IFAC Symposium on Power Plants and Power Systems Control, Kananaskis, Canada, 2006

ROBUST DECENTRALIZED H_∞ CONTROLLER DESIGN FOR POWER SYSTEMS: A MATRIX INEQUALITY APPROACH USING PARAMETER CONTINUATION METHOD

Getachew K. Befekadu and I. Erlich*

Department of Power Engineering, University of Duisburg-Essen, Duisburg 47057, Germany

Abstract: This paper presents a decentralized H_∞ controller design approach for power systems. Initially a centralized H_∞ robust controller, which guarantees the robust stability of the overall system against unstructured and norm bounded uncertainties, is designed. The problem of designing decentralized controller is then reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized to the decentralized controller as the continuation parameter varies monotonically. Moreover, the paper proposes an algorithm to solve such problem using two-stage iterative matrix inequality optimization approach to determine the decentralized controller. The approach is flexible enough to allow designing a reduced-order controller for each subsystem with the same robustness condition of the centralized controller. The approach is demonstrated by designing PSSs for a test system.

Keywords: Decentralized control, interconnected systems, nonlinear systems, optimization methods and robustness.

1. INTRODUCTION

The deregulation of the electricity markets has led to increasing uncertainties concerning the power flow within the network. This is further compounded by the physical expansion of interconnected networks such as those in Europe, which makes more difficult the prediction of system responses to disturbances and severe loading conditions. Furthermore, the ever – increasing utilization of wind energy is also expected to have a significant impact on the load flows as well as the dynamic behaviour of the system. These further have created new challenges in guaranteeing end-to-end reliability of electricity service. Traditional control principles applied to local components do not take into account the continually changing of the dynamic structure of the network. Specifically, secure operations of current power systems heavily rely on the controller schemes that placed in the system to manage disturbances and/or prevent disastrous cascading. These controllers systems are usually static, in the sense that they do not adapt to changing network configurations and operating conditions. Additionally, the design and parameter settings of the control schemes do not take into account the very system dynamic behaviours. Consequently, these and other similar developments prompted both power and control engineers to use new controller design techniques and more accurate model descriptions for the power system with the objective of providing reliable electricity services. Thus, to meet modern power systems requirements, controllers have to guarantee robustness over a wide range of system operating conditions and this further highlights the fact that robustness is one of the major issues in power system controllers design.

Recently, a number of efforts have been made to extend the application of robust control techniques to power systems, such as L_∞ optimization (Vittal, *et al.*, 1995; Venkataraman, *et al.*, 1995), H_∞-optimization (Chen and Malik, 1995; Klein, *et al.*, 1995), structured singular value (SSV or μ) technique (Djukanovic, *et al.*, 1998, 1999) and linear matrix inequalities (LMIs) technique (Siljak, *et al.*,

* Corresponding author.
E-mail address: erlich@uni-duisburg.de
University of Duisburg-Essen, Institute of Electrical Power Engineering and Automation, 47057 Duisburg, Germany.

2002; Siljak, *et al.*, 2004). Interesting robust decentralized controller schemes that are based on the concept of connectively stabilizing a large-scale nonlinear interconnected system for turbine/governor control and exciter control using the LMIs optimization (Boyd, *et al.*, 1994) have been presented in (Siljak, *et al.*, 2002) and (Siljak, *et al.*, 2004), respectively. However, the designed local state feedback controllers need the corresponding state information of the subsystems, which may be either impossible or simply impractical to obtain measurements of the full information for all individual subsystems. Specially, the result presented in (Djukanovic, *et al.*, 1999) uses the sequential μ - synthesis technique where the design procedure is carried-out successively for each local input-output pairs in the system. Though the individual controllers are sequentially designed to guarantee the robust stability and performance of the whole system, the reliability of the decentralized controller depends on the order in which the design procedures for these individual controllers are carried-out. Moreover, a failure in the lower-loop may well affect the stability/performance of the whole system. It is also clear from the nature of the problem that the order of the controller increases for each sequentially designed local controller. Another attempt is also made to apply a linear parameter varying (LPV) technique for designing decentralized power system stabilizers for large power system (Qiu, *et al.*, 2004). The resulting controllers, however, are typically high order - at least as high as the system since the technique relies on H_∞ - optimization; and besides the problem formulation attempts to solve an infinite-dimensional LMI type optimization where the latter problem is computationally very demanding. Furthermore, the approach did not consider the entire interconnection model of the power system in the design formulation.

This paper focuses on the extension of matrix inequalities based H_∞ optimization approach to problems of practical interest in power systems. The design problem considered is the natural extension of the reduced-order decentralized H_∞ dynamic output controller synthesis for power systems. In the design, the decentralized H_∞ dynamic output feedback controller problem is first reformulated as an embedded parameter continuation problem that homotopically deforms from the initially full-order designed centralized H_∞ controller to the desired decentralized controller as the continuation parameter monotonically varies. Moreover, the paper proposes an algorithm to solve such optimization problems using two-stage iterative matrix inequality optimization method to determine the robust decentralized H_∞ controller. The paper also addresses the possibility of extending the approach to design reduced-order decentralized controllers that have practical benefits since high-order controllers when implemented in real-time configurations usually create undesirable effects such as time delays.

The approach has a number of practical relevance among which the following are singled out: i) designing reduced-order decentralized controllers can be incorporated in the approach by explicitly stating the order of each controller in the specified structure and, ii) multi-objective optimization technique can easily be incorporated in the design by minimizing the H_∞ norms of the multiple transfer functions between different input/output channels. Moreover, the paper also presents a general approach that can be used for designing a combination of any order robust PSSs for power systems. The application of this approach to a multi-machine power system allows a coordinated tuning of controllers that incorporate robustness to changes in the operating conditions as well as model uncertainties in the system.

The outline of the paper is as follows. In Section 2, the robust decentralized H_∞ controller design problem is formulated as a matrix inequality problem using parameterized continuation method. The associated computational problem and the extension of the approach to design reduced-order decentralized controllers are also discussed in this section. The application of the approach to design robust decentralized PSSs and simulation results together with performance indices are given in Section 3. Finally, in Section 4, a brief conclusion about the paper is given.

2. OUTLINE OF THE PROBLEM

2.1 System model and problem formulation

Consider the general structure of the *i*th-generator together with the PSS block in a multimachine power system shown in Fig. 1. The input of the *i*th-controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. Let the structure of this *i*th-washout stage be given by:

$$\Delta y_i = \frac{sT_{wi}}{1+sT_{wi}}\Delta\omega_i \qquad (1)$$

After augmenting the washout stage in the system, the *i*th-subsystem, within the framework of H_∞ design, is described by the following state space equation:

$$\begin{aligned}
\dot{\mathbf{x}}_i(t) &= \mathbf{A}_{ii}\,\mathbf{x}_i(t) + \sum_{j\neq i}\mathbf{A}_{ij}\,\mathbf{x}_j(t) + \mathbf{B}_{1i}\,\mathbf{w}_i(t) + \mathbf{B}_{2i}\,\mathbf{u}_i(t)\\
\mathbf{z}_i(t) &= \mathbf{C}_{1i}\,\mathbf{x}_i(t) + \mathbf{D}_{11i}\,\mathbf{w}_i(t) + \mathbf{D}_{12i}\,\mathbf{u}_i(t)\\
\mathbf{y}_i(t) &= \mathbf{C}_{yi}\,\mathbf{x}_i(t) + \mathbf{D}_{y1i}\mathbf{w}_i(t)
\end{aligned} \qquad (2)$$

where $\mathbf{x}_i(t)\in\Re^{n_i}$ is the state variable, $\mathbf{u}_i(t)\in\Re^{m_i}$ is the control input, $\mathbf{y}_i(t)\in\Re^{q_i}$ is the measurement signal, $\mathbf{z}_i(t)\in\Re^{p_i}$ is the regulated variables, and $\mathbf{w}_i(t)\in\Re^{r_i}$ is exogenous signal for the *i*th-subsystem. Moreover, assume that there is no unstable fixed mode (Wang and Davidson, 1973) with respect to $\mathbf{C}_y = \mathrm{diag}\{\mathbf{C}_{y1}, \mathbf{C}_{y2}, \ldots, \mathbf{C}_{yN}\}$, $[\mathbf{A}_{ij}]_{N\times N}$ and $\mathbf{B}_2 = \mathrm{diag}\{\mathbf{B}_{21}, \mathbf{B}_{22}, \ldots, \mathbf{B}_{2N}\}$.

Consider the following decentralized output feedback controller for the system given in (2):

$$\begin{aligned} \dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\,\mathbf{y}_i(t) \\ \mathbf{u}_i(t) &= \mathbf{C}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t) \end{aligned} \tag{3}$$

where $\mathbf{x}_{ci}(t) \in \Re^{n_{ci}}$ is the state of the ith-local controller, n_{ci} is a specified dimension, and $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$, $\mathbf{D}_{ci}$, $i = 1, 2, \cdots, N$ are constant matrices to be determined during the actual design step. In this paper, the design procedure deals with nonzero $\mathbf{D}_{ci}$, however, it can be set to zero, i.e., $\mathbf{D}_{ci} = \mathbf{0}$, so that the ith-local is strictly proper controller.

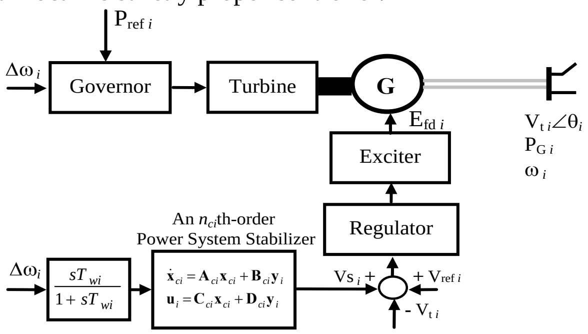

Fig. 1. General structure of the ith-generator together with the PSS in the multimachine power system.

After augmenting the decentralized controller (3) in the system, the state space equation for ith-extended subsystem will have the following form

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_i(t) &= (\tilde{\mathbf{A}}_{ii} + \tilde{\mathbf{B}}_{2i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\tilde{\mathbf{x}}_i(t) \\ &\quad + (\tilde{\mathbf{B}}_{1i} + \tilde{\mathbf{B}}_{2i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\mathbf{w}_i(t) + \sum_{j\neq i}\tilde{\mathbf{A}}_{ij}\,\tilde{\mathbf{x}}_j(t) \\ \mathbf{z}_i(t) &= (\tilde{\mathbf{C}}_{i1} + \tilde{\mathbf{D}}_{12i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\tilde{\mathbf{x}}_i(t) + (\tilde{\mathbf{D}}_{11i} + \tilde{\mathbf{D}}_{12i}\,\mathbf{K}_i\,\tilde{\mathbf{D}}_{y1i})\mathbf{w}_i(t) \end{aligned} \tag{4}$$

where $\tilde{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t) \quad \mathbf{x}_{ci}^T(t)]^T$ is the augmented state variable for the ith-subsystem and

$$\tilde{\mathbf{A}}_{ij} = \begin{bmatrix} \mathbf{A}_{ij} & \mathbf{0}_{n_i\times n_{ci}} \\ \mathbf{0}_{n_{ci}\times n_i} & \mathbf{0}_{n_{ci}\times n_{ci}} \end{bmatrix}, \qquad \tilde{\mathbf{B}}_{1i} = \begin{bmatrix} \mathbf{B}_{1i} \\ \mathbf{0}_{n_i\times r_i} \end{bmatrix},$$

$$\tilde{\mathbf{B}}_{2i} = \begin{bmatrix} \mathbf{0}_{n_i\times n_{ci}} & \mathbf{B}_{2i} \\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci}\times m_i} \end{bmatrix}, \qquad \tilde{\mathbf{C}}_{1i} = \begin{bmatrix} \mathbf{C}_{1i} & \mathbf{0}_{p_i\times n_{ci}} \end{bmatrix}$$

$$\tilde{\mathbf{C}}_{yi} = \begin{bmatrix} \mathbf{0}_{n_{ci}\times n_i} & \mathbf{I}_{n_{ci}\times n_{ci}} \\ \mathbf{C}_{yi} & \mathbf{0}_{q_i\times n_{ci}} \end{bmatrix}, \quad \tilde{\mathbf{D}}_{11i} = \mathbf{D}_{11i},$$

$$\tilde{\mathbf{D}}_{12i} = \begin{bmatrix} \mathbf{0}_{p_i\times n_{ci}} & \mathbf{D}_{12i} \end{bmatrix} \quad \tilde{\mathbf{D}}_{y1i} = \begin{bmatrix} \mathbf{0}_{n_{ci}\times r_i} \\ \mathbf{D}_{y1i} \end{bmatrix}, \quad \mathbf{K}_i = \begin{bmatrix} \mathbf{A}_{ci} & \mathbf{B}_{ci} \\ \mathbf{C}_{ci} & \mathbf{D}_{ci} \end{bmatrix}.$$

Moreover, the overall extended system equation for the system can be rewritten in one state-space equation form as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= (\tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\,\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y)\mathbf{w}(t) \\ \mathbf{z}(t) &= (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1})\mathbf{w}(t) \end{aligned} \tag{5}$$

where

$$\tilde{\mathbf{A}} = [\tilde{\mathbf{A}}_{ij}]_{N\times N}, \qquad \tilde{\mathbf{B}}_1 = \mathrm{diag}\{\tilde{\mathbf{B}}_{11}, \tilde{\mathbf{B}}_{12}, \ldots, \tilde{\mathbf{B}}_{1N}\},$$

$$\tilde{\mathbf{B}}_2 = \mathrm{diag}\{\tilde{\mathbf{B}}_{21}, \tilde{\mathbf{B}}_{22}, \ldots, \tilde{\mathbf{B}}_{2N}\}, \quad \tilde{\mathbf{C}}_1 = \mathrm{diag}\{\tilde{\mathbf{C}}_{11}, \mathbf{C}_{12}, \ldots, \tilde{\mathbf{C}}_{1N}\},$$

$$\tilde{\mathbf{D}}_{11} = \mathrm{diag}\{\tilde{\mathbf{D}}_{111}, \tilde{\mathbf{D}}_{112}, \ldots, \tilde{\mathbf{D}}_{11N}\}, \tilde{\mathbf{D}}_{12} = \mathrm{diag}\{\tilde{\mathbf{D}}_{121}, \tilde{\mathbf{D}}_{122}, \ldots, \tilde{\mathbf{D}}_{12N}\},$$

$$\tilde{\mathbf{C}}_y = \mathrm{diag}\{\tilde{\mathbf{C}}_{y1}, \tilde{\mathbf{C}}_{y2}, \ldots, \tilde{\mathbf{C}}_{yN}\}, \quad \tilde{\mathbf{D}}_{y1} = \mathrm{diag}\{\tilde{\mathbf{D}}_{y11}, \tilde{\mathbf{D}}_{y12}, \ldots, \tilde{\mathbf{D}}_{y1N}\},$$

$$\mathbf{K}_D = \mathrm{diag}\{\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_N\}.$$

Hence, the overall extended system can be rewritten in a compact form as follows:

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}_{cl}\,\tilde{\mathbf{x}}(t) + \mathbf{B}_{cl}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_{cl}\tilde{\mathbf{x}}(t) + \mathbf{D}_{cl}\mathbf{w}(t) \end{aligned} \tag{6}$$

where

$$\mathbf{A}_{cl} = \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad \mathbf{B}_{cl} = \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{D}}_{y1},$$

$$\mathbf{C}_{cl} = \tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad \mathbf{D}_{cl} = \tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1}$$

Consider the following design approach where the controller strategy in (3) internally stabilizes the closed-loop of the transfer function $T_{zw}(s)$ from $\mathbf{w}$ to $\mathbf{z}$ and moreover satisfies a certain prescribed disturbance attenuation level $\gamma > 0$, i.e., $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$.

In the following, the design procedure assumes that the system in (2) is stabilizable with the same prescribed disturbance attenuation level γ via a centralized H_∞ controller of dimension equal to or greater than $n_c := \sum_{i=1}^{N} n_{ci}$ in which each controller input $\mathbf{u}_i$ is determined by the corresponding measured outputs $\mathbf{y}_j, 1 \le j \le N$. The significance of this assumption lies on the fact that the decentralized controllers cannot achieve better performances than that of centralized controllers. In this paper, the centralized H_∞ controller is used for the initial boundary value in the two-stage iterative matrix inequality optimization method.

2.2 Decentralized H_∞ output feedback controller design using parameterized continuation method

Designing a decentralized H_∞ output feedback controller for the system is equivalent to that of finding the matrix $\mathbf{K}_D$ that satisfies an H_∞ norm bound condition on the closed-loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from disturbance $\mathbf{w}$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}}(s)$ must be stable (Gahinet & Apkarian, 1994). The following proposition is instrumental in establishing the existence of decentralized control strategy (3) for the system (2).

Proposition. The system (2) is stabilizable with the disturbance attenuation level $\gamma > 0$ via a decentralized controller (3) composed of N n_{ci} - dimensional local controllers if there exist a matrix $\mathbf{K}_D$ and a positive-definite matrix $\tilde{\mathbf{P}}$ that satisfy the following matrix inequality:

$$\mathbf{\Phi}(\mathbf{K}_D, \tilde{\mathbf{P}}) := \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T\tilde{\mathbf{P}} & \tilde{\mathbf{P}}\tilde{\mathbf{B}}_1 & \tilde{\mathbf{C}}_1^T \\ \tilde{\mathbf{B}}_1^T\tilde{\mathbf{P}} & -\gamma\mathbf{I}_r & \tilde{\mathbf{D}}_{11}^T \\ \tilde{\mathbf{C}}_1 & \tilde{\mathbf{D}}_{11} & -\gamma\mathbf{I}_p \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} + \left\{ \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} \right\}^T \prec \mathbf{0} \quad (7)$$

The condition stated in the above proposition seems to be the same as that of the centralized H_∞ control case (Gahinet & Apkarian, 1994). However, due to the specified structure on the controller (i.e., designing controllers with "block diagonal") makes the problem an NP-hard nonconvex optimization problem. To compute the optimal solution of this problem, the design problem is reformulated as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the design parameter varies monotonically (Richter & Decarlo, 1983). The parameterized family of the problem in (7) is given as follows:

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D, \tilde{\mathbf{P}}) \prec \mathbf{0} \quad (8)$$

with $\lambda \in [0, 1]$ such that at $\lambda = 0$

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_F, \tilde{\mathbf{P}}, 0) = \mathbf{\Phi}(\mathbf{K}_F, \tilde{\mathbf{P}}) \prec \mathbf{0} \quad (9)$$

and at $\lambda = 1$

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, 1) = \mathbf{\Phi}(\mathbf{K}_D, \tilde{\mathbf{P}}) \prec \mathbf{0} \quad (10)$$

where

$$\mathbf{K}_F = \begin{bmatrix} \mathbf{A}_F & \mathbf{B}_F \\ \mathbf{C}_F & \mathbf{D}_F \end{bmatrix} \quad (11)$$

is a constant matrix of the same size as $\mathbf{K}_D$ and composed of the coefficient matrices $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{C}_F$ and $\mathbf{D}_F$ of an n_c- dimensional centralized H_∞ for the disturbance attenuation level γ. The centralized controller $\mathbf{K}_F$ can be obtained via the existing method (e.g., Gahinet & Apkarian, 1994). Thus, the term $(1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D$ in (8) defines a homotopy interpolating centralized H_∞ controller and a desired decentralized H_∞ controller. Thus, the problem of finding a solution of (8) can be embedded in the family of problems as:

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, \lambda) \prec \mathbf{0}, \qquad \lambda \in [0, 1] \quad (12)$$

Therefore, the algorithm based on parameter continuation method for finding the robust decentralized output feedback controller has the following two-stage iterative matrix inequalities optimization.

Algorithm: *A Matrix Inequality Based Parameterized Continuation Method.*

Initialization

Compute the centralized controller $\mathbf{K}_F$ and $\tilde{\mathbf{P}}_0$ that guarantees a disturbance attenuation level γ

Set $\lambda_0 = 0$, $k = 0$, $M = K$ (Large Number)

and $\mathbf{K}_{D0} = \mathbf{0}$ (Zero Matrix)

Repeat until $\lambda_k = 1$, **do**

$k \leftarrow k + 1$

$\lambda_k \leftarrow \lambda_{k-1} + 1/M$

(1) Compute $\mathbf{K}_{Dk}$ that satisfies

$$\tilde{\mathbf{\Phi}}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk}, \tilde{\mathbf{P}}_{k-1}, \lambda_k) \prec \mathbf{0}$$

(2) Compute $\tilde{\mathbf{P}}_k$ that satisfies

$$\tilde{\mathbf{\Phi}}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk}, \tilde{\mathbf{P}}_k, \lambda_k) \prec \mathbf{0}$$

End do

Remark 1: If the problem in Step-1 of the above Algorithm fails to be feasible, then the step-length should be changed in order to compute $\mathbf{K}_{Dk}$ that satisfies $\tilde{\mathbf{\Phi}}(\mathbf{K}_{Dk}, \tilde{\mathbf{P}}_{k-1}, \lambda_k) \prec \mathbf{0}$ for the remaining interval.

The above two-stage iterative matrix inequality optimization method involves: i) computing a continuous family of decentralized feedback $\mathbf{K}_{Dk}$ starting from $\mathbf{K}_{D0} = \mathbf{0}$ at $\lambda = 0$, i.e., an LMI problem for each iteration in Step-1, and ii) computing the positive definite matrix $\tilde{\mathbf{P}}_k$ in Step-2. This two-stage iterative algorithm can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers such as MATLAB *LMI Toolbox* (Gahinet, *et al*., 1995).

2.3 Designing a reduced-order decentralized controller.

The algorithm proposed in the previous section can only be applied when the dimension of the decentralized H_∞ controller is equal to the order of the plant, i.e., $n_c = n$. However, it is possible to compute directly a reduced-order decentralized controller, i.e., $n_c < n$ by augmenting the matrix $\mathbf{K}_D$ as

$$\hat{\mathbf{K}}_D = \left[\begin{array}{cc|c} \hat{\mathbf{A}}_D & 0_{n_n\times(n-n_c)} & \hat{\mathbf{B}}_D \\ * & -\mathbf{I}_{(n-n_c)} & ** \\ \hline \hat{\mathbf{C}}_D & 0_{m\times(n-n_c)} & \hat{\mathbf{D}}_D \end{array}\right] \quad (13)$$

where the notation $*$, $**$ are any submatrices, and $\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$ are the reduced-order decentralized controller matrices. Note that the n-dimensional controller defined by $\hat{\mathbf{K}}_D$ of (13) is equivalent to the n_c- dimensional decentralized controller described by state-space representation of ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) if the controller and observable parts are extracted. Next, define the matrix function $\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}, \lambda)$ (which is similar to (8)) as

$$\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}, \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}) \prec \mathbf{0} \quad (14)$$

Then, one can apply the algorithm proposed in the previous section with $\mathbf{K}_F$ of n-dimensional centralized H_∞ controller. In this case, at $\lambda = 0$ set the matrix $\hat{\mathbf{K}}_D$ to zero except $(2, 2)$ - block $-\mathbf{I}_{(n-n_c)}$ and proceed with computing $\hat{\mathbf{K}}_{D_k}$ for each λ_k. If the algorithm succeeds, then the matrices ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) extracted from the obtained $\hat{\mathbf{K}}_D$ at $\lambda = 1$, comprise the desired decentralized H_∞ controller.

Remark 2: The approach outlined in the previous section considers the problem of designing decentralized controllers for the full-order system. It is also possible first to reduce the order of the system by applying a model reduction technique, and then designing a decentralized controller for the system.

3. SIMULATION RESULTS

The robust decentralized controllers design approach presented in the previous section is now applied to a four machine test system for designing robust PSS. This system, which is shown in Fig. 2, has been specifically designed to study the fundamental behaviour of large interconnected power systems including inter-area oscillations in power systems (Klein, *et al*., 1991). The system has four generators and each generator is equipped with IEEE standard exciter and governor controllers. The parameters for the standard exciter and governor controllers used in the simulation were taken from (Kundur, 1994). Moreover, the generators for these simulations are all represented by their fifth-order models with rated voltage of 15.75 kV. To demonstrate the applicability of the proposed approach second order PSSs are designed, although it is possible to extend the method to any order and/or combinations of PSS blocks in the design procedure without any difficulty. After including the washout filter in the system, the design problem is reformulated as an embedded parameter continuation problem that deforms from the designed centralized H_∞ controller to the decentralized one as the design parameter varies to its range space. The design procedure has been carried out for the base loading condition of $[\mathrm{P_{L1}} = 1600\ \mathrm{MW},\ \mathrm{Q_{L1}} = 150\ \mathrm{Mvar}]$ and $[\mathrm{P_{L2}} = 2400\ \mathrm{MW},\ \mathrm{Q_{L2}} = 120\ \mathrm{Mvar}]$. The speed of each generator, the output of the PSS together with the terminal voltage error signal, which are the input to the regulator of the exciter, are used as regulated signals within the framework of the design. Moreover, the output of the washout block, i.e., measured output signal, is used as an input signal for the PSS in the system.

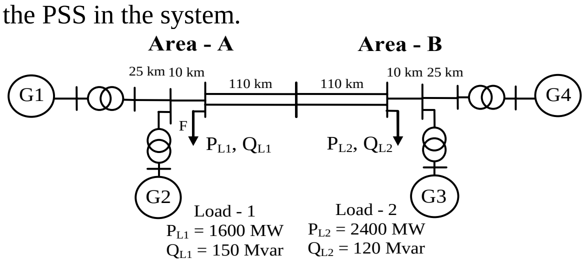

Fig. 2. One-line diagram of the test system.

A three-phase fault with different fault durations was applied at different fault locations and operating conditions to verify the performance of the proposed robust PSSs. For a short circuit of 150 ms duration at node F in Area-A, the transient responses of generator G2 with and without the PSS in the system are shown in Fig. 3. This generator which is the most disturbed generator in the system due to its relative nearness to the fault location shows a good damping behaviour after the PSS included in the system. The robust decentralized PSSs designed through this approach are also given in Table 1.

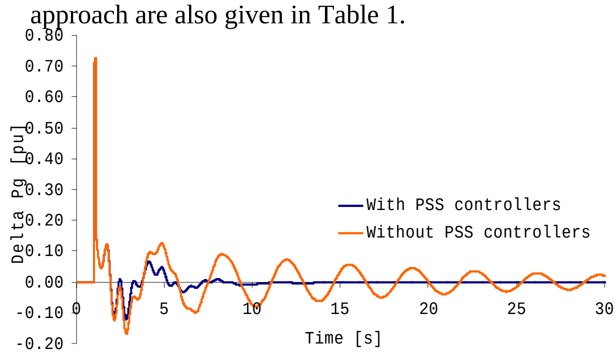

Fig. 3. The transient responses of Generator G2 to a short circuit at node F.

Table 1. The robust decentralized power system stabilizers for the system with $M = 256$

Generator G_i	The Corresponding PSS
1	$\frac{s^2+8.294s+20.122}{s^2+14.724s+29.431}$
2	$\frac{s^2+19.610s+21.010}{s^2+37.050s+22.344}$
3	$\frac{s^2+17.704s+22.120}{s^2+37.611s+17.862}$
4	$\frac{s^2+6.211s+6.508}{s^2+10.070s+11.832}$

To further assess the effectiveness of the proposed approach regarding robustness, the transient performance indices were computed for different loading conditions at node 1 $[\mathrm{P_{L1}}, \mathrm{Q_{L1}}]$ and node 2 $[\mathrm{P_{L2}}, \mathrm{Q_{L2}}]$ while keeping constant total load in the system. These transient performance indices, which are used to investigate the behaviour of the system during any fault and/or sudden load changes, are then normalized to the transient performance indices of the base operating condition for which the designed procedure has been carried out. Specifically, these normalizations are computed according to the following formula:

$$I_N = \frac{I_{DLC}}{I_{BLC}} \tag{15}$$

where I_{DLC} is the transient performance index for different loading condition, I_{BLC} is the transient performance index for base loading condition for which the design has been carried out. Moreover, the transient performance indices for generator powers P_{Gi}, generator terminal voltages V_{ti} and excitation voltages $E_{fd\,i}$ following a short circuit of 150 ms duration at node F in Area-A are computed using the following equations.

$$I^{P_G} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| P_{Gi}(t) - P_{Gi}^0 \right| dt \tag{16a}$$

$$I^{V_t} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| V_{ti}(t) - V_{ti}^0 \right| dt \tag{16b}$$

$$I^{E_{fd}} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| E_{fd\,i}(t) - E_{fd\,i}^0 \right| dt \tag{16c}$$

The normalized transient performance indices for different loading conditions are shown in Fig. 4. It can be seen from Fig 4 that the indices for $I_N(E_{fd})$,

$I_N(P_G)$ and $I_N(V_t)$ are either near unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behaviour exhibits robustness for all loading conditions.

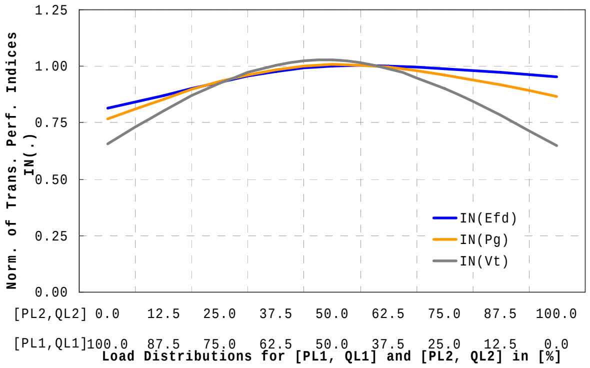

Fig. 4. Plot of the normalized transient performance indices.

Remark 3: The value of the normalized transient performance index $I_N(.)$ gives a qualitative measure of the dynamic behaviour of the system during any fault and/or sudden load changes. A value much greater than unity means that the system behaves poorly as compared to the base operating condition.

4. CONCLUSION

A framework for robust PSS design that takes into account model uncertainties and changes in the operating conditions has been presented. The applicability of the approach has been demonstrated through design example in a four-machine test system. The design problem is first reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized H_∞ controller to the decentralized one as the continuation parameter varies monotonically. The corresponding optimization problem is solved using two-stage matrix inequality optimization method. The possibility of extending to design a reduced-order decentralized controller has also been addressed properly. The latter which has additional practical benefits since high-order controllers when implemented in real-time configurations may create undesirable effects such as time delays. Moreover the approach is flexible enough to allow the inclusion of additional design parameters such as the order of the controller and for each input/output channel in the system. An additional benefit of this approach is that all the controllers are linear and use minimum local feedback information.

REFERENCES

Boyd, S. El Ghaoui, L. Feron, E. and Balakrishnan, V. (1994). *Linear matrix inequalities in systems and control theory*. Volume 15 of Studies in Applied Mathematics, SIAM.

Chen S. and Malik O. P. (1995). H_∞ optimization based power system stabilizer design. *IEE Proceedings Generation, Transmission and Distribution*. 142(3), 179–184.

Djukanovic, M. M. H. Khammash and V. Vittal, (1998). Application of the structured singular value theory for robust stability and control analysis in multimachine power systems, Part-I: Framework development and Part II: Numerical simulation and results. *IEEE Transactions on Power Systems*, 13(4), 1311–1316.

Djukanovic, M. M. H. Khammash and V. Vittal, (1999). Sequential synthesis of structured singular value based decentralized controllers in power systems, *IEEE Trans. Power Syst*., vol.14(2), 635–641.

Gahinet, P. and Apkarian, P. (1994). A linear matrix inequality approach to H_∞ control. *International Journal on robust and nonlinear control*. 4, 421-448.

Gahinet, P. Nemiroviski, A. Laub, A. L. and Chilali, M. (1995). *LMI Control Toolbox*. Cambridge, MA: Mathworks Inc.

Klein, M. Rogers, G. J. and Kundur, P. (1991). A fundamental study of inter-area oscillations in power systems. IEEE Transactions on Power Systems. 6(3), 914-921.

Klien, M. Le, X. L. Rogers, G. J. and Farrokhpay, S. (1995). H_∞ damping controller design in large power systems. *IEEE Transactions on Power Systems*, 10(1), 158–166.

Kundur, P. (1994). *Power system stability and control*. McGraw-Hill Inc.

Qiu W., Vittal, V and Khammash M. H., (2004) Decentralized power system stabilizer design using linear parameter varying approach, *IEEE Trans. Power Syst*. vol. 19(4), 1951-1960.

Richter, S. L. and Decarlo, R. A. (1983). Continuation Methods: Theory and Applications. *IEEE Transactions on Circuits and Systems*, 30(6), 347-352.

Siljak, D. D. Stipanovic, D. M. and Zecevic, A. I. (2002). Robust decentralized turbine/governor control using linear matrix inequalities. *IEEE Transactions on Power Systems*, 17(3), 715-722.

Siljak, D.D. Zecevic, A. I. and Neskovic, G. (2004). Robust decentralized exciter control with linear feedback. *IEEE Transactions on Power Systems*, 19(2), 1096–1103.

Venkataraman, S., Khammash, M. H. and Vittal, V. (1995). Analysis and synthesis of HVDC controls for stability of power systems. *IEEE Transactions on Power Systems*, 10(4), 1933-1938.

Vittal, V., Khammash, M. H. and Pawloski, C. D. (1995). *Systems and control theory for power systems*. In Chow, J. H., Kokotovic, P. V. and Thomas, R. J. editors, 64 of Mathematics and its Applications, IMA, 399-413.

Wang, S. H. and Davidson, E. J. (1973). On stabilization of decentralized control systems, *IEEE Trans. Automat. Contr*., vol. 18(5) 473-478.