These units may have prohibited operating zones due to faults in the machines themselves or auxiliary equipments [14]. Operation in these zones would lead to instability problems. Hence, a feasible operating point of these generators should not be in one of these zones. This constraint can be written as follows:

$$P_i \in \begin{cases} P_i^{\min} \leq P_i \leq PZ_{i,1}^l \\ PZ_{i,k-1}^u \leq P_i \leq P_{i,k}^l \\ PZ_{i,pz_i}^u \leq P_i \leq P_i^{\max} \end{cases} \quad \forall i \in \mathbf{s}_{GPZ} \quad (4)$$

where  $P_i^{\min}$  and  $P_i^{\max}$  are minimum and maximum generation limits;  $PZ_{i,k}^l$  and  $PZ_{i,k}^u$  are lower and upper bounds of prohibited zone  $k$  of unit  $i$ , respectively;  $\mathbf{s}_{GPZ}$  is the set of generator buses with the prohibited operating zone.

- Generator reactive power ( $Q_G$ ) limits

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad \forall i \in \mathbf{s}_G \quad (5)$$

The set  $\mathbf{s}_G$  contains generator buses.

- Transformer tap position (*tap*) limits

$$tap_i^{\min} \leq tap_i \leq tap_i^{\max} \quad \forall i \in \mathbf{s}_T \quad (6)$$

The set  $\mathbf{s}_T$  contains under-load tap changers.

- Bus voltage ( $U$ ) limits

$$U_i^{\min} \leq U_i \leq U_i^{\max} \quad \forall i \in \mathbf{s}_B \quad (7)$$

The set  $\mathbf{s}_B$  contains all buses in the power system.

- Power line flow ( $S$ ) limits

$$|S_i| \leq |S_i^{\max}| \quad \forall i \in \mathbf{v}_L \quad (8)$$

The set  $\mathbf{v}_L$  contains all transmission lines.

- Voltage stability margin ( $\lambda$ ) limit

The loading parameter  $\lambda$  is used as the voltage stability margin (VSM). It is defined by the percentage of increase in generation and load from a base condition until the system reaches the voltage collapse point. A feasible operating condition should guarantee minimum VSM denoted by  $\lambda^{\max}$  as:

$$\lambda \geq \lambda^{\max} \quad (9)$$

Continuation power flow [15] is the method to determine  $\lambda$  for each operating condition.

$h(\mathbf{x}, \mathbf{u})$  is a set of nonlinear equality constraints representing power flow equations as follows:

$$0 = P_{Gi} - P_{Di} - U_i \sum_{j=1}^N U_j Y_{ij} \cos(\theta_i - \theta_j - \delta_{ij}) \quad (10)$$

$$0 = Q_{Gi} - Q_{Di} - U_i \sum_{j=1}^N U_j Y_{ij} \sin(\theta_i - \theta_j - \delta_{ij}) \quad (11)$$

where  $P_{Gi}$  and  $Q_{Gi}$  are active and reactive power generation at bus  $i$ , respectively;  $P_{Di}$  and  $Q_{Di}$  are active and reactive power load at bus  $i$ , respectively;  $Y_{ij}$  is the admittance matrix corresponding to the  $i$  row  $j$  column and  $\delta_{ij}$  is the difference in the voltage angle between the  $i$  and  $j$  buses;  $N$  is the total number of buses.

### 3 THE IMPROVED DIFFERENTIAL EVOLUTION ALGORITHM

#### 3.1 Basics of differential evolution

Differential evolution (DE) initially proposed in [16] is a popular evolutionary algorithm (EA) gaining significant interests during recent years due to its simplicity, efficiency and robustness. DE shares a number of similar concepts to other EAs. The evident distinction is the mutation step used to generate a new candidate solution. A parent individual moves to its new location in the search space by adding the weighted vector of difference between the other two vectors randomly chosen from the population. For an individual  $i$  at generation  $G$ , the mutant vector  $\mathbf{v}_i^G$  is computed by:

$$\mathbf{v}_i^G = \mathbf{x}_{r_0}^G + F_i(\mathbf{x}_{r_1}^G - \mathbf{x}_{r_2}^G) \quad (12)$$

where  $r_0 \neq r_1 \neq r_2 \neq i$  are distinct integers uniformly chosen from the set  $\{1, 2, \dots, NP\}$ ;  $NP$  is the population size;  $F_i$  is the mutation strategy for the individual  $i$  usually in the range  $(0, 1)$ .

To balance the search capabilities between “diversification” and “intensification”, the “DE/current-to-best/1” mutation strategy can be applied as follows:

$$\mathbf{v}_i^G = \mathbf{x}_i^G + F_i(\mathbf{x}_{best}^G - \mathbf{x}_i^G) + F_i(\mathbf{x}_{r_1}^G - \mathbf{x}_{r_2}^G) \quad (13)$$

where  $r_1 \neq r_2 \neq i$  are distinct integers uniformly chosen between 1 and  $NP$ . This strategy helps accelerate the convergence because of the additional information given by the global best vector.

After the mutation, a trial vector  $\mathbf{u}_i^G$  is generated by mixing the components of target vector and mutant vector as follows:

$$u_{i,j}^G = \begin{cases} v_{i,j}^G & \text{if } rand \leq CR_i \text{ or } j = j_{rand} \\ x_{i,j}^G & \text{otherwise} \end{cases} \quad (14)$$

$j=1, 2, \dots, D$

where  $rand$  is a uniform random number in  $[0, 1]$ ;  $CR_i \in [0, 1]$  corresponds to the crossover rate of the vector  $i$ ;  $j_{rand}$  is an integer randomly chosen between 1 and  $D$ ;  $D$  is the total number of variables.

The better vector between the parent  $\mathbf{x}_i^G$  and the trial  $\mathbf{u}_i^G$  is selected to be a member of population in the next generation  $G+1$  according to their fitness values  $fit(\cdot)$ . For a minimization problem, the selected vector is given by:

$$\mathbf{x}_i^{G+1} = \begin{cases} \mathbf{u}_i^G & \text{if } fit(\mathbf{u}_i^G) < fit(\mathbf{x}_i^G) \\ \mathbf{x}_i^G & \text{otherwise} \end{cases} \quad (15)$$

### 3.2 Control parameter adaptation

Research in evolutionary computation has demonstrated that search performance of an EA depends on the suitable setting of control parameters. In DE there are three control parameters namely the mutation factor  $F$ , the crossover rate  $CR$  and the population size  $NP$ . Although there are some guidelines for appropriate settings for these parameters such as [17, 18], the interaction between the parameter setting and the search performance is quite complicated. Inappropriate control parameters may lead to false or slow convergence. Classical trial-and-error tuning is generally computationally intensive and thereby time consuming. Consequently, automatic adaptation of control parameters is a solution to overcome such inconvenience such as [19, 20].

Followed the same line of idea proposed in [13], this paper extends the original JADE is extended by allowing the population size  $PS$  to be adaptively adjusted. This algorithm is named by the authors the JADE with variable population size (JADE-vPS). Figure 1 shows the variable encoding scheme of JADE-vPS with  $PS$  at

generation  $G$ . An individual is represented by a row vector consisting of four components: the vector of candidate solution  $\mathbf{x}$ , two scalars  $F$  and  $CR$  and the new variable namely  $NPn$ . In JADE-vPS, only  $NP \leq PS$  is updated by the evolutionary operation.

$\mathbf{x}_1^G$	$F_1^G$	$CR_1^G$	$NPn_1^G$
$\mathbf{x}_2^G$	$F_2^G$	$CR_2^G$	$NPn_2^G$
$\mathbf{x}_3^G$	$F_3^G$	$CR_3^G$	$NPn_3^G$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbf{x}_{PS}^G$	$F_{PS}^G$	$CR_{PS}^G$	$NPn_{PS}^G$

Figure 1: Encoding scheme of JADE-vPS

At the beginning,  $PS^{\text{ini}}$  individuals are randomly initialized within the boundaries. In our implementation,  $PS^{\text{ini}}$  is set to  $3 \cdot D$  where  $D$  is the problem dimension. At the initial stage,  $NP=PS$  individuals are updated by the same evolutionary procedure as JADE. Upon completion of every generation, the variable  $NP$  for the next generation is determined. If  $NP > PS$ , then  $(PS-NP)$  new individuals have to be inserted. In our current version,  $PS$  is bounded in  $[PS^{\text{min}}, PS^{\text{max}}]$  representing the minimum ( $0.5 \cdot PS^{\text{ini}}$ ) and maximum ( $2 \cdot PS^{\text{ini}}$ ) allowable population size, respectively. For each individual  $i$ , the normalized number of individuals is defined by:

$$NPn_i^G = randn(\mu_{NPn}^G, 0.1) \quad (16)$$

where  $\mu_{NPn}^G$  is the mean value of all  $NPn$  variables.

When the algorithm starts,  $\mu_{NPn}$  is initialized to 0.5. The mean value of  $NPn$  for the next generation is updated by:

$$\mu_{NPn}^{G+1} = (1 - c_1)\mu_{NPn}^G + c_1 \cdot mean_A(\mathbf{s}_{NPn}) \quad (17)$$

where  $c_1$  is a small constant (in this study set to 0.01);  $\mathbf{s}_{NPn}$  is the vector storing the normalized number of individuals corresponding to the successful candidates;  $mean_A(\cdot)$  is the arithmetic mean operator for a vector. The individual  $i$  is defined to be successful if the first condition of (15) is achieved. Then, the number of individual that the algorithm will evolve in the generation  $G+1$  is determined by:

$$NP^{G+1} = PS^{\text{min}} + round\{\mu_{NPn}^{G+1}(PS^{\text{max}} - PS^{\text{min}})\} \quad (18)$$

where  $round\{\cdot\}$  is the round-up function. As stated earlier,  $(NP^{G+1} - PS^G)$  individuals may have to be inserted to the population. There are two simple alternatives to address this issue. First, the best individuals can be duplicated to fulfill the requirement. The other solution is to insert new individuals which can be randomly close to the local best solutions to intensify the search process or in the other hyper-spheres excluding the local bests. The latter strategy in some cases is beneficial to

prevent the population trapping to a local optimum. In this paper, we adopt the first mechanism in adding local best solutions. By doing this, as discussed earlier the search process could be biased toward a not truly optimal direction. In this paper, we have restricted the number of local best solutions  $PS^{added}$  that can be inserted to the population by offsetting  $(NP^{G+1} - PS^G)$  according to:

$$PS^{added} = \min\{(NP^{G+1} - PS^G), 0.05 \cdot PS^{ini}\} \quad (19)$$

Hence the population size of the next generation becomes:

$$PS^{G+1} = PS^G + PS^{added} \quad (20)$$

To select  $NP$  individuals that will be updated at the generation  $G$ , a vector denoted  $\mathbf{r}_{pi}$  whose elements are random permutation between 1 and  $PS$  is generated. Then, the first  $NP$  elements of  $\mathbf{r}_{pi}$  indicate the individuals on which the algorithm will operate.

For each individual  $i = \mathbf{r}_{pi}(k); k = 1, 2, \dots, NP$ , the mutant vector is generated by:

$$\mathbf{v}_i^G = \mathbf{x}_i^G + F_i(\mathbf{x}_{best,p}^G - \mathbf{x}_i^G) + F_i(\mathbf{x}_{r1}^G - \mathbf{x}_{r2}^G) \quad (21)$$

where  $r1 \neq r2 \neq i$  are two distinct integers randomly chosen from  $\{1, 2, \dots, PS\}$ ;  $\mathbf{x}_{best,p}^G$  is randomly chosen from the top best 100p % of the current population denoted as  $X_P$  at generation  $G$  with probability  $p \in [0, 1]$ .

$\mathbf{x}_{r2}^G$  is a vector randomly chosen from the set of  $X_P \cup X_A$  where  $X_A$  is the solution archive storing the local best solutions that the algorithm has discovered so far.

The remaining processes of JADE-vPS stay unchanged from the original JADE.

### 3.3 Adaptive penalty method

To eliminate the parameter tuning process, the self-learning penalty function proposed in [21] is applied in this paper. The elegant feature of this technique is that it is parameter-less and capable of adjusting the penalized fitness function at different stages of the search process. The fitness evaluation function is computed by:

$$fit(\mathbf{x}) = d(\mathbf{x}) + p(\mathbf{x}) \quad (22)$$

where  $d(\mathbf{x})$  and  $p(\mathbf{x})$  represent the distance term and the penalty term of an individual  $\mathbf{x}$ , respectively. The distance value can be determined by:

$$d(\mathbf{x}) = \begin{cases} v'(\mathbf{x}) & \text{if } r_f = 0 \\ \sqrt{f'(\mathbf{x})^2 + v'(\mathbf{x})^2} & \text{otherwise} \end{cases} \quad (23)$$

where  $r_f$  is the ratio of the feasible individuals in the population;  $f'(\mathbf{x})$  is the normalized objective value and  $v'(\mathbf{x})$  is the total normalized constraint violations. The penalty term  $p(\mathbf{x})$  is set up to ensure that an infeasible individual will receive the appropriate fitness value at different stages of the search process in terms of the feasibility and objective value. For example, when the

population lacks feasible individuals, infeasible individuals with lower constraint violations should receive relatively less penalty irrespective to the objective value. On the other hand when there are a reasonable number of feasible individuals, infeasible individuals with lower objective value are assigned lower penalty. This idea can be mathematically expressed by:

$$p(\mathbf{x}) = (1 - r_f)X(\mathbf{x}) + r_f Y(\mathbf{x}) \quad (24)$$

where

$$X(\mathbf{x}) = \begin{cases} 0 & \text{if } r_f = 0 \\ v'(\mathbf{x}) & \text{otherwise} \end{cases} \quad (25)$$

$$Y(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \text{ is feasible} \\ f'(\mathbf{x}) & \text{if } \mathbf{x} \text{ is infeasible} \end{cases} \quad (26)$$

### 3.4 Handling prohibited operating zone constraints

The power generation outputs have to be within its feasible operating boundaries (4). This constraint becomes hard to maintain because of the existence disjoint search spaces. Therefore, every element of power generation vector must be checked. If the power production of the generator  $j$  is in the prohibited operating zone  $k$ , the following rule for adjusting the power production is applied.

$$P_j = \begin{cases} PZ_{j,k-1}^u + rand \cdot (PZ_{j,k}^l - PZ_{j,k-1}^u) & \text{if } d_l < d_u \\ PZ_{j,k}^u + rand \cdot (PZ_{j,k+1}^l - PZ_{j,k}^u) & \text{otherwise} \end{cases} \quad (27)$$

$$d_l = |P_j - PZ_{j,k}^l| \text{ and } d_u = |P_j - PZ_{j,k}^u| \quad (28)$$

The variables  $d_l$  and  $d_u$  measure the absolute distances from the power generation inside the prohibited zone  $k$  to the corresponding lower and upper bounds, respectively.

### 3.5 Termination criteria

In this paper, all of the testing algorithms stop if the number of fitness evaluation  $feval$  reaches the maximum allowable number  $feval^{max}$ .

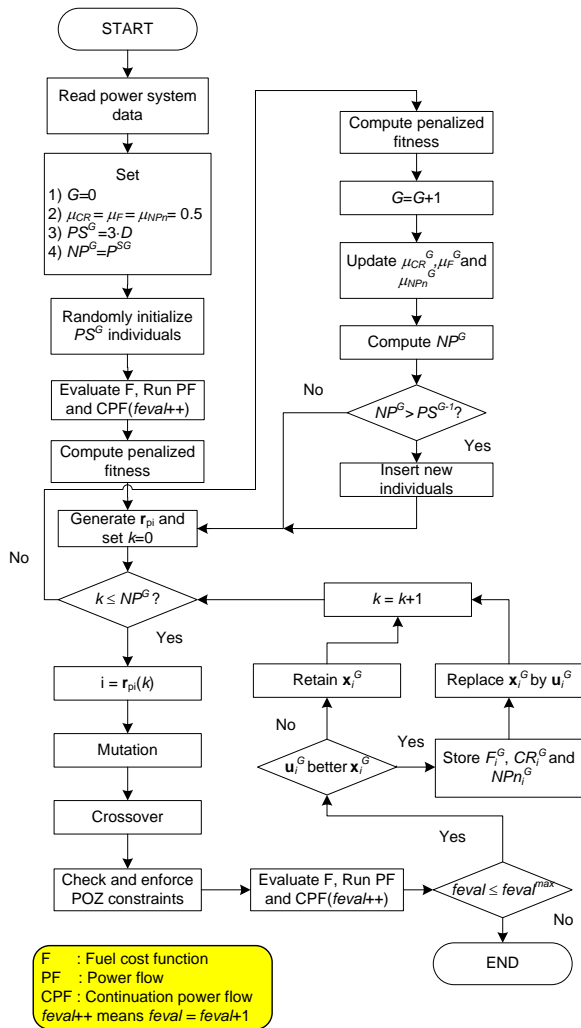
### 3.6 JADE-vPS algorithm for VSCOPF

The implementation steps of JADE-vPS applied to solve the VSCOPF can be depicted as in Figure 2.

## 4 SIMULATION RESULTS

In this section, the modified IEEE-30 bus test system has been used to demonstrate effectiveness of the JADE-vPS algorithm. There are 6 generation buses, 21 load buses, 4 transformers and 41 transmission lines in this test system [22]. All four transformers are assumed to be equipped with the on-load tap changer with 13 steps ( $\pm 6\%$  of the nominal). Modified from the original test system, shunt reactive power elements are connected to buses 15, 16, 17, 18, 20, 22, 23, 25 and 30. A feasible operating condition is obtained if the voltage level is maintained between  $\pm 10\%$ , apparent line flows do not exceed the nominal rating and minimum VSM of 6% is guaranteed. The PSAT software package [23] is used for power flow and continuation power flow com-

putation. All six generators are assumed to have prohibited operating zones. For all generating units, the valve point effect and prohibited operating zones are considered. The parameters of the fuel cost function of each unit are given in Table 1. The operating capacity and ranges of prohibited zones are listed in Table 2. According to the problem formulation presented in section 2, this problem has 16 (12 continuous and 4 discrete) control variables.



**Figure 2:** Flowchart of JADE-vPS for VSCOPF

Coefficients	Generating unit					
	1	2	3	4	5	6
$a_i$	0	0	0	0	0	0
$b_i$	2000	1750	2000	6500	9000	9000
$c_i$	375	1750	12500	1668	7500	7500
$e_i$	150	100	200	100	150	150
$f_i$	62.83	89.76	295.68	418.88	753.99	554.4

**Table 1:** Fuel cost characteristic of each generating unit

Unit	$[P_i^{\min}, P_i^{\max}]$	Prohibited operating zone		
		1	2	3
1	[0.5,2.5]	[1,1.25]	[1.75,2]	[2.1,2,25]
2	[0.2,1.6]	[0.35,0.4]	[0.5,0.75]	[1.1,1.25]
3	[0.15,1]	[0.3,0.5]	[0.7,0.85]	-
4	[0.1,0.7]	[0.2,0.35]	[0.5,0.6]	-
5	[0.1,0.6]	[0.2,0.35]	[0.5,0.6]	-
6	[0.12,0.8]	[0.3,0.4]	[0.5,0.6]	-

$[P_i^{\min}, P_i^{\max}]$ : Min. and Max. limit of generator  $i$

**Table 2:** Generator limits and boundaries of prohibited operating zones

To verify the performance of the JADE-vPS, different DE algorithms were applied as the reference algorithms in order to compare the results. The testing algorithms consist of:

1. DE1 is the classical DE with the “DE/rand/1” mutation strategy [see (12)];
2. DE2 is the classical DE with the “DE/current-to-best/1” mutation strategy [see (13)];
3. JADE-PSx is the adaptive JADE with the population size  $x$  and the external solution archive;
4. JADE-vPS is the modified variant of JADE with the variable population size and the external solution archive;
5. SaDE is the self-adaptive DE proposed in [19].

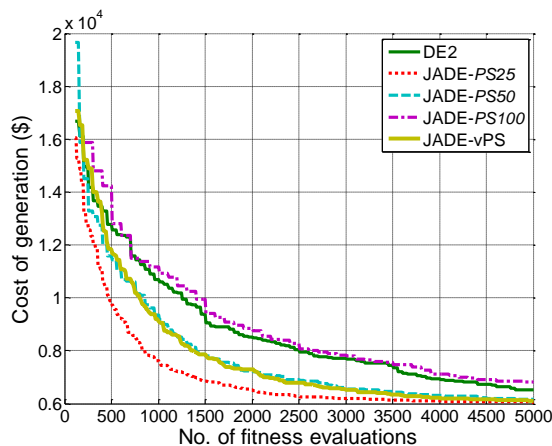
The codes of the above algorithms are written in MATLAB 7.6 programming language and executed on the Pentium Core 2 Quad with CPU 2.83 GHz and RAM 3.25 GB. The maximum number of fitness evaluation  $feval^{\max}$  is 5000. The initial population size for all testing algorithms is 50. The mutation factor  $F$  and crossover rate  $CR$  of DE1 and DE are fixed to typical settings of 0.7 and 0.2, respectively. To fairly compare the two algorithms, the population size of JADE is set to 25 and 100 representing  $PS^{\min}$  and  $PS^{\max}$ , respectively. Statistical values of the final results computed from ten independent trials are given in Table 3.

Algorithm	Statistical values			
	Min.	Avg.	Max.	Std.
DE1	6111.03	8425.89	11329.32	1590.57
DE2	6206.01	6516.50	6874.96	185.58
JADE-PS25	<b>5957.69</b>	<b>6058.50</b>	6424.77	135.24
JADE-PS50	6025.43	6156.95	6263.32	76.27
JADE-PS100	6541.07	6829.51	7025.58	176.94
JADE-vPS	6029.66	6110.61	<b>6167.73</b>	<b>48.41</b>
SaDE	6425.22	7089.61	8192.98	697.57

Min.: Minimum Avg.: Average  
 Max: Maximum Std.: Standard deviation

**Table 3:** Statistical results of different algorithms from 10 independent trials

The classical mutation scheme DE1 shows great difficulties in solving this problem as reflected by a very high standard deviation. Then, DE2 improves the solution quality due to the second term in (21). It is obvious from the simulation results that the parameter adaptation plays a significant role in improving the solution quality. The maximum result of JADE-vPS is also the lowest among all testing algorithms. From these results, it is interesting to notice that JADE-PS25 ( $PS < 2 \cdot D$ ) can find the cheapest result. The average of ten results of this method is also the minimum. We could also observe influences of the population size to final results of VSCOPF. It is not always necessary that the large population size is beneficial to the search process as JADE-PS100 gives the worst results among the original JADE.

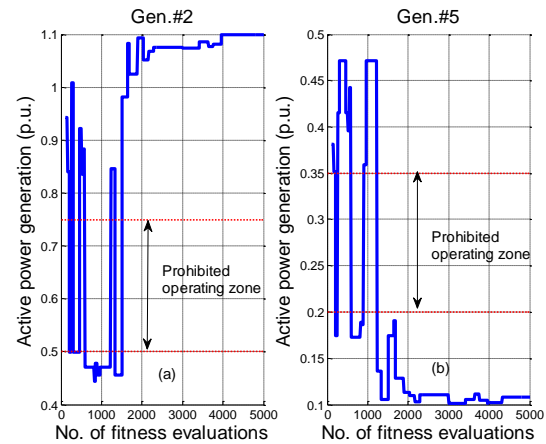


**Figure 3:** Average convergence property of selected testing methods

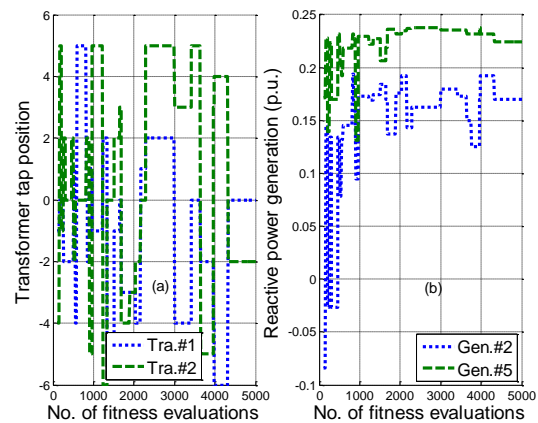
The average convergence characteristic of five selected testing algorithms is shown in Figure 3. The convergence rate of JADE-PS25 is the fastest and finally arriving to the least average cost. The proposed JADE-vPS performs nearly similar to JADE-PS50. On the other hand, the performance of JADE-PS100 is behind its counterpart with the power sizes and also DE2. We postulate the reason that a large population size degrades the final accuracy as follows. The search landscape of this problem could be uni-modal with disjoint spaces. To deal with the disjoint areas, we have implemented a strategy to move the violating vector component to the closet feasible operating zone. This strategy would help facilitate the search process and alleviate the searching burden of the optimizer. Therefore, diversification of individuals may be not required for this problem. This issue is beyond the scope of this paper and thereby not investigated here.

During the optimization process, the decision variables corresponding to the global best individual are recorded. Based on the best final result of JADE-vPS, Figure 4 shows the variation of active power output of generators 2 and 5. It can be clearly seen that the handling method for prohibited operating zones in (27) works well because there is no solution falling into these zones. Because the active power output is directly associated with the objective function, the variable

changes very rapidly at the beginning of the search to respond to the minimum cost requirement. The variables settle once the algorithm converges to the optimal or quasi-optimal solution.



**Figure 4:** Variation of active power output of two selected generators



**Figure 5:** Variation of selected control variables (a) transformer tap position (b) reactive power output

Although transformer taps positions and generator reactive power outputs do not directly influence the cost objective, they are included in the VSCOPF because they are crucial controls for security and voltage stability constraints. The variation of these two variables during the optimization process is depicted in Figure 5.

The variation of the corresponding VSM is shown in Figure 6. It can be observed that the control variables could be adjusted to significantly enhance VSM of the system. However, those control options do not satisfy the cost objective and security constraints. Therefore, the stability constraint should be properly considered in any OPF problems.

As discussed earlier, the success of JADE-vPS is primarily owing to the control parameter adaptation. The evolution of mean values of  $F$ ,  $CR$  and  $NP$  is depicted Figure 7. We can observe that the algorithm automatically adapts the parameters to suitable values at different stages of the search. For instance, the parameter  $F$  increases from the initial value of 0.5 and gradual-

ly reaches 0.8 toward the end. We notice also that the suitable  $NP$  at different stages is also less than 50. This conforms to the simulation results that the JADE works best with the population size of 25.

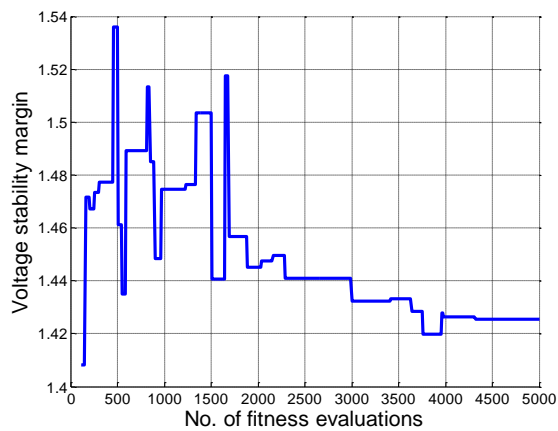


Figure 6: Variation of voltage stability margin

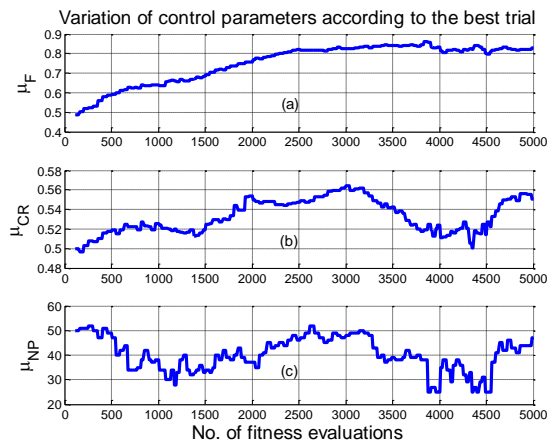


Figure 7: Variation of JADE-vPS control parameters

## 5 CONCLUSION

In this paper, a new adaptive differential evolution algorithm namely JADE-vPS that eliminates the need of control parameter tuning is proposed to solve the VSCOPF problem. The proposed methodology provides offline solutions to optimal active and reactive control settings that minimize the economic cost objective while maintain security and stability constraints. Simulation results of JADE-vPS are compared to those of the two classical DEs and an adaptive DE and the original JADE with different population sizes. These results demonstrate the superiority of JADE-vPS in providing robust results. A synergy of evolutionary algorithms can be introduced in the future work to improve the results.

## REFERENCE

[1] H. W. Dommel and W. F. Tinney, "Optimal Power Flow Solutions," *IEEE Trans. Power App. Syst.*, vol. PAS-87, no. 10, pp. 1866-1876, Oct. 1968.

[2] E. Vaahedi, J. Tamby, Y. Mansour, W. Li, and D. Sun, "Large Scale Voltage Stability Constrained Optimal Var Planning and Voltage Stability Applications Using Existing OPF/Optimal Var

Planning Tools," *IEEE Trans. Power Syst.*, vol. 14, no. 1, pp. 65-74, Feb. 1999.

[3] W. D. Rosehart, C. A. Canizares, and W. H. Quintana, "Effect of Detailed Power System Models in Traditional and Voltage-Stability-Constrained Optimal Power Flow Problems," *IEEE Trans. Power Syst.*, vol. 18, no. 1, pp. 27-35, Feb. 2003.

[4] J. A. Momoh, M. E. El-Hawary, and R. Adapa, "A review of selected optimal power flow literature to 1993 part I & II," *IEEE Trans. Power Syst.*, vol. 14, no. 1, pp. 96-111, Feb. 1999.

[5] M. Todorovski and D. Rajcic, "An initialization procedure in solving optimal power flow by genetic algorithm," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 480-487, May 2006.

[6] Q. H. Wu, Y. J. Cao, and J. Y. Wen, "Optimal reactive power dispatch using an adaptive genetic algorithms," *Int. J. Electr. Power Energy Syst.*, vol. 20, no. 8, pp. 563-569, 1998.

[7] H. Yoshida, K. Kawata, Y. Fukuyama, S. Takamura, and Y. Nakanishi, "A Particle Swarm Optimization for Reactive Power and Voltage Control Considering Voltage Security Assessment," *IEEE Trans. Power Syst.*, vol. 15, no. 4, pp. 1232-1239, Nov. 2000.

[8] A. A. A. Esmín, G. Lambert-Torres, and A. C. Z. d. Souza, "A hybrid particle swarm optimization applied to loss power minimization," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 859-866, May 2005.

[9] C. H. Liang, C. Y. Chung, K. P. Wong, X. Z. Duan, and C. T. Tse, "Study of differential evolution for optimal reactive power flow," *IEE Proc. Gener. Trans. Distrib.*, vol. 1, no. 2, pp. 253-260, 2007.

[10] M. Varadarajan and K. S. Swarup, "Network loss minimization with voltage security using differential evolution," *Elec. Power Syst. Research*, vol. 78, pp. 815-823, 2008.

[11] Q. H. Wu and J. T. Ma, "Power system optimal reactive power dispatch using evolutionary programming," *IEEE Trans. Power Syst.*, vol. 10, no. 3, pp. 1243-1249, 1995.

[12] M. Tripathy and S. Mishra, "Bacteria Foraging-Based Solution to Optimize Both Real Power Loss and Voltage Stability Limit," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 240 - 248 Feb. 2007.

[13] J. Zhang and A. C. Sanderson, "JADE: Adaptive Differential Evolution with Optional External Archive," *IEEE Trans. Evol. Compt.*, vol. 13, no. 5, pp. 945-958, Oct. 2009.

[14] S. O. Orero and M. R. Irving, "Economic dispatch of generators with prohibited operating zones: a genetic algorithm approach " *IEE Gener. Transm. Distrib.*, vol. 143, no. 6, pp. 529 - 534 Nov. 1996.

[15] V. Ajjarapu, *Computational Techniques for Voltage Stability Assessment and Control*: Springer Verlag, 2006.

[16] R. Storn and K. Price, "Differential evolution - a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization* vol. 11, pp. 341-359, 1997.

[17] J. Liu and J. Lampinen, "On setting the control parameter of the differential evolution method," in the *8th International Conference on Soft Computing (MENDEL)*, Brno, Czech Republic, 2002.

[18] D. Zaharie, "Critical values for the control parameters of differential evolution algorithms," in *8th International Mendel Conference on Soft Computing*, Brno, Czech Republic, 2002.

[19] J. Brest, S. Greiner, B. Boskovic, M. Mernik, and V. Zumer, "Self-Adapting Control Parameters in Differential Evolution: A Comprehensive Study on Numerical Benchmark Problems," *IEEE Trans. Evol. Compt.*, vol. 10, no. 6, pp. 646-657, Dec. 2006.

[20] A. K. Qin, V. L. Huang, and P. N. Suganthan, "Differential Evolution Algorithm With Strategy Adaptation for Global Numerical Optimization," *IEEE Trans. Evol. Compt.*, vol. 13, no. 2, pp. 398-417, April 2009.

[21] B. Tessema and G. G. Yen, "A self adaptive penalty function based algorithm for constrained optimization," in *Proc. IEEE Congr. Evolut. Comput.*, July 2006, pp. 246-253.

[22] "Power Systems Test Case Archive," [Online] <http://www.ee.washington.edu/research/pstca/>.

[23] F. Milano, L. Vanfretti, and J. C. Morataya, "An Open Source Power System Virtual Laboratory: The PSAT Case and Experience," *IEEE Trans on Education*, vol. 51, no. 1, pp. 17-23, Feb. 2008.